CONTENTS

A research project on each unit.

UNIT 1

The set of integers

Lessons of the unit :

Scan the QR code to solve an interactive test on each lesson.

UNIT AIMS

By the end of this unit, student should be able to :

- recognize the set of integers ($\mathbb{Z}$)
- represent the integers on the number line.
- recognize the absolute value.
- order and compare integers.
- add and subtract integers.
- recognize the properties of addition in $\mathbb{Z}$
- multiply and divide integers.
- recognize the properties of multiplication in $\mathbb{Z}$
- recognize the repeated multiplication.
- use the rules of multiplying and dividing numbers with equal bases.
- recognize the numerical patterns.
- describe the pattern and complete it.

LESSON 1

Set of integers "ℤ"

You know that 0 is the smallest natural number.

Now, the question is : Are there any numbers less than 0 ?

To find the answer, let us see the following examples :

- **Temperature :**

  In Canada, sometimes the temperature records **30° C below zero.**

  In this case, you can say that :

  The temperature is $-30°$ C

- **Diving :**

  In Ras Mohammed (about 12 km. from Sharm El-Sheikh), the normal diving depth is **10 m. below sea level.**

  In this case, you can say that :

  The depth is -10 m.

The numbers -30 and -10 are not contained in the set of natural numbers, these numbers are called **negative numbers. Each of them is less than zero.**

The natural numbers and the negative numbers form together one set called **"the set of integers"** and it is denoted by **"$\mathbb{Z}$"**

Notice that :

0 is an integer.

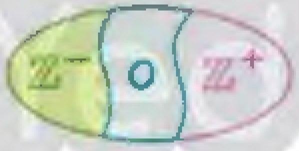
i.e. $\boxed{\mathbb{Z} = \{\cdots, -3, -2, -1, 0, 1, 2, 3, \cdots\}}$

The set of integers $\mathbb{Z}$ is formed from three sets, which are :

- The set of negative integers $\mathbb{Z}^-$ where : $\mathbb{Z}^- = \{-1, -2, -3, -4, \cdots\}$
- The set containing the number zero which is $\{0\}$
- The set of positive integers $\mathbb{Z}^+$ where : $\mathbb{Z}^+ = \{1, 2, 3, 4, \cdots\}$

i.e. $\mathbb{Z} = \{\cdots, -3, -2, -1, 0, 1, 2, 3, \cdots\}$

$\mathbb{Z} = \mathbb{Z}^- \cup \{0\} \cup \mathbb{Z}^+$



As shown in the opposite figure.

The following diagram shows the relation between $\mathbb{Z}$, $\mathbb{Z}^+$, $\mathbb{Z}^-$ and $\mathbb{N}$

Set of integers $\mathbb{Z}$

- Set of negative integers $\mathbb{Z}^-$
- $\{0\}$
- Set of positive integers $\mathbb{Z}^+$

Set of natural numbers $\mathbb{N}$ ($\{0\}$ and $\mathbb{Z}^+$)

LESSON 1

From the previous diagram , we deduce that :

[1] $\mathbb{N} \subset \mathbb{Z}$, $\mathbb{Z}^+ \subset \mathbb{Z}$, $\mathbb{Z}^- \subset \mathbb{Z}$, $\{0\} \subset \mathbb{Z}$

[2] $\mathbb{Z} = \mathbb{N} \cup \mathbb{Z}^-$

[3] $\mathbb{Z}^+ \cap \mathbb{Z}^- = \varnothing$

[4] $\mathbb{Z} - \mathbb{N} = \mathbb{Z}^-$, $\mathbb{N} - \mathbb{Z} = \varnothing$

Remarks

1. The integer zero is neither positive nor negative.
   i.e. $0 \notin \mathbb{Z}^+$ and $0 \notin \mathbb{Z}^-$
2. The set of non-negative integers = $\{0 , 1 , 2 , \cdots\} = \{0\} \cup \mathbb{Z}^+ = \mathbb{N}$
3. The set of non-positive integers = $\{0 , -1 , -2 , -3 , \cdots\} = \{0\} \cup \mathbb{Z}^-$
4. The set of odd integers = $\{\cdots , -3 , -1 , 1 , 3 , \cdots\}$
5. The set of even integers = $\{\cdots , -4 , -2 , 0 , 2 , 4 , \cdots\}$

Example 1

Put the suitable sign " $\in$, $\notin$, $\subset$ or $\not\subset$ " :

[a] -2 ☐ $\mathbb{N}$

[b] -3 ☐ $\mathbb{Z}$

[c] 0 ☐ $\mathbb{Z}^-$

[d] 0 ☐ $\mathbb{Z}$

[e] $\frac{2}{3}$ ☐ $\mathbb{Z}$

[f] $\{0.2 , 5\}$ ☐ $\mathbb{Z}$

[g] $\{2 , -3\}$ ☐ $\mathbb{N}$

[h] $\{6 , -7\}$ ☐ $\mathbb{Z}$

[i] $\mathbb{Z}^-$ ☐ $\mathbb{Z}$

[j] $\mathbb{N}$ ☐ $\mathbb{Z}$

Solution

[a] $\notin$	[b] $\in$	[c] $\notin$	[d] $\in$	[e] $\notin$
[f] $\not\subset$	[g] $\not\subset$	[h] $\subset$	[i] $\subset$	[j] $\subset$

Try by yourself

- Put the suitable sign " $\in$, $\notin$, $\subset$ or $\not\subset$ " :

[a] $\{0\}$ ☐ $\mathbb{Z}^{+}$
[b] -7 ☐ $\mathbb{Z}$
[c] 10 ☐ $\mathbb{Z}^{-}$
[d] $\{2.5 , -4\}$ ☐ $\mathbb{Z}$

Example (2)

Write an integer to represent each of the following situations :

[a] A profit of L.E. 25
[b] A loss of L.E. 3
[c] 10 degrees below 0
[d] An increase of P.T. 75
[e] 6 m. above sea level.
[f] 19 m. below ground.
[g] A building is 12 m. high.
[h] 4 steps backward.

Solution

[a] 25
[b] -3
[c] -10
[d] 75
[e] 6
[f] -19
[g] 12
[h] -4

Try by yourself

- Write an integer to represent each of the following situations :

[a] A temperature of 3 degrees below zero. (..........)
[b] A bank deposit of L.E. 100 (..........)
[c] A loss of 5 yards in a football. (..........)
[d] A withdrawal of L.E. 25 (..........)
[e] A decrease of 5 kg. (..........)
[f] A gain of 2000 pounds. (..........)

LESSON **1**

Representation of the integers on the number line

Every integer can be represented by one point on the number line as follows :

From the number line above, notice that :

- The **negative** integers are to the left of zero.
- The point that represents 0 is called **"the origin"**.
- The **positive** integers are to the right of zero.

• The set of integers is **an infinite set**, so it extends to infinity right to zero and left to zero.

Example 3

Represent each of the following sets of numbers on the number line :

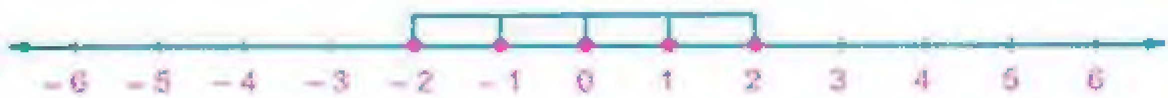
[a] {4 , −2 , 0 , 3 , −5}

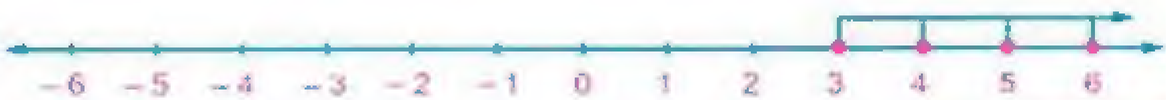
[b] {−2 , −1 , 0 , 1 , 2}

[c] {3 , 4 , 5 , ...}

Solution

[a]

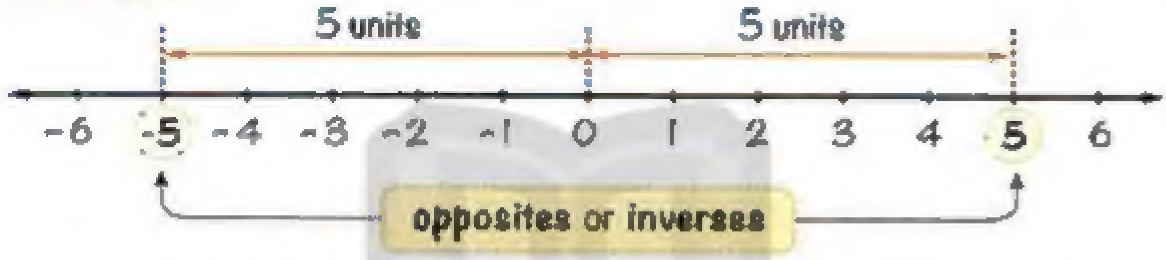

[b]


[c]


Opposites (inverses) and absolute value

On the number line , any two numbers that are at the same distance from 0 and on two opposite positions of it are called **opposites** or **inverses**.

For example :



Each of the integers 5 and − 5 has the same distance away from 0

Therefore 5 and − 5 are **opposites**.

i.e. The **opposite** of 5 is − 5 and the **opposite** of − 5 is 5

The absolute value

1. The absolute value of a number is its distance from 0 on the number line.
2. The absolute value of any number x is denoted by $|x|$
3. The absolute value of any number **(except 0)** is always positive.
4. The absolute value of 0 is 0

For example :
- $|4| = 4$ is read as : "The absolute value of 4 is 4"
- $|-4| = 4$ is read as : "The absolute value of − 4 is 4"
- $|0| = 0$ is read as : "The absolute value of 0 is 0"

Example 4

Write the opposite (inverse) of each of the following integers :

[a] − 2 [b] 0 [c] 8 [d] − 33

Solution

[a] 2 [b] 0 [c] − 8 [d] 33

LESSON 1

Example 5

Find each of the following :

[a] $|-9|$

[b] $|3|$

[c] $|0|$

[d] $|-3|+|5|$

[e] $|-2|+|-7|$

[f] $|-6|-|6|$

[g] $|-3|\times|-2|$

[h] $|-10|\div|2|$

Solution

[a] $|-9|=9$

[b] $|3|=3$

[c] $|0|=0$

[d] $|-3|+|5|=3+5=8$

[e] $|-2|+|-7|=2+7=9$

[f] $|-6|-|6|=6-6=0$

[g] $|-3|\times|-2|=3\times2=6$

[h] $|-10|\div|2|=10\div2=5$

Try by yourself

- Find each of the following :

[a] $1+|2|=$ ....

[b] $|8|=$ ....

[c] $|4|+|-5|=$ ....

[d] $|-10|+|-5|=$ ....

[e] $|-7|\times|2|=$ ....

[f] $|-15|\div|3|=$ ....

Example 6

Find the value of x :

[a] $|x|=5$ [b] $|x|=10$ [c] $|x|=0$

Solution

[a] Since $|x|=5$, then $x=5$ or $x=-5$

[b] Since $|x|=10$, then $x=10$ or $x=-10$

[c] Since $|x|=0$, then $x=0$

Exercise 1

Set of integers "$\mathbb{Z}$"

Interactive test

From the school book

1 Put the suitable sign "$\in$, $\notin$, $\subset$ or $\not\subset$":

a -3 ☐ $\mathbb{N}$

b $\{-5\}$ ☐ $\mathbb{Z}$

c Zero ☐ $\mathbb{Z}^+$

d $\{1, -2\}$ ☐ $\mathbb{N}$

e $\mathbb{Z}^-$ ☐ $\mathbb{Z}$

f $\{-3, \frac{7}{11}\}$ ☐ $\mathbb{Z}$ (Red Sea 2015)

g $\frac{13}{5}$ ☐ $\mathbb{Z}$

h $|-65|$ ☐ $\mathbb{Z}^-$

i $\mathbb{N}$ ☐ $\mathbb{Z}$

j $\{2, 5, \frac{3}{7}\}$ ☐ $\mathbb{Z}$

k $\frac{7}{12-6}$ ☐ $\mathbb{Z}$

l $\left|\frac{11-5}{3}\right|$ ☐ $\mathbb{Z}$

2 Complete :

a $\mathbb{Z}^+ \cup \{0\} \cup \mathbb{Z}^- =$ .... (Suez 2015)

b $\mathbb{Z} = \mathbb{N} \cup$ .... (Luxor 2012)

c $\mathbb{Z}^+ \cap \mathbb{Z}^- =$ .... (El-Dakahlia 2017)

d $\mathbb{Z} - \mathbb{N} =$ ....

e $\mathbb{Z} - \mathbb{Z}^- =$ .... (Qena 2013)

f $\mathbb{Z}^+ \cup \{0\} =$ .... (El-Kalyoubia 2016)

g $\mathbb{Z}^+ \cup$ .... $= \mathbb{N}$ (South Sinai 2013)

h $\mathbb{Z}^+ \cup \mathbb{N} =$ .... (El-Kalyoubia 2011)

i $\mathbb{Z} \cap \mathbb{N} =$ .... (El-Menia 2012)

j $\mathbb{N} - \mathbb{Z}^+ =$ ....

k $\mathbb{N} - \mathbb{Z} =$ .... (Alexandria 2013)

LESSON **1**

l $\mathbb{Z} = \mathbb{Z}^{+} \cup \mathbb{Z}^{-} \cup$ .... (El-Sharkia 2014)

m $\mathbb{Z}^{-} \cap \mathbb{N} =$ .... (Port Said 2016)

n The set of odd integers $\cup$ the set of even integers = ...

o The complement of $\mathbb{Z}^{-}$ with respect to $\mathbb{Z}$ = ...

p The complement of $\mathbb{Z}^{+}$ with respect to $\mathbb{N}$ = ...

3 Choose the correct answer :

a $\{2\}$ .... $\mathbb{Z}$ (Giza 2014) ($\in$ or $\notin$ or $\subset$ or $\not\subset$)

b $\frac{2}{7}$ .... $\mathbb{Z}$ (Souhag 2015) ($\notin$ or $\in$ or $\subset$ or $\not\subset$)

c $|-9|$ .... $\mathbb{Z}^{+}$ (El-Beheira 2017) ($\in$ or $\notin$ or $\subset$ or $\not\subset$)

d $\frac{6-6}{8}$ .... $\mathbb{Z}$ (El-Monofia 2014) ($\in$ or $\notin$ or $\subset$ or $\not\subset$)

e $|-5| + 3$ .... $\mathbb{Z}$ (Alexandria 2016) ($\in$ or $\notin$ or $\subset$ or $\not\subset$)

f $|-5| + |7| =$ .... (South Sinai 2014) (12 or 2 or -2 or -12)

g $|-3| + |-2| =$ .... (El-Kalyoubia 2011) (-5 or 5 or -1 or 1)

h $|-2| + |2| =$ .... (North Sinai 2017) (zero or 1 or -4 or 4)

i If $b = |-7|$, then $b =$ .... (El-Menia 2011) (-7 or 7 or 0 or 14)

j $\mathbb{Z} - \mathbb{Z}^{-} =$ .... (El-Monofia 2011) ($\mathbb{Z}^{-}$ or $\mathbb{Z}^{+}$ or $\mathbb{N}$ or $\{zero\}$)

k $\mathbb{N} \cup \mathbb{Z} =$ .... (El-Menia 2016) ($\mathbb{Z}$ or $\mathbb{N}$ or $\mathbb{Z}^{-}$ or $\mathbb{Z}^{+}$)

l $\mathbb{N} \cup \mathbb{Z}^{-} =$ .... (Assiut 2013) ($\mathbb{Z}^{-}$ or $\mathbb{Z}^{+}$ or $\mathbb{Z}$ or $\mathbb{N}$)

m $\mathbb{Z}^{+} - \mathbb{Z}^{-} =$ .... (Aswan 2013) ($\mathbb{Z}^{+}$ or $\varnothing$ or $\mathbb{N}$ or $\{0\}$)

n If $X \subset \{2 , -3\} \cap \{5 , -3\}$, then $X =$ .... (Giza 2011)

($\{2\}$ or $\{-3\}$ or $\{-5\}$ or $\{5\}$)

4 Write an integer to represent each situation :

a A temperature is 12 C° below zero. (........)

b She's diving 10 m. below sea level. (........)

c Ahmed withdraws 6000 pounds from his bank account. (........)

d The tree is 4 m. high. (........)

e 3 steps forward. (........)

f A bank deposit of L.E. 750 (........)

g A loss of L.E. 20 (........)

h A gain of 7 kilograms. (........)

i A profit of L.E. 100 (........)

j A decrease of L.E. 200 (........)

5 Complete the following using one of the words (positive - negative - zero) :

a Moving forwards is represented by .... numbers , while moving backwards is represented by .... numbers.

b Moving to the right is represented by .... numbers , while moving to the left is represented by .... numbers.

c Lowering than sea level is represented by .... numbers , height above sea level is represented by .... numbers.

d Sea level is represented by the number ....

6 Represent each of the following on the number line :

a 3 , −4 , 1 , −2

b 6 , −3 , 0 , −1 , 3 , 5

c −4 , −5 , −6 , ...

d −1 , 0 , 1 , ...

LESSON 1

7 Write the opposite (inverse) of each integer :

a -3 | b 12 | c 0

d $|9|$ | e $|-34|$ | f $-|-8|$

8 Represent each number and its inverse on the number line :

a 3 b -4 c 0 d -99

9 Find each of the following :

a $|-3|+|2|$ | b $|-2|+|-13|$ | c $|-100|-|-50|$

d $|-6|+7$ | e $|-12|-|12|$ | f $|0|+|-7|$

g $|-5|-5$ | h $|-3|\times|-5|$ | i $|-10|\times|2|$

j $|-30|+|-5|$ | k $|0|\times|-3|$ | l $8\times|-11|$

10 Find the value of x :

a $|x|=5$ | b $|x|=12$ | c $|x|=0$

d $|-4|=x$ | e $|3|=x$ | f $|-101|=x$

11 Mark (true) or (false) and give the reason :

a Zero $\in \mathbb{Z}^-$ (.........)

Because : ....

b $\varnothing = \mathbb{Z}^- \cap \mathbb{Z}$ (.........)

Because : ....

c $\mathbb{Z}^+ \cup \mathbb{N} = \mathbb{Z}^+$ (.........)

Because : ....

d $\{-17\} \in \mathbb{Z}$ (......)

Because : ....

e $\mathbb{Z}^+$ is the set of counting numbers. (......)

Because : ....

f Zero is the smallest positive number. (......)

Because : ....

12 In each of the following , find the value of x to get a true statement :

a $-4 \in \{7 , x , -3\}$

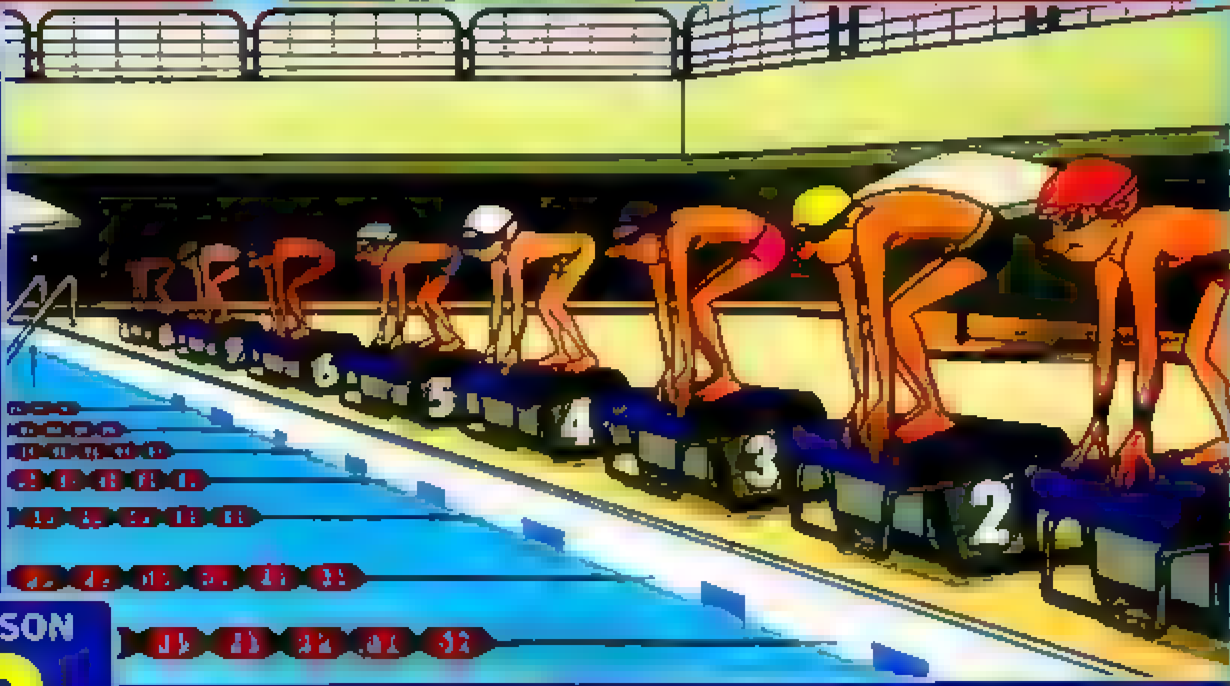
b $-5 \in \{-1 , 0 , -3 , x\}$

c $x \in \{-2\}$

d $x \in \{2 , 5 , -3\} \cap \{5 , -2 , -3\}$

e $\{2 , x\} \cup \{-4 , 0 , 4\} = \{0 , -2 , 2 , -4 , 4\}$

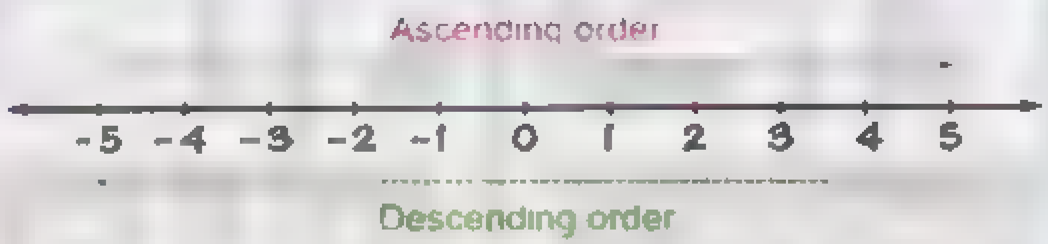
f $|-5| \notin \{x , -5 , 3\}$



LESSON 2

Ordering and comparing integers

The set of integers is represented on the number line as shown in the following figure :



From the number line above, notice that :

- **The numbers increase from left to right and decrease from right to left.**

The numbers are in an ascending order from left to right, and they are in a descending order from right to left.

For example :

- 1 (which is less than 4) is to the left of 4
- 3 (which is less than 7) is to the left of 7

- This relationship holds true for all numbers on the number line, even when we go to the negative side.

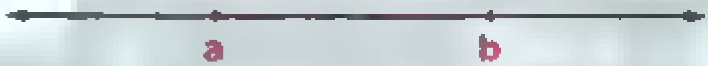
For example :

- -3 is less than 2, because -3 is to the left of 2
- 5 is less than -3, because -5 is to the left of -3

i.e. $\boxed{\cdots < -3 < -2 < -1 < 0 < 1 < 2 < 3 < \cdots}$

Generally

For any two integers a and b, if the point representing a is to the left of the point representing b, then a < b



For example : $-4 < -1$

because the point representing -4 lies on the left of the point representing -1

Similarly : • $-5 < 2$ • $2 < 3$ • $-2 < 0$ • $0 < 1$

Remarks

1. Any positive integer is greater than any negative integer.
2. Zero is smaller than any positive integer and is greater than any negative integer.
   For example : $0 < 5$ and $0 > -5$
3. The least positive integer is "1" and we cannot determine the greatest positive integer.
4. The greatest negative integer is "-1" and we cannot determine the least negative integer.

LESSON 2

Example (1)

Put (< , > or =) :

[a] 4 ☐ 3 | [b] 5 ☐ – 10 | [c] – 12 ☐ – 4
[d] 0 ☐ – 2 | [e] – 8 ☐ – 7 | [f] |– 3| ☐ 3
[g] |– 4| ☐ 2 | [h] |– 3| ☐ |0| | [i] – 7 ☐ – |– 6|

Solution

[a] > | [b] > | [c] <
[d] > | [e] < | [f] =
[g] > | [h] > | [i] <

Try by yourself

- Put (< , > or =) :

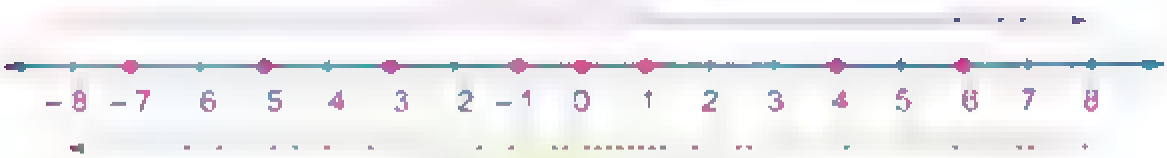
[a] 6 ☐ 2 | [b] 2 ☐ – 3 | [c] 0 ☐ – 1
[d] – 4 ☐ – 8 | [e] – 100 ☐ 1 | [f] |– 10| ☐ 9
[g] |– 5| ☐ |– 6| | [h] |– 6| ☐ 6 | [i] – |– 4| ☐ – 2

Example (2)

Arrange the following integers once in an ascending order and another in a descending order :

4 , – 5 , 1 , – 3 , 0 , 6 , – 7 and – 1

Solution



The ascending order is : – 7 , – 5 , – 3 , – 1 , 0 , 1 , 4 and 6

The descending order is : 6 , 4 , 1 , 0 , – 1 , – 3 , – 5 and – 7

Try by yourself

[1] Arrange the following numbers in an ascending order :

$4 , -3 , 6 , 0$ and -7

[2] Arrange the following numbers in a descending order :

$-7 , -9 , 0 , -4 , 2$ and -11

Example (3)

Write , using the listing method , each of the following sets :

[a] The set of integers greater than -3

[b] The set of integers less than or equal to 2

[c] The set of integers more than -7 and less than -3

[d] The set of non-positive odd integers.

[e] $X = \{x : x \in \mathbb{Z} , x < -4\}$

[f] $X = \{x : x \in \mathbb{Z} , -2 \leq x < 3\}$

Solution

[a] $\{-2 , -1 , 0 , 1 , 2 , \cdots\}$

[b] $\{2 , 1 , 0 , -1 , -2 , \cdots\}$

[c] $\{-6 , -5 , -4\}$

[d] $\{-1 , -3 , -5 , \cdots\}$

[e] $\{-5 , -6 , -7 , \cdots\}$

[f] $\{-2 , -1 , 0 , 1 , 2\}$

Exercise 2

Ordering and comparing integers

Interactive test

📖 From the school book

1 Put [< , > or =] :

a $3 \square -3$

b $4 \square 3$

c $-3 \square -4$

d $-8 \square 4$

e $0 \square -1$

f $-2 \square 0$

g 📖 $3 \square -6$

h $|-4| \square |0|$

i $8 \square |-8|$

j 📖 $|-13| \square 3$

k $-15 \square |-2|$

l 📖 $-|-4| \square 2$

m 📖 $3 + |-3| \square 8$

n $|-9| - |5| \square |-4|$

o $|-7| \square -7$

2 Complete the following :

a The number .... is neither positive nor negative.

b The smallest positive integer is .... and the greatest negative integer is .... (Kafr El-Sheikh 2017)

c The smallest non-negative integer is .... (Damietta 2016)

d The largest non-positive integer is ....

e The set of integers between -3 and 2 = ... (El-Menia 2011)

f The set of integers less than 1 and more than -4 is { .... } (Giza 2013)

3 Choose the correct answer :

a -7 .... $-|-9|$ (Damietta 2013) (> or = or < or ≤)

b (-4) .... $|-4|$ (South Sinai 2013) (> or < or = or ≥)

c An integer included between -2 and 3 is .... (Giza 2017)

(3 or -3 or -4 or -1)

d The integer which comes just before the number -5 is .... (Damietta 2011) (-6 or -4 or 4 or 6)

- The integer which comes just next the number 23 is ......

(El-Sharkia 2014) (25 or 22 or 23 or 24)

f The number of integers between – 2 and 2 = ... (El Beheira 2012)

(2 or 3 or 4 or 5)

4 Arrange in an ascending order each of the following :

a 1 , – 5 , – 1 and 3 (El-Sharkia 2014)

b – 7 , – 9 , – 4 and – 1

c – 8 , 12 , | – 8 | , – 15 and 19 (Suez 2012)

d 6 , – 60 , 2 , – 17 , – 22 and 0

5 Arrange in a descending order each of the following :

a – 9 , 0 , 7 and – 15 (Red Sea 2013)

b 8 , – 13 , – 19 , 0 and – 15

c – 28 , – 35 , 33 , – 37 and 2

d 1 , – 11 , 3 , – 1 , – 8 and 5

6 Write the previous integer and the next integer of each of the following integers :

a – 9 b 13 c 23 d zero

7 Write the integers between each two integers of the following :

a – 4 , 2 b – 1 , 5 c – 7 , 0

8 Write, using the listing method each of the following sets :

a The set of integers greater than – 2

b The set of integers smaller than 0

c The set of integers greater than – 3 and smaller than 2

d The set of integers between – 4 and 3 (Matrouh 2015)

e The set of negative integers whose absolute value of each is greater than 4

LESSON 2

f The set of non-negative integers.

g The set of non-positive even integers.

h $X = \{x : x \in \mathbb{Z}, x \leq -2\}$

i $X = \{x : x \in \mathbb{Z}, -1 < x < 1\}$

9 Complete the following :

a $-7, -6, -5$, ..., ..., ...

b $-2, 0, 2, 4$, ..., ..., ...

c $-50, -40, -30$, ..., ..., ...

10 Use the following table to answer the following questions :

City	Highest temperature	Lowest temperature
New York	8	-2
Paris	3	-5
Cairo	16	8
Mosco	-5	-15
London	5	-4
Brazil	10	3

a Order the highest temperatures of all the cities from the greatest to the smallest.

b Order the lowest temperatures of all the cities from the smallest to the greatest.

11 Solve the following :

a A diver is at -5 m. and a balloon is at 5 m. Which is closer to sea level ?

b A helicopter is at an altitude of 1000 ft. , and a diving bell is at -750 ft. Which is further from sea level ?

LESSON 3

Adding and subtracting integers

First: Adding integers

Adding integers by using the number line

The number line can help you in visualizing adding and subtracting integers. Just think of addition and subtraction as directions on the number line.

- To add a positive integer move to the right on the number line. →
- To add a negative integer move to the left on the number line. ←

First: Adding two positive integers

Example (1)

Use the number line to find the sum 3 + 5

Solution

To find the sum 3 + 5 using the number line , do as follows :

−5 −4 −3 −2 −1 0 1 2 3 4 5 6 7 8 9

- Start at 3
- Move 5 units to the right.

This movement takes you to 8 So, 3 + 5 = 8

LESSON 3

Notice that :

The sum of two positive integers is a positive integer.

B Adding two negative integers

Example 2

Use the number line to find the sum (– 4) + (– 3)

Solution

To find the sum (– 4) + (– 3) using the number line , do as follows :

- Start at – 4
- Move 3 units to the left.

This movement takes you to (– 7) So, (– 4) + (– 3) = – 7

Notice that :

The sum of two negative integers is a negative integer.

C Adding a positive integer and a negative integer

Example 3

Use the number line to find the sum 7 + (– 8)

Solution

To find the sum 7 + (– 8) using the number line , do as follows :

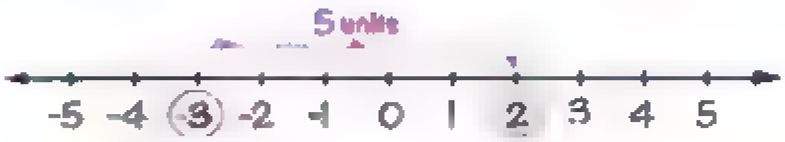
- Start at 7
- Move 8 units to the left.

This movement takes you to (– 1) So, 7 + (– 8) = – 1

Example (4)

Use the number line to find the sum (– 3) + 5

Solution



- Start at (– 3)
- Move 5 units to the right

This movement takes you to 2 **So,** (– 3) + 5 = 2

Notice that :

The sign of the sum of two integers with different signs is the sign of the integer with the largest absolute value.

Adding integers without using the number line

1 To add integers having the **same sign**, keep the same sign and add the absolute value of each number.

For example :

- 4 + 7 = 11

(The sum of two positive integers is a positive integer)

- (– 2) + (– 6) = – (| – 2 | + | – 6 |) = – (2 + 6) = – 8

(The sum of two negative integers is a negative integer)

2 To add integers with **different signs**, keep the sign of the number with the largest absolute value and subtract the smallest absolute value from the largest.

For example :

- 7 + (– 1) = 6

– The sum is a positive number because | 7 | > | – 1 |

– The sum is 6 because the difference of the absolute values of the two integers is : | 7 | – | – 1 | = 7 – 1 = 6

LESSON 3

- $(-6) + 4 = -2$
  - The sum is a negative number because $|-6| > |4|$
    The sum is -2 because the difference of the absolute values of the two integers is : $|-6| - |4| = 6 - 4 = 2$

Possibility of addition in $\mathbb{Z}$

From the previous examples, notice that **the sum of two integers is always an integer.**

Generally

If we add any two elements of $\mathbb{Z}$, the result will be an element of $\mathbb{Z}$

It means that : Addition of two integers is always possible in $\mathbb{Z}$

Example (5)

Find the sum :

[a] $(-4) + (-5)$ [b] $-2 + 6$ [c] $-7 + 1$ [d] $(-9) + (-6)$

Solution

[a] $(-4) + (-5) = -(|-4| + |-5|) = -(4 + 5) = -9$

[b] $-2 + 6 = +(|6| - |-2|) = +(6 - 2) = +4 = 4$

[c] $-7 + 1 = -(|-7| - |1|) = -(7 - 1) = -6$

[d] $(-9) + (-6) = -(|-9| + |-6|) = -(9 + 6) = -15$

Note : You can find the result directly.

Calculator :

You can use a calculator to check your answers.

Example : $-78 + (-105) = \ldots\ldots$

(–) 7 8 + ((–) 1 0 5) = -183

Try by yourself

- **Find the sum :**

[a] $(-5) + (-10) = \dots\dots$ [b] $-8 + 2 = \dots\dots$

[c] $7 + (-9) = \dots\dots$ [d] $-4 + 15 = \dots\dots$

Properties of addition in $\mathbb{Z}$

① Closure property

$\mathbb{Z}$ is a closed set under addition.

It means that the sum of any two elements of $\mathbb{Z}$ is always an element of $\mathbb{Z}$

For example :

- $-4 \in \mathbb{Z}$, $-2 \in \mathbb{Z}$

, then : $(-4) + (-2) = -6 \in \mathbb{Z}$

② Commutative property

If a and b are two integers , then : $a + b = b + a$

For example :

- $-3 + 8 = 5$, $8 + (-3) = 5$

i.e. $-3 + 8 = 8 + (-3)$

③ Associative property

If a , b and c are three integers , then : $a + b + c = (a + b) + c = a + (b + c)$

For example :

- $6 + (-4) + (-3) = [6 + (-4)] + (-3) = 2 + (-3) = -1$

Also, $6 + (-4) + (-3) = 6 + [(-4) + (-3)] = 6 + (-7) = -1$

i.e. $6 + (-4) + (-3) = [6 + (-4)] + (-3) = 6 + [(-4) + (-3)]$

LESSON 3

④ The existence of the additive identity (neutral) element in $\mathbb{Z}$

For any integer a , we have : $a + 0 = 0 + a = a$

i.e. Zero is the additive identity element in $\mathbb{Z}$

For example :

- $2 + 0 = 0 + 2 = 2$
- $-3 + 0 = 0 + (-3) = -3$

⑤ The existence of additive inverse (opposite) property

For every integer (a) there is an additive inverse (– a)
Where : $a + (-a) = 0$

For example :

- The additive inverse of 3 is – 3 , because $3 + (-3) = 0$
- The additive inverse of – 4 is 4 , because $-4 + 4 = 0$

Notice that :

- The additive inverse of zero is zero because $0 + 0 = 0$
- The additive inverse of a is (– a) and also the additive inverse of (– a) is a
  i.e. The additive inverse of (– a) is $-(-a) = a$

For example :
The additive inverse of – 6 is $-(-6) = 6$

Example 6

Use the properties of addition in $\mathbb{Z}$ to find :

[a] $8 + 10 + (-8)$

[b] $24 + (-19) + (-24) + 9$

Solution

[a] $8 + 10 + (-8) = 8 + (-8) + 10$ (Commutative property)
$= [8 + (-8)] + 10$ (Associative property)
$= 0 + 10$ (Additive inverse property)
$= 10$ (Additive identity)

[b] $24 + (-19) + (-24) + 9 = 24 + (-24) + (-19) + 9$ (Commutative property)

$= [24 + (-24)] + [(-19) + 9]$ (Associative property)

$= 0 + (-10)$ (Additive inverse property)

$= -10$ (Additive identity)

Try by yourself

- **Use the properties of addition in $\mathbb{Z}$ to find each of the following :**

[a] $6 + 10 + (-6)$ [b] $23 + (-64) + 77 + (-36)$

Second: Subtracting Integers

Subtracting an integer means adding its opposite.

For example :

- Subtracting (-2) from 8, means adding the opposite of (-2) to 8

[The opposite of (-2) is 2]

$8 - (-2)$

Keep Change Opposite

$8 + (+2) = 10$

- Subtracting 3 from (-5), means adding the opposite of 3 to (-5)

i.e. $(-5) - 3 = (-5) + (-3) = (-8)$

Keep Change Opposite

- Subtracting (-1) from (-6), means adding the opposite of (-1) to (-6)

i.e. $(-6) - (-1) = (-6) + 1 = (-5)$

Keep Change Opposite

- Subtracting (-9) from 0, means adding the opposite of (-9) to 0

i.e. $0 - (-9) = 0 + 9 = 9$

Keep Change Opposite

LESSON 3

"Possibility of subtraction in $\mathbb{Z}$"

From the previous examples, notice that the result of the subtraction of two integers is always an integer.

Generally

If we subtract any two elements of $\mathbb{Z}$, the result will be an element of $\mathbb{Z}$

It means that : Subtraction of two integers is always possible in $\mathbb{Z}$

Example (7)

Find the result of each of the following :

[a] $-5-2$ [b] $6-10$ [c] $0-6$ [d] $-6-(-12)$

Solution

[a] $-5-2=-5+(-2)=-7$

[b] $6-10=6+(-10)=-4$

[c] $0-6=0+(-6)=-6$

[d] $-6-(-12)=-6+(12)=6$

Example (8)

Find the result of each of the following :

$5-3$ and $3-5$

What do you notice ? what does that mean ?

Solution

$5-3=5+(-3)=2$, $3-5=3+(-5)=-2$

We notice that : $5-3 \neq 3-5$

i.e. Subtraction operation is not commutative in $\mathbb{Z}$

Example 9

Find the result of each of the following :

$5-(3-1)$ and $(5-3)-1$

What do you notice ? what does that mean ?

Solution

$5-(3-1)=5-[3+(-1)]=5-2=5+(-2)=3$

, $(5-3)-1=[5+(-3)]-1=2-1=2+(-1)=1$

We notice that : $5-(3-1)\neq(5-3)-1$

I.e. $\boxed{\text{Subtraction operation is not associative in } \mathbb{Z}}$

From the previous , we can deduce the following properties of subtraction in $\mathbb{Z}$:

1. $\mathbb{Z}$ is closed under subtraction operation.
   i.e. The result of subtracting any two integers is an integer.
2. The subtraction operation in $\mathbb{Z}$ is not commutative
3. The subtraction operation in $\mathbb{Z}$ is not associative.

Example 10

If $a=-2$, $b=3$ and $c=-1$, then find the value of :

[a] $a+b+c$ [b] $a-c-(-b)$

Solution

[a] $a+b+c=-2+3+(-1)=1+(-1)=0$

[b] $a-c-(-b)=-2-(-1)-(-3)=-2+1+3=-1+3=2$

Exercise 3

Adding and subtracting integers

Interactive test

From the school book

1 Use the number line to find :

a $5 + 2$ b $4 + (-3)$ c $-4 + (-2)$

d $-7 + 4$ e $8 - 4$ f $5 - (-3)$

g $-3 - 3$ h $-4 + 4$ i $-10 + 2$

2 Find the result of each of the following :

a $4 + 2$ b $(-2) + (-1)$ c $-5 + 9$

d $9 + (-8)$ e $0 + (-5)$ f $18 + (-18)$

g $-6 + 0$ h $-48 + 34$ i $-10 + (-10)$

3 Find the result of each of the following :

a $7 - 5$ b $3 - 9$ c $-7 - 3$

d $19 - (-11)$ (Giza 2013) e $-3 - (-4)$ f $-9 - 8$

g $0 - 7$ h $0 - (-3)$ i $-5 - 0$

j $-73 - (-73)$ k $33 - |-11|$ l $|-14| - |-28|$

4 Choose the correct answer :

a The additive identity in $\mathbb{Z}$ is .... (South Sinai 2011)

(0 or 1 or -1 or 2)

b $15 + 8 - 15 =$ .... (Giza 2015) (-15 or 8 or 15 or 23)

c $3 - |-3| =$ .... (El-Gharbia 2017) (0 or 1 or 3 or 6)

d $\left|\frac{5-8}{3}\right| =$ .... (El-Gharbia 2013) (1 or 6 or -6 or -2)

e $|-5| + \text{.......} = 0$ (El-Dakahlia 2011) (-5 or 5 or 0 or 1)

f The additive inverse of (-5) is .... (-10 or 5 or 0 or -5)

g $4 + (-6) >$ .... (2 or 0 or -2 or -4)

h If $X = -1$, $Y = 2$, then the value of $X + Y =$ .... (2 or 3 or 1 or -1)

5 Write the property used in each of the following :

a $-5 + 3 = 3 + (-5)$ (........................)

b $6 + (-6) = 0$ (........................)

c $0 + (-7) = -7$ (........................)

d $(-10 + 5) + 3 = -10 + (5 + 3)$ (........................)

e $-a + a = 0$ (........................)

6 Complete each of the following :

a $4 + (-3) = (-3) +$ ....

b $5 + \text{........} = 0$

c $(-7) + \text{........} = 0$

d $(-8) + \text{........} = (-8)$

e $6 + (-6) =$ ....

f $2 - (-3) =$ ....

g $|-17| - 12 =$ .... (El Monofia 2017)

h $0 + \text{........} = |-7|$

i $-2 + (\text{........} + 5) = -2$

j $\big(5 + (-8)\big) + 7 = 5 + (\text{........} + 7)$

k The additive inverse of 8 is ....

l The additive inverse of (-4) is ....

m The additive inverse of $|-6|$ is ....

n The additive inverse of zero is ....

o The additive identity element in $\mathbb{Z}$ is ....

p The result of subtracting 7 from (-2) is ....

LESSON 3

q The result of subtracting -5 from 3 is ....

r If $a + b = b + c$, then $c =$ ....

s If $a + (-3) = b + a$, then $b =$ ....

t If $a + b = b$, then $a =$ ....

u If $a + b = 0$, then a is ....

7 Find the value of (n) in each of the following :

a $-8 + 0 = n$

b $-6 + n = -6$

c $n + 6 = 0$

d $5 + n = 8$

e $-6 + n = -9$

f $27 + (-27) = n$

8 Use the properties of addition in $\mathbb{Z}$ to find :

a $-5 + (-6) + 5$

b $10 + (-5) + (-2)$

c $-7 + 2 + (-13)$

d $(-17) + 19 + 17$ (Kafr El-Sheikh 2016)

e $15 + (-3) + 25$

f $5 + (-3) + 7 + (-9)$

g $25 + (-8) + (-25) + 7$ (El-Monofia 2013)

h $55 + (-255) + 45 + 225$

i $-74 + 65 + 74 + (-65)$ (El-Beheira 2015)

j $113 - 120 + 17$ (Red Sea 2013)

k $2015 + 180 + (-1015)$

l $63 + 54 + 37 + 46$ ([illegible] 2012)

9 Find each of the following :

a $3 + 7 + 6$

b $(-6) + (-2) + (-1)$

c $-3 + 6 + (-2)$

d $4 - 7 - 5$

e $-3 + 7 - 5$

f $-17 - 13 + 10$

g $-6 - (-3) - 5$

h $-9 + 7 - 3$

i $(-3 + 5) - (-6)$

j $-9 - (4 - 7)$

10 If $a = 3$, $b = -4$ and $c = -2$, then find the value of :

a $a + b$ | b $b + c$ | c $a - b$

d $b - c$ | e $a + b + c$ | f $a - b + c$

11 Check the property of closure of the addition and subtraction on the following sets of numbers :

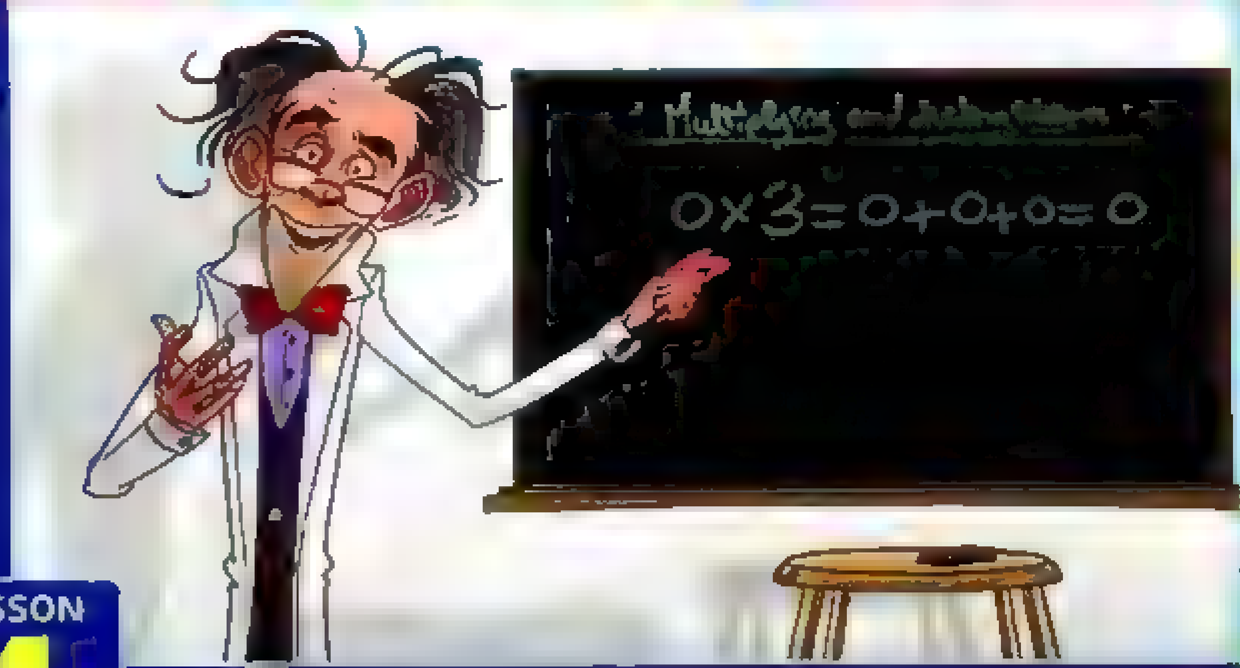
a $X = \{-1 , 0 , 1\}$ b $Y = \{-2 , -1 , 0 , 1 , 2\}$

12 Temperature is recorded in St. Catherine $-3°C$ at three o'clock after midnight , while it is recorded $11°C$ in the afternoon
Calculate the increase in temperature.

13 The temperature of the North polar water layer is $-1°C$, the temperature rises $5°C$ in the North Atlantic deep water layer.
What is the temperature of that layer ?

14 A submarine at a depth of 90 metres below sea level. It rose 60 metres.
Use the appropriate calculation to calculate the new depth of the submarine.

15 Ramy deposited a sum of money amounting to L.E. 6220 , then he withdrew an amount of L.E. 1211 , and then he deposited an another amount of L.E. 2110 How much is the balance of Ramy in the bank ?



LESSON 4

Multiplying and dividing integers

First Multiplying integers

You know that multiplication is a repeated addition.

For example :

$3 \times 2 = 3 + 3$

- Since the sum of integers is always an integer, so the product of two integers is also an integer

For example :

- $2 \times 3 = 2 + 2 + 2 = 6 \in \mathbb{Z}^+$
- $(-3) \times 4 = (-3) + (-3) + (-3) + (-3) = -12 \in \mathbb{Z}^-$
- $4 \times (-2) = (-2) + (-2) + (-2) + (-2) = -8 \in \mathbb{Z}^-$
- $(-3) \times (-2) = -\big(3 \times (-2)\big) = -\big((-2) + (-2) + (-2)\big) = -(-6) = 6 \in \mathbb{Z}^+$
- $0 \times 3 = 0 + 0 + 0 = 0$

From the previous examples , notice that :

1 When we multiplied two integers, the product was always an integer.

Generally

If we multiply any two elements of $\mathbb{Z}$, the result will be an element of $\mathbb{Z}$

It means that : Multiplication of two integers is always possible in $\mathbb{Z}$

2
- When we multiplied **two positive** integers, the product was **positive**.
- When we multiplied **two negative** integers, the product was **positive**.
- When we multiply two integers **one positive** and the **other negative**, the product was **negative**.

Generally

- **If the signs are the SAME, then the product is POSITIVE.**
  i.e. $(+) \times (+) = (+)$ and $(-) \times (-) = (+)$
- **If the signs are DIFFERENT, then the product is NEGATIVE.**
  i.e. $(+) \times (-) = (-)$ and $(-) \times (+) = (-)$

3 If we multiply any integer by 0 , the product will be 0

For example :

- $4 \times 0 = 0$
- $0 \times 3 = 0$
- $-2 \times 0 = 0$
- $0 \times -5 = 0$

Example 1

Find the product for each of the following :

[a] $(-8) \times (-1)$ | [b] 5×-2 | [c] 0×-7

[d] $2 \times |-4|$ | [e] $-|-3| \times 5$ | [f] $-(-4) \times 6$

Solution

[a] $(-8) \times (-1) = 8$

[b] $5 \times -2 = -10$

[c] $0 \times -7 = 0$

[d] $2 \times |-4| = 2 \times 4 = 8$

[e] $-|-3| \times 5 = -3 \times 5 = -15$

[f] $-(-4) \times 6 = 4 \times 6 = 24$

LESSON 4

Try by yourself

- Find the product for each of the following :

[a] $-3 \times 5 =$ ....
[b] $(-4) \times (-6) =$ ....
[c] $-9 \times 0 =$ ....
[d] $6 \times (-3) =$ ....
[e] $(-100) \times |-2| =$ ....
[f] $(-31) \times 3 =$ ....

Properties of multiplication in $\mathbb{Z}$

1 Closure property

$\mathbb{Z}$ is a closed set under multiplication.

It means that : The product of any two elements of $\mathbb{Z}$ is always an element of $\mathbb{Z}$

For example :

- $-3 \in \mathbb{Z}$, $-2 \in \mathbb{Z}$
 , then : $-3 \times (-2) = 6 \in \mathbb{Z}$

2 Commutative property

If a and b are two integers , then : $a \times b = b \times a$

For example :

- $(-3) \times (-4) = 12$, $(-4) \times (-3) = 12$

i.e. $(-3) \times (-4) = (-4) \times (-3)$

3 Associative property

If a , b and c are three integers , then : $a \times b \times c = (a \times b) \times c = a \times (b \times c)$

For example :

- $4 \times (-3) \times (-2) = \big(4 \times (-3)\big) \times (-2) = (-12) \times (-2) = 24$

Also, $4 \times (-3) \times (-2) = 4 \times \big((-3) \times (-2)\big) = 4 \times 6 = 24$

i.e. $4 \times (-3) \times (-2) = \big(4 \times (-3)\big) \times (-2) = 4 \times \big((-3) \times (-2)\big)$

④ The existence of the multiplicative identity (neutral) element in $\mathbb{Z}$

For any integer a , we have : $1 \times a = a \times 1 = a$

i.e. the number "1" is the multiplicative identity (neutral) element in $\mathbb{Z}$

For example :

- $1 \times 3 = 3 \times 1 = 3$
- $(-2) \times 1 = 1 \times (-2) = -2$

⑤ Multiplication is distributed over addition and subtraction in $\mathbb{Z}$

If a , b and c are three integers , then :

- $a \times (b + c) = a \times b + a \times c$ and $(b + c) \times a = b \times a + c \times a$
- $a \times (b - c) = a \times b - a \times c$ and $(b - c) \times a = b \times a - c \times a$

For example :

- $2 \times (-4 + 7)$
  $= 2 \times 3$
  $= 6$
- $2 \times (-4) + 2 \times 7$
  $= -8 + 14$
  $= 6$

i.e. $2 \times (-4 + 7) = 2 \times (-4) + 2 \times 7 = 6$

- $3 \times (5 - 7)$
  $= 3 \times (-2)$
  $= -6$
- $3 \times 5 - 3 \times 7$
  $= 15 - 21$
  $= -6$

i.e. $3 \times (5 - 7) = 3 \times 5 - 3 \times 7 = -6$

Example (2

Use the properties of multiplication of integers to find :

[a] $(-4) \times 57 \times (-25)$

[b] $8 \times 2 \times 125 \times (-50)$

Solution

[a] $(-4) \times 57 \times (-25) = (-4) \times (-25) \times 57$ (Commutative property)

$= ((-4) \times (-25)) \times 57$ (Associative property)

$= 100 \times 57$

$= 5700$

LESSON 4

[b] $8 \times 2 \times 125 \times (-50) = 8 \times 125 \times 2 \times (-50)$ (Commutative property)

$= (8 \times 125) \times (2 \times (-50))$ (Associative property)

$= 1000 \times (-100)$

$= -100\,000$

Example (3)

Use the distribution property to find the value of each of the following :

[a] $3 \times (-4) + 3 \times 5$ [b] $5 \times 7 + 5 \times (-7)$

[c] $15 \times (-17) + 35 \times (-17) - 50 \times (-17)$

Solution

[a] $3 \times (-4) + 3 \times 5 = 3 \times ((-4) + 5) = 3 \times 1 = 3$

[b] $5 \times 7 + 5 \times (-7) = 5 \times (7 + (-7)) = 5 \times 0 = 0$

[c] $15 \times (-17) + 35 \times (-17) - 50 \times (-17)$

$= (15 + 35 - 50) \times (-17) = (50 - 50) \times (-17) = 0 \times (-17) = 0$

Example (4)

Find each of the following by two methods :

[a] $5 \times (-3 + (-5))$ [b] $120 \times 19 + 120 \times (-19)$

Solution

[a] First method :

$5 \times (-3 + (-5))$

$= 5 \times (-3) + 5 \times (-5)$

$= -15 + (-25) = -40$

Second method :

$5 \times (-3 + (-5))$

$= 5 \times (-8)$

$= -40$

[b] First method :

$120 \times 19 + 120 \times (-19)$

$= 120 \times (19 + (-19))$

$= 120 \times 0 = 0$

Second method :

$120 \times 19 + 120 \times (-19)$

$= 2280 + (-2280)$

$= 0$

Calculator :

You can use a calculator to check your answers.

Example : $-15 \times (-12) =$

(–) 1 5 × ((–) 1 2) = 180

Second Dividing integers

Possibility of division in $\mathbb{Z}$

Notice that :

• $6 \div 2 = 3$	because	$2 \times 3 = 6$	$(3 \in \mathbb{Z})$
• $-15 \div 3 = -5$	because	$3 \times (-5) = -15$	$(-5 \in \mathbb{Z})$
• $18 \div (-9) = -2$	because	$(-9) \times (-2) = 18$	$(-2 \in \mathbb{Z})$
• $(-24) \div (-4) = 6$	because	$(-4) \times 6 = -24$	$(6 \in \mathbb{Z})$

Since the result of the division $5 \div 3$ is not an integer , because there is no integer multiplied by 3 gives 5 , so we can say that :

The division is not always possible in $\mathbb{Z}$ or $\mathbb{Z}$ is not a closed set under division.

The following rules are applied when dividing integers is possible :

1. The quotient of two integers with the SAME sign is POSITIVE.

i.e. ⊕ ÷ ⊕ = ⊕ and ⊖ ÷ ⊖ = ⊕

For example :

• $8 \div 2 = 4$ • $-10 \div (-2) = 5$

2. The quotient of two integers with DIFFERENT signs is NEGATIVE.

i.e. ⊕ ÷ ⊖ = ⊖ and ⊖ ÷ ⊕ = ⊖

For example :

• $40 \div (-5) = -8$ • $-42 \div 6 = -7$

LESSON 4

Notice that :

- $0 \div 2 = 0$
- $0 \div (-4) = 0$
- $0 \div (-100) = 0$

i.e. the quotient of zero divided by any non zero integer is zero.

Notice that :

Division by 0 has no meaning.

For example :

$-5 \div 0$ has no meaning because there is no number when multiplied by zero gives (-5)

Try by yourself

- State whether the quotient is positive , negative or 0 :

[a] $-6 \div (-2)$	(..........)	[b] $-10 \div 5$	(..........)
[c] $20 \div (-5)$	(..........)	[d] $0 \div 3$	(..........)
[e] $0 \div (-5)$	(..........)	[f] $144 \div 12$	(..........)

Example 5

Find the result of each of the following :

$6 \div (-3)$ and $(-3) \div 6$

Are the results equal ?

Solution

$6 \div (-3) = -2$, while $(-3) \div 6$ is not possible in $\mathbb{Z}$

So, $6 \div (-3) \neq (-3) \div 6$

It means that : The division operation in $\mathbb{Z}$ is not commutative.

Example 6

Find the result of each of the following :

$(36 \div (-6)) \div 2$ and $36 \div ((-6) \div 2)$

Are the results equal ?

Solution

$(36 \div (-6)) \div 2$	$36 \div ((-6) \div 2)$
$= (-6) \div 2$	$= 36 \div (-3)$
$= -3$	$= -12$

So, $(36 \div (-6)) \div 2 \neq 36 \div ((-6) \div 2)$

It means that : The division operation in $\mathbb{Z}$ is not associative.

From the previous , we can deduce the following properties of division in $\mathbb{Z}$:

1. Division is not always possible in $\mathbb{Z}$ or $\mathbb{Z}$ is not closed under division.
2. Division in $\mathbb{Z}$ is not commutative.
3. Division in $\mathbb{Z}$ is not associative.

Example 7

If $a = 6$, $b = -2$ and $c = -6$, then find the result of each of the following :

[a] $4a \div 2b$ [b] $(a \times b) \div c$

[c] $(a + c) \div b$ [d] $(a - c) \div b$

Solution

[a] $4a \div 2b = [4 \times 6] \div [2 \times (-2)] = 24 \div (-4) = -6$

[b] $(a \times b) \div c = [6 \times (-2)] \div (-6) = -12 \div (-6) = 2$

[c] $(a + c) \div b = [6 + (-6)] \div (-2) = 0 \div (-2) = 0$

[d] $(a - c) \div b = [6 - (-6)] \div (-2) = (6 + 6) \div (-2) = 12 \div (-2) = -6$

Exercise 4

Multiplying and dividing integers

Interactive test

From the school book

1 Multiply :

a 3×5

b -6×2

c $(-125) \times (-4)$ *(El-Menia 2015)*

d $0 \times (-10)$

e $9 \times (-1)$

f -9×7

g $(-131) \times (-3)$

h $200 \times (-12)$

i $-(-6) \times (-2)$

j $(-5) \times |-4|$ *(Ismailia 2016)*

k $-|10| \times |-3|$

l $|0| \times |4|$

2 Divide :

a $8 \div 2$

b $-64 \div 8$

c $49 \div (-7)$

d $(-36) \div (-4)$ *(Aswan 2014)*

e $0 \div 10$

f $77 \div (-11)$

g $-100 \div 25$

h $-\frac{18}{2}$

i $(-18) \div (-3)$ *(El-Monofia 2012)*

j $\frac{18}{-6}$

k $|-45| \div |-5|$

l $-|42| \div 6$

3 Write the property of multiplication in the set $\mathbb{Z}$ in each of the following :

a $-12 \times 1 = -12$ (……………)

b $-5 \times (9 \times 7) = (-5 \times 9) \times 7$ (……………)

c $5 \times (-2) = (-2) \times 5$ (……………)

d $(-2 \times 6) + (-2 \times 9) = -2 \times (6 + 9)$ (……………)

4 Find the value of (X) in each of the following :

a $-8 \times 4 = X \times -8$

b $-16 \times X = -16$

c $X \times (9 + 5) = (-4 \times 9) + (-4 \times 5)$

d $-7 \times X = 0$

e $x \times (5 \times (-13)) = (-9 \times 5) \times (-13)$

f $(-8) \times (-3) = x$

g $-9 \div 3 = x$

h $8 \times x = -48$

i $-3\,x = 27$

j $5\,x = 45$

k $x \times 9 = -45$

l $-18 \div x = -9$

5 Complete :

a The additive neutral element in $\mathbb{Z}$ is .... , while the multiplicative neutral element in $\mathbb{Z}$ is ....

b The sum of two negative integers is a .... integer , while the product of two negative integers is a .... integer.

c The quotient of two integers having different signs when the division operation is possible in $\mathbb{Z}$ is a .... integer.

d $5 \times .... = 0$

e $|-24| + (-8) = ...$ (Ismailia 2011)

f $5 \times (-|42|) = ...$ (Port Said 2015)

g $-4 \times [3 + (-1)] = ...$ (Assiut 2016)

h $[9 + (-5)] \times |-11| = ...$ (The New Valley 2017)

i $-7 \times .... = -56$

j $........ \times 9 = -3 \times 21$

k If $a = 3$, $b = -2$, then the value of $3\,a\,b = ...$

l If $x = |-12|$, $y = -3$, then $x + y = ...$

m $A \times (B + C) = ... + A \times C$

n If $a \times b = a$, and $a \neq 0$, then $b = ...$

o If $a + b = a$, and $a \neq 0$, then $b = ...$

p If $a \div b = 1$, then $b = ...$

q If $a \div b = -1$, then b is the .... of a

LESSON 4

6 Choose the correct answer :

a $6 \times (-3) = \dots\dots$ (El-Menia 2012) (18 or −18 or 9 or −9)

b $(-8) \div (-4) = \dots\dots$ (El-Menia 2013) (2 or −2 or 4 or 32)

c $72 \div (-6) = \dots\dots$ (El-Sharkia 2013) (−12 or 12 or 6 or −6)

d If $x = |-2|$, $y = -3$, then $x\,y = \dots\dots$ (Giza 2016)

(−5 or 5 or 6 or −6)

e $[8 + (-3)] \times (-3) = \dots\dots$ (El-Monofia 2013)

(15 or −15 or 72 or −72)

f $\text{Zero} \div (-3) = \dots\dots$ ($\frac{1}{3}$ or −3 or 1 or zero)

g $\text{Zero} \times (-1) \times (-2) \times (-3) = \dots\dots$ (El-Gharbia 2014)

(0 or −6 or −5 or 6)

h $6 \div 3 \times 2 - 1 = \dots\dots$ (1 or 2 or 3 or 4)

i If n is a negative integer , which of the following is the smallest ?

(Red Sea 2014) ($3 + n$ or $3\,n$ or $\frac{-3}{n}$ or $3 - n$)

j If $a + b = \text{zero}$ where $a \neq b$, then $a \times b \dots\dots$ zero (Suez 2017)

(= or > or < or ≥)

7 Use the properties of multiplication of integers to find :

a $5 \times 17 \times 2$

b $50 \times (-45) \times 2$

c $4 \times (-5) \times 3 \times (-2)$

d $(-2) \times (-3) \times 5 \times (-1)$

e $4 \times (-16) \times 25$

f $8 \times 77 \times (-125)$

8 Use the distributive property to find the result of each of the following :

a $3 \times (-2) + 3 \times 5$

b $75 \times 37 + 75 \times 63$ (El-Dakahlia 2017)

c $(-5) \times (-6) + 2 \times (-6)$

d $147 \times 69 - 47 \times 69$

e $112 \times 17 + 112 \times (-17)$ ([illegible])

f $(-35) \times (-42) + (-35) \times 52$

g $32 \times 18 - 32 \times 34 + 32 \times 17$

h $45 \times (-16) + (-47) \times (-16) + (-16)$

i $(-3) \times 4 - (-3) \times 5 - 3$

9 Find the result of each of the following :

a $(-5) \times (3 + 7)$

b $12 \times (5 - 9)$

c $[8 + (-5)] \times 6$ (El-Kalyoubia 2011)

d $[5 + (-3)] \times (-11)$ ([illegible])

e $6 \times (-6 + 0)$ (Damietta 2011)

f $(-7) \times (6 - 2 - 8)$

g $(5 + 3 - 8) \times (-4)$

h $(-5 + 3) \div 2$

i $45 \div [3 - (-6)]$

j $[25 \times (-2)] \div (-5)$

10 If $x = 2$, $y = 1$ and $z = 5$, then find the value of : $3x - 2y + z$

(Kafr El Sheikh 2011)

11 Find the value of : $x - 2y + 4$, when $x = 8$ and $y = -2$ ([illegible])

12 If $x = 3$, $y = -1$ and $z = -2$, calculate the value of : $(2x + y) \times 3z$

(El Dakahlia 2011)

LESSON **5**

Repeated multiplication

You know how to factorize a number by writing it as the **repeated multiplication**.

For example :

$$16 = 2 \times 2 \times 2 \times 2$$

(multiplied by itself 4 times)

i.e. $2 \times 2 \times 2 \times 2$ is another form of writing the number 16

- A third form of writing the number 16 is 2^4

i.e. Instead of writing $2 \times 2 \times 2 \times 2$, we can write 2^4

It is read as :

"2 to the power 4" OR "2 to the fourth power"

2 is called "the base" and 4 is called "the power"

, "the exponent" or "the index"

Power

Base 2^4

Generally

If a is an integer and $n \in \mathbb{Z}^+$, then $a \times a \times a \times \cdots$ to n times = a^n where a is called the base and n is called the power , index or exponent.

For example :

- $7 \times 7 = 7^2$, it is read as "7 to the power 2" OR "7 to the second power".
- $6 \times 6 \times 6 = 6^3$, it is read as "6 to the power 3" OR "6 to the third power".
- $(-3) \times (-3) \times (-3) \times (-3) \times (-3) = (-3)^5$, it is read as "– 3 to the power 5" OR "– 3 to the fifth power".

Notice that :

- **The second power of a number is called the square of this number.**
  For example : 7^2 is read as "the square of 7"
- **The third power of a number is called the cube of this number.**
  For example : 6^3 is read as "the cube of 6"

Remarks

1. Any number to the first power is that number itself.
   For example : • $9^1 = 9$ • $(-3)^1 = -3$ • $x^1 = x$
2. Any number except 0 to the zero power is 1
   For example : • $5^0 = 1$ • $(-7)^0 = 1$ • $a^0 = 1$ where $a \neq 0$
3. If the base is one and $n \in \mathbb{Z}$, then $1^n = 1$
   For example : • $1^5 = 1$ • $1^{12} = 1$
4. If $a \in \mathbb{Z}$ and $n \in \mathbb{Z}^+$, then $(-a)^n = \begin{cases} (a)^n & \text{if n is even} \\ -(a)^n & \text{if n is odd} \end{cases}$
   i.e. • A **negative integer** raised to the **power** of an **even** integer gives a **positive** integer.
   • A **negative integer** raised to the **power** of an **odd** integer gives a **negative** integer.
   For example : • $(-4)^2 = 4^2$ • $(-4)^3 = -(4)^3$

LESSON 5

Example (1)

Find the value of each of the following :

[a] 2^5 [b] $(-5)^4$ [c] $(-3)^3$ [d] $-(6)^2$

Solution

[a] $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$

[b] $(-5)^4 = 5^4 = 5 \times 5 \times 5 \times 5 = 625$

[c] $(-3)^3 = -(3)^3 = -(3 \times 3 \times 3) = -27$

[d] $-(6)^2 = -(6 \times 6) = -36$

Calculator :

You can use a calculator to check your answers.

Example : $(-7)^3 = \dots$

((-) 7) $x^\square$ 3 = -343

Example (2)

Find the value of each of the following :

[a] $(-5)^2 \times 2^2$

[b] $(-2)^3 + (-3)^2$

[c] $(-1)^{11} + (-1)^{10}$

[d] $3^2 + 3^3$

Solution

[a] $(-5)^2 \times 2^2 = 5^2 \times 2^2$

$= (5 \times 5) \times (2 \times 2) = 25 \times 4 = 100$

[b] $(-2)^3 + (-3)^2 = -(2)^3 + 3^2$

$= -(2 \times 2 \times 2) + (3 \times 3) = -8 + 9 = 1$

[c] $(-1)^{11} + (-1)^{10} = -(1)^{11} + 1^{10}$

$= -1 + 1 = 0$

[d] $3^2 + 3^3 = (3 \times 3) + (3 \times 3 \times 3)$

$= 9 + 27 = 36$

Example (3)

If $a = 3$, $b = -1$ and $c = 2$, find the value of each of the following :

[a] $a^2 + c^3$ [b] $a^2 - 2\,a\,b$ [c] $(a - b)^c$

Solution

[a] $a^2 + c^3 = 3^2 + 2^3 = (3 \times 3) + (2 \times 2 \times 2) = 9 + 8 = 17$

[b] $a^2 - 2\,a\,b = 3^2 - 2 \times 3 \times (-1) = 9 - (-6) = 9 + 6 = 15$

[c] $(a - b)^c = [3 - (-1)]^2 = [3 + 1]^2 = 4^2 = 4 \times 4 = 16$

Rules of powers

Rule (1)

You know that : $3^2 = 3 \times 3$ and $3^5 = 3 \times 3 \times 3 \times 3 \times 3$

So, $3^2 \times 3^5 = (3 \times 3) \times (3 \times 3 \times 3 \times 3 \times 3)$

$= 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^7$

i.e. $3^2 \times 3^5 = 3^{2+5} = 3^7$

By using a similar way , you can find also that : $(-5)^6 \times (-5)^3 = (-5)^9$

Generally

If $a \in \mathbb{Z} - \{0\}$, $n \in \mathbb{Z}^+$, $m \in \mathbb{Z}^+$, then :

$$\boxed{a^m \times a^n = a^{m+n}}$$

i.e. In case of multiplying numbers with equal bases , keep the base and add the powers.

For example :

- $3^2 \times 3^3 = 3^{2+3} = 3^5 = 243$
- $(-2)^4 \times (-2)^2 = (-2)^{4+2} = (-2)^6 = 2^6 = 64$

LESSON 5

- $x^5 \times x^7 = x^{5+7} = x^{12}$
- $y \times y^4 \times y^3 = y^{1+4+3} = y^8$
- $a^3 \times b^2 \times a^5 \times b = (a^3 \times a^5) \times (b^2 \times b) = a^8 \times b^3$
- $(-2)^3 \times 2^2 = -(2)^3 \times 2^2 = -2^5 = -32$

Try by yourself

- **Complete :**

[a] $3^2 =$ ....

[b] $(-7)^2 =$ .....

[c] $2^3 \times 2^3 =$ ....

[d] $(-3)^2 \times (-3)^3 =$ ......

[e] $2^2 \times (-2)^2 =$

[f] $3^2 \times (-3)^3 =$ $=$

Rule 2

You know that : $3^6 = 3 \times 3 \times 3 \times 3 \times 3 \times 3$ and $3^4 = 3 \times 3 \times 3 \times 3$

So, $\frac{3^6}{3^4} = \frac{3\times3\times3\times3\times3\times3}{3\times3\times3\times3} = 3\times3 = 3^2$

i.e. $\frac{3^6}{3^4} = 3^{6-4} = 3^2$

By using a similar way , you can find that : $\frac{(-5)^8}{(-5)^3} = (-5)^5$

Generally

If a is an integer and $a \neq 0$, $n \in \mathbb{Z}^+$, $m \in \mathbb{Z}^+$, $m \geq n$, then :

$$\boxed{\frac{a^m}{a^n} = a^{m-n}}$$

i.e. To divide two numbers with equal bases, keep the base and subtract the powers.

For example :

- $\frac{2^5}{2^2} = 2^{5-2} = 2^3 = 8$
- $\frac{(-3)^8}{(-3)^6} = (-3)^{8-6} = (-3)^2 = 9$
- $\frac{a^9}{a^6} = a^{9-6} = a^3$ where $a \neq 0$

Example 4

Find the value of each of the following :

[a] $\frac{2^7 \times 2^4}{2^8}$

[b] $\frac{(-5)^3 \times (-5)^6}{(-5)^7}$

[c] $\frac{7^3 \times 7^5}{7^2 \times 7^6}$

[d] $\frac{(-3)^7}{3^2 \times 3^3}$

Solution

[a] $\frac{2^7 \times 2^4}{2^8} = \frac{2^{7+4}}{2^8} = \frac{2^{11}}{2^8} = 2^{11-8} = 2^3 = 8$

[b] $\frac{(-5)^3 \times (-5)^6}{(-5)^7} = \frac{(-5)^9}{(-5)^7} = (-5)^{9-7} = (-5)^2 = 25$

[c] $\frac{7^3 \times 7^5}{7^2 \times 7^6} = \frac{7^8}{7^8} = 7^{8-8} = 7^0 = 1$

[d] $\frac{(-3)^7}{3^2 \times 3^3} = \frac{-(3)^7}{3^5} = -(3)^{7-5} = -(3)^2 = -9$

Remember that :
$a^0 = 1$, where $a \neq 0$

Try by yourself

- **Complete :**

[a] $\frac{2^6}{2^2} = \dots\dots = \dots\dots$

[b] $\frac{(-6)^8}{(-6)^5} = \dots\dots = \dots\dots$

[c] $\frac{3^4 \times 3^5}{3^7 \times 3^2} = \dots\dots = \dots\dots$

[d] $\frac{(10)^4 \times (-10)^3}{(-10)^5} = \dots\dots = \dots\dots$

Exercise 5

Repeated multiplication

Interactive test

From the school book

1 Complete :

a $7 \times 7 \times 7 = 7^{\cdots}$

b $6 \times 6 \times 6 \times 6 = \cdots^{\cdots}$

c Two to the seventh power = $2^{\cdots}$

d Ten to the twelfth power = $\cdots^{\cdots}$

e Four cubed = $\cdots^{\cdots}$

f Five squared = $\cdots^{\cdots}$

g Seven to the zero power = $\cdots^{\cdots}$

h $2^3 + 2^2 = \cdots$ (Cairo 2013)

i $5^{zero} = \cdots$ (El-Kalyoubia 2014)

j $(-7)^{zero} + (7)^{zero} = \cdots$ (El-Gharbia 2015)

k $(-1)^{10} + (-1)^{11} = \cdots$ (El-Fayoum 2017)

l $\frac{a^m}{a^n} = a^{\cdots}$ where $m , n \in \mathbb{Z}^+ , m > n$ (Suez 2016)

2 Find the value of each of the following :

a 2^3 | b 3^2 | c 5^3

d 10^4 | e $(-7)^2$ | f $(-2)^3$

g $(-6)^3$ | h $-(9)^2$ | i 5^0

j $(-8)^0$ | k $(-1)^{50}$ | l $(-1)^{51}$

3 Find the value of each of the following :

a $2^2 \times 2^3$ | b $(10)^3 \times (-10)^4$ | c $(-5)^3 \times 5^2$

d $-(2)^4 \times 2^2$ | e $b^8 \times b^2$ | f $7 \times 7^3 \times 7^2$

4 Find the value of each of the following :

a $3^7 \div 3^4$ | b $3^4 \div 3^3$ | c $5^6 \div 5^6$

d $(-6)^5 \div (-6)^3$ | e $(-5)^5 \div 5^3$ | f $a^8 \div a^3$, $a \neq 0$

5 Find the value of each of the following :

a $2^3 \times 3^2$ | b $(-5)^2 \times 2^2$ | c $(-1)^2 \times (-2)$ ([illegible] 2014)

d $(-4)^3 \times (-1)^5$ | e $(-5)^3 \times (-1)^{17}$ | f $(-1) \times 2^3$

g $-(4)^2 \times (-2)^3$ | h $2^3 + 2^2$ | i $(-2)^3 + 3^2$

j $(-2)^4 + (-3)^3$ | k $(-1)^{30} + (-1)^{31}$ | l $(-1)^{50} + (-1)^{100}$

m $2^4 + 3^3 - 4^2$ | n $3^4 \times 3^2 \times 2^2 \times 2^3$ | o $(-2)^2 \times (2^3)^2$

6 Find the value of each of the following :

a $\dfrac{5 \times 5^3}{5^4}$ ([illegible])

b $\dfrac{7^4 \times 7^5}{7^7}$ ([illegible])

c $\dfrac{3^7}{3^3 \times 3^2}$ (Suez 2015)

d $\dfrac{2^6 \times 2^5}{2^3 \times 2}$ (El-Menia 2016)

e $\dfrac{(-3)^3 \times (-3)^4}{(-3)^5}$ (North Sinai 2017)

f $\dfrac{(-5)^5 \times (-5)^4}{(-5)^6}$

g $\dfrac{(-3)^4 \times (-3)^5}{(-3)^6 \times (-3)}$ (Sohag 2013)

h $\dfrac{(-8)^3 \times 8^4}{(-8)^7}$

i $\dfrac{3^2 \times (-3)^5}{3^4}$ (Aswan 2012)

j $\dfrac{(-2)^7 \times (-2)^5}{2^{10}}$ (Kafr El-Sheikh 2017)

k $\dfrac{(-2)^6 \times 2^4}{2^7 \times 2}$ (Ismailia 2012)

l $\dfrac{9^6 \times (-9)^3}{9^2 \times (-9)^5}$

m $\dfrac{(-5)^{10} \times (-5)^8}{-5 \times (-5)^5 \times (-5)^{11}}$

n $\dfrac{(-3)^6}{(-3)^3} + \dfrac{(-4)^5}{(-4)^3}$ (El-Dakahlia 2015)

o $\dfrac{2^2 + 2^3}{2^4}$

p $\dfrac{2^6}{2^3} + (-1)^5$ (South Sinai 2013)

LESSON 5

7 Simplify each of the following to its simplest form :

a $\frac{a^6 \times a^3}{a^5}$ where $a \neq 0$

b $\frac{a^{12}}{a^9 \times a^2}$ where $a \neq 0$

c $\frac{x^8}{x^3 \times x^3}$ where $x \neq 0$

d $\frac{x \times x^3 \times x^{10}}{x^2 \times x^7}$ where $x \neq 0$

8 Simplify each of the following to its simplest form :

a $\frac{5^4 \times 3^3}{3^2 \times 5^2}$

b $\frac{(-2)^5 \times 3^7}{3^3 \times (-2)^3}$

c $\frac{(-4)^4 \times (-3)^2}{4^2 \times (-3)}$

d $\frac{x^5 \times y^6}{y^3 \times x^2}$ where $x , y \neq 0$

9 Arrange in an ascending order :

a $(-2)^5$, $(-3)^4$, $(-4)^0$, $(-1)^{15}$ and 3^2

b 2^3 , 3^2 , $(-2)^3$, $(100)^0$ and $(-1)^5$

10 Arrange in a descending order :

a $(-2)^3$, $(-2)^2$, $(-2)^0$ and $(-1)^5$

b 10^2 , $(-1)^5$, 1000 and $(1000)^{zero}$

c 10^2 , $(-1)^5$, 100^2 , $(-10)^3$ and 1000000

11 Put [< , > or =] :

a 2^5 .... 5^2

b 4^2 8

c $(-5)^2$ 25

d $(-4)^5$ .... $(-4)^2$

e $(-6)^2$ .... (-12)

f 9^2 .... $(-3)^4$

g $(-5)^3$ .... $(-1)^0$

h $(-1)^6$ .... $(-1)^7$ (Damietta 2012)

i $|-8|$ .... 2^3

j $2^2 + 2^3$ .... $2^2 \times 2^3$

12 Choose the correct answer :

a $(-5)^2$ .... $\mathbb{N}$ $(\in$ or $\notin$ or $\subset$ or $\not\subset)$

b $(-3)^5$ .... $\mathbb{N}$ $(\in$ or $\notin$ or $\subset$ or $\not\subset)$

c $(-11)^0$ .... $\mathbb{N}$ $(\in$ or $\notin$ or $\subset$ or $\not\subset)$

d The additive inverse of $(-8)^0$ is .... $(8$ or -8 or 1 or $-1)$

e The additive inverse of $(-1)^3$ is .... $(1$ or -1 or 3 or $-3)$

f $3^2 \times 3^3 = 3^{\cdots}$ (Giza 2014) $(5$ or 3 or 2 or $1)$

g $2^3 \div 2^2 =$ .... (El-Beheira 2013) $(2$ or 8 or 16 or $32)$

h $3^7 \div 3^7 =$ .... (Ismailia 2014) $(0$ or 1 or 3 or $7)$

i $2^6 \times 2^2 \div 2^7 =$ .... (El-Gharbia 2014) $(2^8$ or 2^{12} or 2^5 or $2)$

j $2^5 \div 2^5 = 3^{\cdots}$ (El-Dakahlia 2017) $(2$ or 10 or zero or $1)$

k $(3)^0 + (-3)^0 =$ .... (South Sinai 2012) $(6$ or 0 or 1 or $2)$

l $(-1)^3 - 1 =$ .... (Cairo 2017) $(-2$ or 0 or 1 or $2)$

m $(-3)^3 + (-3)^2 =$ .... (Souhag 2013) $((-3)^5$ or $(-3)^6$ or -18 or $18)$

n $3^2 + 3^2 + 3^2 =$ .... (Aswan 2015) $(2^6$ or 4^6 or 3^3 or $2^9)$

o $(-5)^2 + 5 =$ .... $(5^2$ or 20 or 15 or $30)$

p $\frac{7^5}{7^4} + 1 =$ .... $(7$ or 1 or 8 or $7^2)$

q $(-2)^{20} \div 2^{15} =$ .... $(2^5$ or $(-2)^5$ or 2^{35} or $(-2)^{35})$

r If $3^5 \div 3^a = 3^0$, then a = ... $(4$ or 5 or -5 or $0)$

LESSON **5**

s If $x = 1$, $y = -2$, then the negative number in the following is ……

[illegible] $(x + y^2$ or $x^2 - y$ or $x^2 + y$ or $x^2 + y^2)$

t If F is an odd number , then the even number in the following is ……

[illegible] $(F^2$ or $F^2 + F$ or $2F + 1$ or $F^3)$

13 If $n = 2$, then find the value of each of the following :

a n^2 | b $3n^7$ | c $n^4 + 5$

d $n^3 - 1$ | e $\frac{n^6}{8}$ | f $2n^5 + 1$

14 If $a = 2$ and $b = -3$, find the value of each of the following :

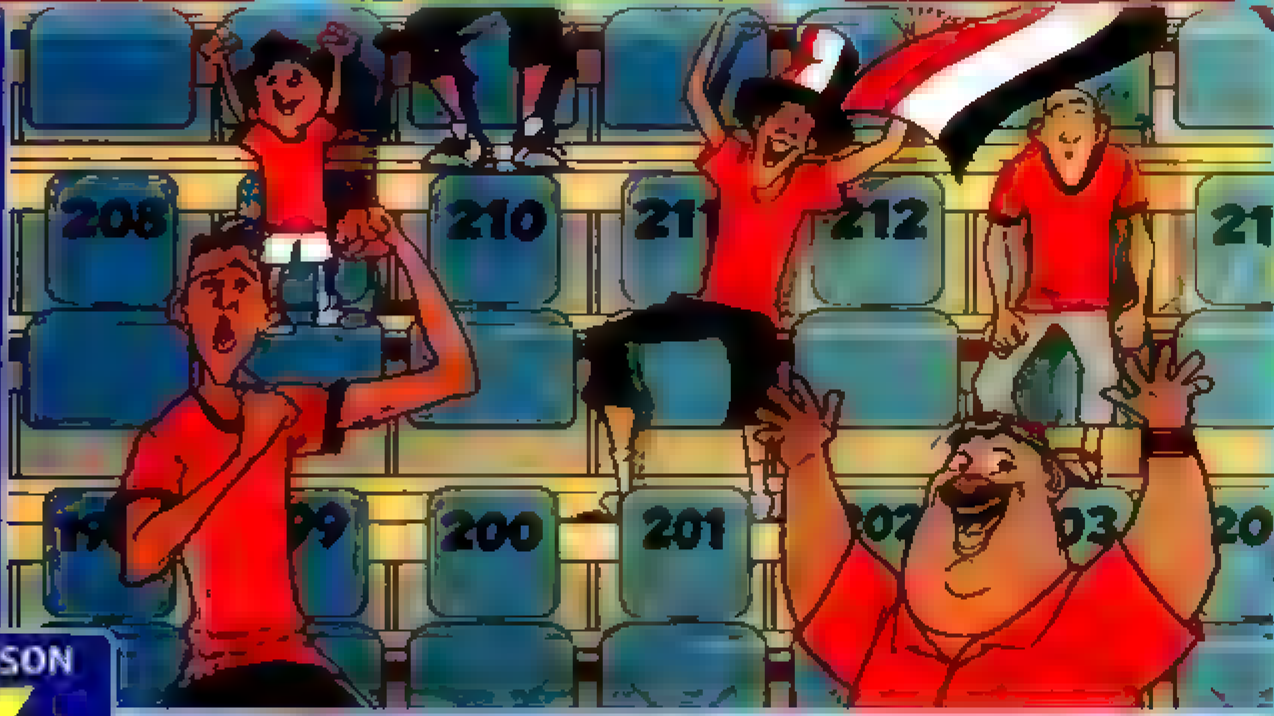
a $3a^2b$ | b $2a + 3b$ | c $a^2 + b^2 + ab$ [illegible]

15 If $a = 3^2$, $b = 2^3$ Find the value of : $(a - b)^{10}$ [illegible]

16 Use the distributive property to calculate the value of each of the following :

a $(17)^2 + 17 \times 83$ | b $33 \times 23 - (23)^2$

c $(27)^2 + 27 \times (-17)$ | d $(23)^2 + 23 \times 78 - 23$



LESSON 6

Numerical patterns

Numerical pattern is a sequence of numbers according to a particular rule.

For example :

- The set of natural numbers $\mathbb{N}$ represents a numerical pattern where "each number is more than its preceding by one".

- The set of even numbers = $\{0, 2, 4, 6, ...\}$ represents a numerical pattern where "each number is more than its preceding by 2"
- Also , the set of odd numbers = $\{1, 3, 5, 7, ...\}$ represents a numerical pattern where "each number is more than its preceding by 2"

Pascal's triangle

- Pascal's triangle is one of the most interesting numerical patterns.
- In this triangle , we notice that each **row begins and ends with number (1)**
- After the second row , each number is the **sum** of the **two numbers** just to the left and right of it in the row above.

1 → Row 0

1 1 → Row 1

1 2 1 → Row 2

1 3 3 1 → Row 3

1 4 6 4 1 → Row 4

1 5 10 10 5 1 → Row 5

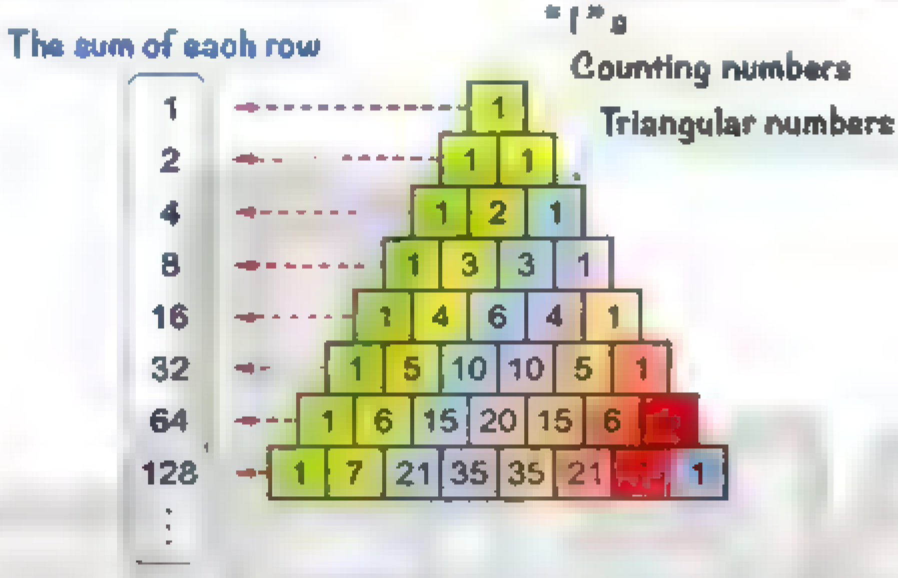
Pascal's triangle

For example : 1 + 3 = 4 and is represented by the red triangle.

LESSON 6

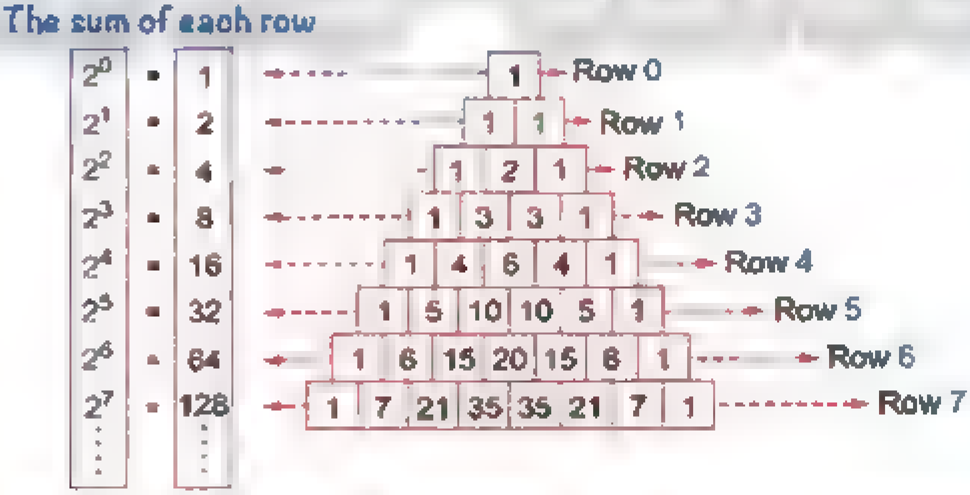
- There are a lot of numerical patterns that we can get from Pascal's triangle as shown in the following :

1. The first diagonal is of course just "1" s and the next diagonal has the counting numbers (1 , 2 , 3 , .. etc.) , the third diagonal has the triangular numbers.



2. The sum of each row is twice of its preceding or the sum of the numbers in any row is equal to 2 raised to the n^{th} power or 2^n , where "n" is the index number of the row.

For example :



- Try by helping your teacher to discover another numerical patterns in Pascal's triangle.

Describing of the pattern

Means discovering the rule of the pattern and expressing it in words.

Example (1)

Describe each of the following patterns , then complete in the same pattern :

[a] 1 , 3 , 5 , 7 , ... , ...

[b] 95 , 85 , 75 , 65 , ... , ...

[c] 3 , 6 , 12 , 24 , ... , ...

[d] 1 , −2 , 4 , −8 , 16 , ... , ...

Solution

[a] $1 \xrightarrow{+2} 3 \xrightarrow{+2} 5 \xrightarrow{+2} 7 \xrightarrow{+2} 9 \xrightarrow{+2} 11$

Description of the pattern

Each number is more than its preceding by 2

[b] $95 \xrightarrow{-10} 85 \xrightarrow{-10} 75 \xrightarrow{-10} 65 \xrightarrow{-10} 55 \xrightarrow{-10} 45$

Description of the pattern

Each number is less than its preceding by 10

[c] $3 \xrightarrow{\times 2} 6 \xrightarrow{\times 2} 12 \xrightarrow{\times 2} 24 \xrightarrow{\times 2} 48 \xrightarrow{\times 2} 96$

Description of the pattern

Each number is twice of its preceding.

LESSON 6

×(−2) ×(−2) ×(−2) ×(−2) ×(−2) ×(−2)

[d] 1 , − 2 , 4 , −8 , 16 , −32 , 64

Description of the pattern

Each number equals its preceding multiplied by (− 2)

Try by yourself

- **Complete in the same pattern :**

[a] 1 , 4 , 7 , 10 , ... , ...

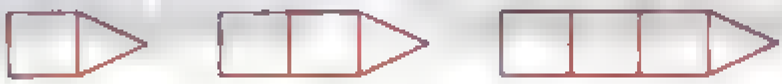
[b] 2 , 8 , 32 , ... , ...

[c] 6 , 2 , − 2 , ... , ...

[d] 128 , 64 , 32 , ... , ...

Example (2)

Write the number of line segments in each shape , then write the numerical pattern and describe it :



Solution

Number of line segments : 6 , 9 , 12

The numerical pattern : 6 , 9 , 12 , 15 , 18 , ...

Description of the pattern : each number is more than its preceding by 3

Exercise 6

Numerical patterns

Interactive test

From the school book

1 Complete in the same pattern :

a 7 , 10 , 13 , ... , ... (Assiut 2017)

b − 10 , − 8 , − 6 , − 4 , ... , ...

c − 15 , − 12 , − 9 , ... , ... (El-Sharkia 2011)

d − 2 , − 4 , − 6 , − 8 , ... , ... (Beni Suef 2013)

e 2 , 4 , 8 , 16 , ... , ...

f 3 , 9 , 27 , ... , ... (Souhag 2012)

2 Complete in the same pattern :

a 1 , − 1 , − 3 , − 5 , ... , ...

b 3 , − 6 , 12 , − 24 , ... , ... (Alexandria 2011)

c − 6 , − 4 , − 2 , ... , ... (Kafr El-Sheikh 2016)

d 9 , 6 , 3 , 0 , ... , ...

e 16 , 12 , 8 , 4 , ... , ... (Ismailia 2011)

f − 3 , 9 , − 27 , ... , ...

3 Complete in the same pattern :

a 2 , − 6 , 18 , − 54 , ... , ...

b − 5 , − 10 , − 15 , − 20 , ... , ...

c 3 , − 3 , 3 , − 3 , ... , ...

d 1 , 3 , 6 , 10 , ... , ...

e 8 , 4 , 2 , ... , ... , $\frac{1}{4}$

f 1 , 1 , 2 , 3 , 5 , 8 , ... , ... (Aswan 2013)

LESSON 6

4 Complete the following numerical patterns by writing three consecutive numbers :

a 6 , 14 , 22 , 30 , 38 , ... , ... , ...

b $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, ... , ... , ...

c 2 , 3 , 5 , 8 , 13 , ... , ... , ...

d 1 , 4 , 9 , 16 , 25 , ... , ... , ...

5 Discover the rule of the numerical pattern , then complete in the same pattern :

a 160 , 80 , 40 , 20 , ... , ...

b 1 , 3 , 7 , 15 , ... , ...

c 1 , 4 , 5 , 9 , 14 , ... , ...

d $\frac{1}{3}$, $\frac{2}{3}$, 1 , $\frac{4}{3}$, ... , ...

6 Discover the rule of the numerical pattern and write the missing numbers in each case :

a 4 , 7 , ... , 13 , 16 , ... , ...

b 7 , ... , 15 , 19 , 23 , ... , ...

c 0.5 , 1 , ... , 2 , 2.5 , ... , ...

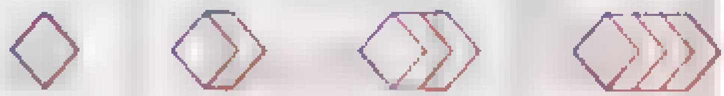
d 128 , 64 , ... , 16 , 8 , ... , ...

e .... , 15 , 12 , 9 , ... , ...

7 Complete the following table :

The numerical pattern	Description of the pattern
3 , 7 , 11 , 15 , 19 , 23 , ...	......
......	Each number is more than its preceding by 5
$\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, 1 , $\frac{5}{4}$, ...	......
......	Each number is less than its preceding by 4
3 , 9 , 27 , 81 , ...	......

8 Write the number of line segments below each shape , and then write the numerical pattern and describe it :

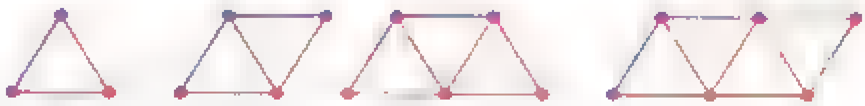


Number of line segments : ....

The numerical pattern : ......

Description of the pattern : ......

9 Write the number of triangles below each shape , and then write the numerical pattern and describe it :



Number of triangles : ....

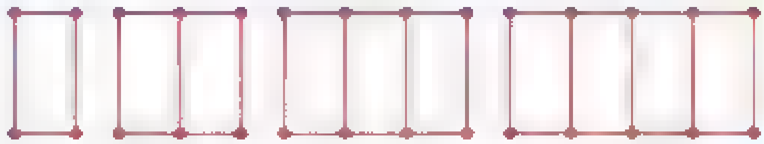
The numerical pattern : ......

Description of the pattern : ......

Using the number of line segments , write another pattern and describe it

LESSON 6

10 Deduce the pattern rule expressing the following design , then write the numerical pattern :

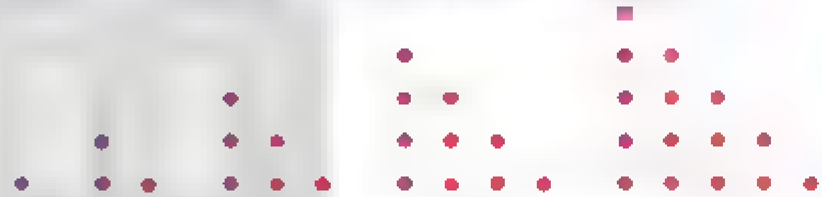


Number of line segments : ....

The numerical pattern : ....

The pattern rule : ....

11 Write the number of dots below each figure of the following :

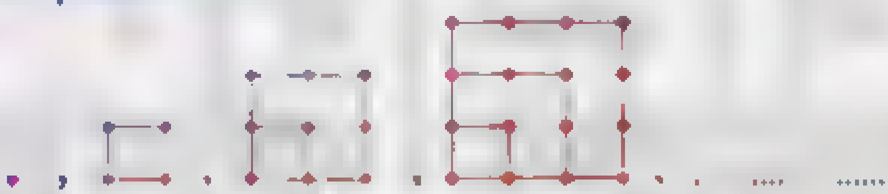


Number of dots : ....

The numerical pattern : ....

The rule of the pattern : ....

12 Look at the pattern of dots :



a How many dots will be there in the 5th shape ?

b Draw the 5th shape in the pattern.

13 An Egyptian land company reclaims 6 feddans per day to become prepared and ready for agriculture.

How many days do the company require to reclaim about 50 feddans ?

Write the numerical pattern which expresses this and describe it.

14 Khaled decided to lose weight at the rate of 3 kg. monthly.

If he is 90 kg. heavy right now , then how many months does he need to reach 69 kg. ?

Write expressing the numerical pattern and describe it.

Test on Unit One

Answer the following questions :

1 Choose the correct answer from the given ones :

a $\mathbb{N} \cup \mathbb{Z}^- = \ldots\ldots$ ($\mathbb{Z}^-$ or $\mathbb{Z}^+$ or $\mathbb{Z}$ or $\varnothing$)

b The smallest positive integer is ...... (0 or 1 or -1 or 2)

c $(-4) \ldots\ldots |-4|$ ($>$ or $<$ or $=$ or $\geq$)

d $5^{zero} + (-5)^{zero} = \ldots\ldots$ (zero or 5 or 2 or 10)

e If $a = 2$, $b = -3$, then $2ab = \ldots\ldots$ (-12 or -18 or -2 or 12)

2 Complete :

a $19 - (-11) = \ldots\ldots$

b The additive inverse of the integer (-17) is ......

c $\mathbb{Z}^+ \cap \mathbb{Z}^- = \ldots\ldots$

d $(-64) \div 8 = \ldots\ldots$

e The additive identity element in $\mathbb{Z}$ is ......

3 a Arrange the following numbers in an ascending order :

-9 , 17 , $|-9|$, -15 and $(-4)^2$

b Express each of the following sets using the listing method :

(1) The set of integers less than -2

(2) $X = \{x : x \in \mathbb{Z} , -3 < x \leq 2\}$

TEST ?

4 a Use the properties of addition in $\mathbb{Z}$ to find :

(1) $(-15) + 23 + 15$

(2) $36 + (-72) + 64 + (-28)$

b Find the value of the following in the simplest form : $\frac{(-3)^4 \times 3^5}{3^7}$

5 a Complete in the same pattern :

(1) 7 , 3 , − 1 , ... , ...

(2) $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{12}$, $\frac{1}{24}$, ...

b Use the properties of multiplication in $\mathbb{Z}$ to find :

(1) $5 \times (-4) \times 2$

(2) $45 \times 117 - 45 \times 17$

Activity of Unit One

1. Use Excel program to find the quotient of two integers and print the sheet.

2. Use Excel program to verify that : $a^m \times a^n = a^{m+n}$ and print the sheet.

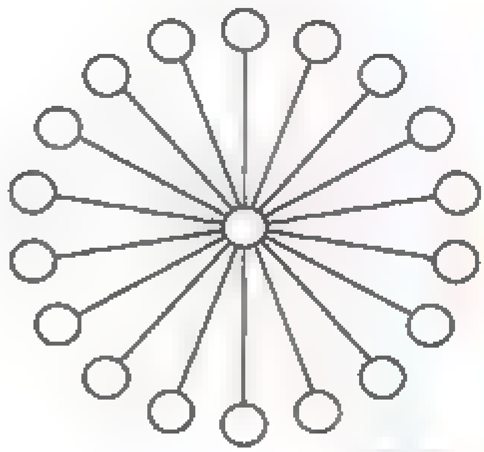
3. Use Excel program to verify that : $a^m \div a^n = a^{m-n}$, $m \geq n$, $a > 0$ and print the sheet.

4. Watch the weather forecast which describes the state of the weather in some cities , and register some cities of temperature less than zero and other cities of temperature greater than zero in the following table :

City					
Temperature					

- How many cities are of temperature less than zero ?
- Consider yourself a resident of one of the cities where temperature is greater than zero , and you will travel to the city of temperature less than zero.
  (a) Calculate the difference in temperature between the two cities.
  (b) Describe the preparations needed to travel to this city.

5. Copy the diagram , and arrange the integers – 1 through – 19 in the circles so that the sum of the numbers on each line is – 30



A research project

on unit one

Project aims

- Estimating the mathematics role in daily life.
- Linking mathematics with history.

Do a research project on the following topic

"Numbers and counting have become an integral part of our everyday life".

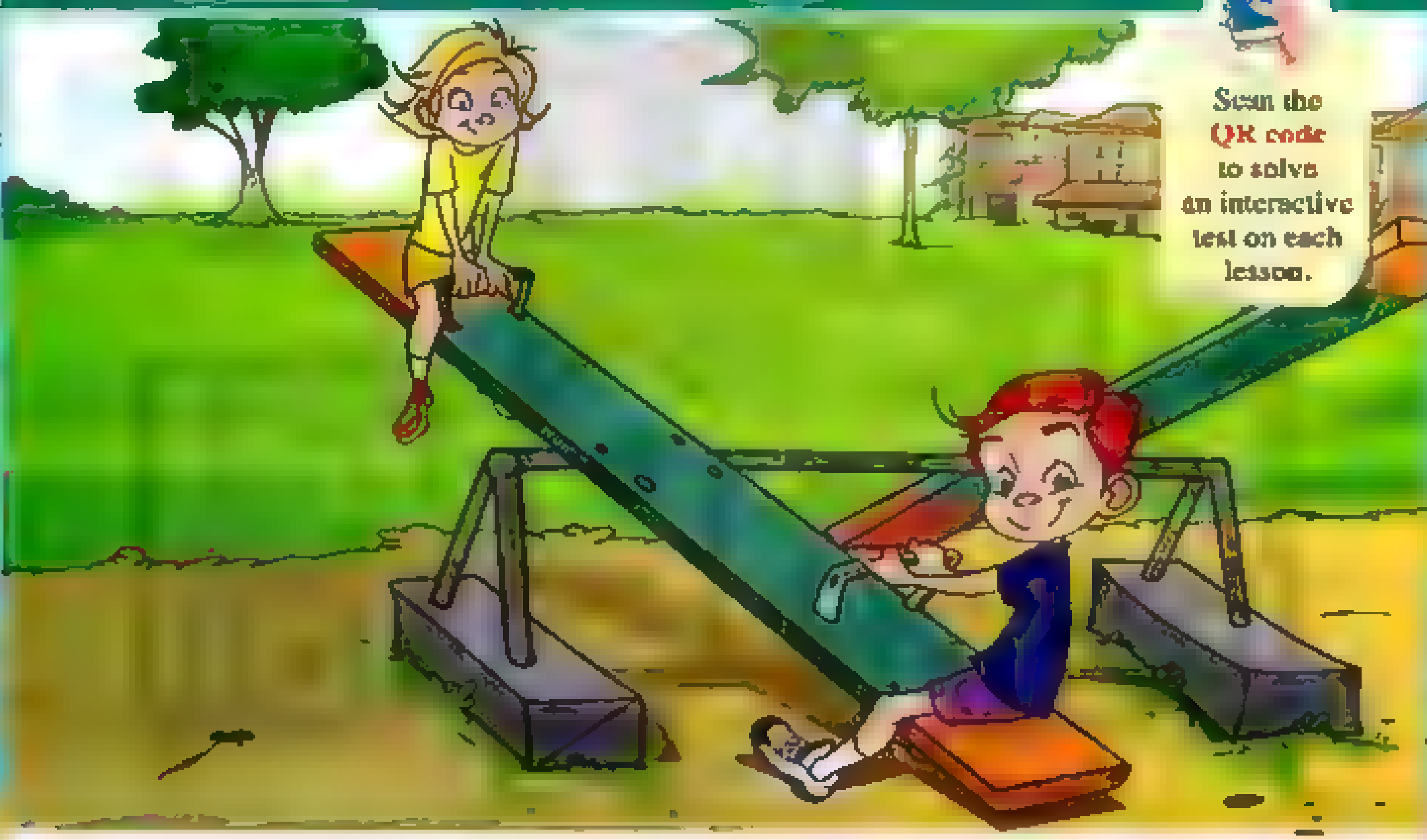
Discuss the following points using available resources

- Write a brief history of numbers and how early humans kept count.
- Write a brief note about Roman numerals, Ancient Egyptian numerals and Arabic numerals.
- How big a role did numbers play in our daily life?

UNIT

2 Equations and inequalities

Lessons of the unit

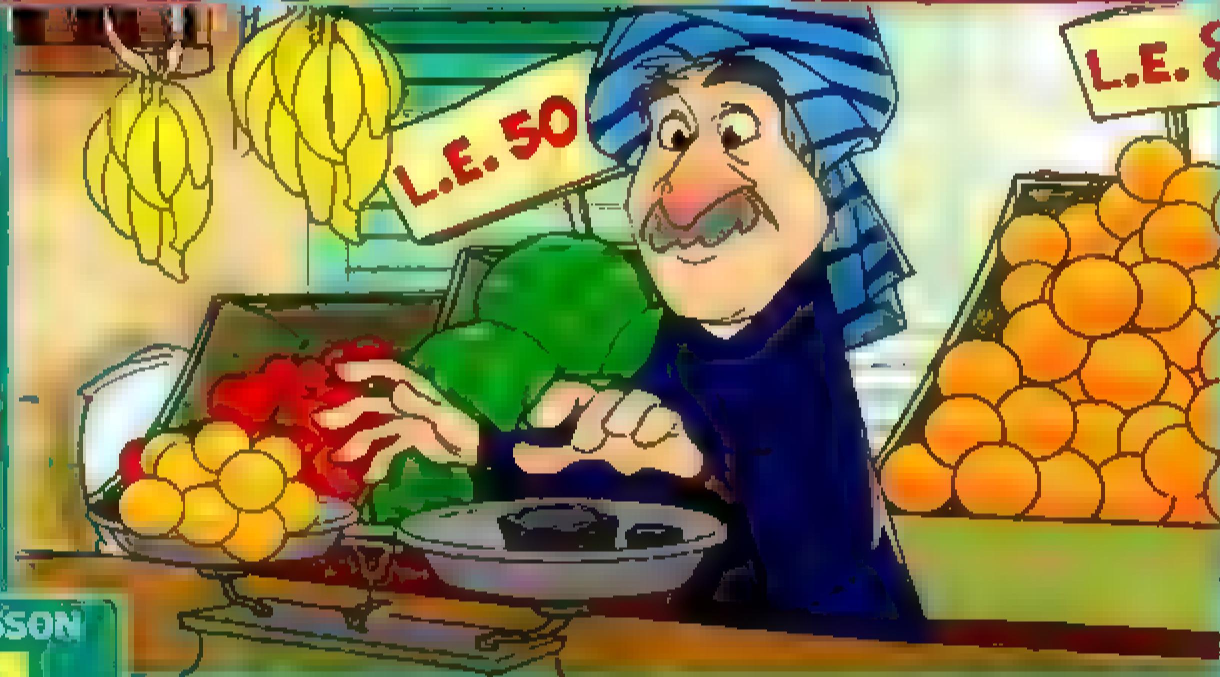


Scan the QR code to solve an interactive test on each lesson.

UNIT AIMS

By the end of this unit, student should be able to

- recognize the equation and the inequality
- express symbolically each of the equation and the inequality
- determine the degree of each of the equation and the inequality.
- find the solution set of each of the equation and the inequality using the substitution set method
- recognize the properties of each of the equations and the inequalities.
- find the solution set of each of the first degree equation in one unknown and the first degree inequality in one unknown using the properties of each one of them, and represent it on the number line.



LESSON 1

Equation and inequality of the first degree

The equation

- The figure below represents a pair of scales.

One of its pans contains a bag of orange and a weight of 2 kg.

The other pan contains a weight of 5 kg.

Fig (1)

- If we denote the weight of the bag by x kg., then the whole weight in the left pan will be $(x + 2)$ kg.
- The weights in the two pans are equal when $x + 2 = 5$, or when $x = 3$ (because 3 is the unique number if added to 2 gives 5)

It means that the weight of the bag of orange is 3 kg. when the two pans contain equal weights.

Remarks

1. $x + 2 = 5$ is called **"an equation"**.
2. The letter "x" in the equation is called **"the unknown"** or **"the variable"**.
3. The number "3" is called **"the solution"** of the equation $x + 2 = 5$, and $\{3\}$ is called **"the solution set"** (S.S.) of the equation.
4. The solution of the equation is the number which **"satisfies"** the equation. i.e. **which makes the two sides of the equation equal.**

Definition : An equation is a mathematical statement that has two expressions separated by an equal sign. One or both of the expressions contains one unknown (or more).

Examples of equations :

- $x + 4 = 9$
- $8 - 3x = 5$
- $x + 2y = -1$
- $x + 1 = 3 - 2x$
- $x^2 + 5 = 14$
- $x^3 + 4x^2 = 0$

Example (1)

For the equation $2x + 1 = 7$, check each element of the set $\{1, 2, 3\}$ to find whether it is a solution of this equation or not.

Solution

- When $x = 1$, the left side $2x + 1 = 2 + 1 = 3 \neq$ the right side $(\neq 7)$
  Therefore, 1 is not a solution of the equation.
- When $x = 2$, the left side $2x + 1 = 4 + 1 = 5 \neq$ the right side $(\neq 7)$
  Therefore, 2 is not a solution of the equation.
- When $x = 3$, the left side $2x + 1 = 6 + 1 = 7 =$ the right side
  Therefore, 3 is a solution of the equation.

Remark

In the previous example, the set $\{1, 2, 3\}$ is called **"substitution set"**.

LESSON 1

Example (2

Find the solution set of the equation
$x + 2 = 5$ if the substitution set is :
[a] $\{-2 , 3 , 4\}$ [b] $\{-1 , 1 , 2\}$

Notice that :
($\because$) means since
($\therefore$) means then or therefore

Solution

Substitute in the left hand side of the equation for x by the elements of the substitution set as follows :

[a] If the substitution set is $\{-2 , 3 , 4\}$

- When $x = -2$ $\therefore$ The left hand side $= -2 + 2 =$ zero $\neq 5$
  i.e. -2 is not a solution to the equation.
- When $x = 3$ $\therefore$ The left hand side $= 3 + 2 = 5$
  i.e. 3 is a solution to the equation.
- When $x = 4$
  $\therefore$ The left hand side $= 4 + 2 = 6 \neq 5$
  i.e. 4 is not a solution to the equation.
  $\therefore$ The solution set of the equation is $\{3\}$

Notice that :
The solution set is a subset of the substitution set.

[b] If the substitution set is $\{-1 , 1 , 2\}$

- When $x = -1$ $\therefore$ The left hand side $= -1 + 2 = 1 \neq 5$
  i.e. -1 is not a solution to the equation.
- When $x = 1$ $\therefore$ The left hand side $= 1 + 2 = 3 \neq 5$
  i.e. 1 is not a solution to the equation.
- When $x = 2$ $\therefore$ The left hand side $= 2 + 2 = 4 \neq 5$
  i.e. 2 is not a solution to the equation.
  $\therefore$ All the elements of the substitution set do not satisfy the equation.
  i.e. the equation has no solution in the substitution set $\{-1 , 1 , 2\}$
  $\therefore$ The S.S. $= \varnothing$

Remember that :
$\varnothing$ is the empty set (null set) which has no elements.

Try

- If the substitution set is $\{2 , 8 , -1 , 5 , -3\}$, tick ✓ in front of the number which represents a solution to the equation : $2x - 3 = 7$

• -1 ☐ • 2 ☐ • 5 ☐ • 8 ☐ • -3 ☐

The inequality

In the previous example , if we add 2 kg. to the left pan in fig. (1) , the scales will be inclined as in the opposite figure, then the left side $(x + 4)$ kg. is greater than the right side (5 kg.)

Fig (2)

So, we can express that mathematically by

$$x + 4 > 5$$

If we add 3 kg. to the right pan in fig. (1) , the scales will be inclined as in the opposite figure , then the left side $(x + 2)$ kg. is less than the right side (8 kg.)

Fig (3)

So, we can express that mathematically by

$$x + 2 < 8$$

Notice that :

The signs of the inequality are :

- " < " read as "is less than"
- " > " read as "is greater than"
- " ≤ " read as "is less than or equal to"
- " ≥ " read as "is greater than or equal to"

LESSON 1

- Each of these mathematical statements $x + 4 > 5$ and $x + 2 < 8$ is called an inequality because there is a sign of inequality ("<" less than or ">" greater than) between the two sides.

Definition : An inequality is a mathematical statement that has two expressions separated by an inequality sign (< or >). One (or both) of the expressions contains one unknown (or more).

Examples of inequalities :

- $x + 4 > 9$
- $8 - 3x < 5$
- $x + 2y > -1$
- $x + 1 > 3 - 2x$

Example (3

Find the solution set of the inequality $x + 1 > 5$ if the substitution set is {3 , 4 , 5 , 6}

Solution

Substitute in the left hand side of the inequality for x by the elements of the substitution set as follows :

- When $x = 3$ $\therefore$ The left hand side = 3 + 1 = 4 is not greater than 5
  i.e. 3 is not a solution to the inequality.
- When $x = 4$ $\therefore$ The left hand side = 4 + 1 = 5 is not greater than 5
  i.e. 4 is not a solution to the inequality.
- When $x = 5$ $\therefore$ The left hand side = 5 + 1 = 6 is greater than 5
  i.e. 5 is a solution to the inequality.
- When $x = 6$ $\therefore$ The left hand side = 6 + 1 = 7 is greater than 5
  i.e. 6 is a solution to the inequality.

$\therefore$ The S.S. = {5 , 6}

Try by yourself

- If the substitution set is $\{2, 3, 4, 5, 8\}$, tick $\boxed{\checkmark}$ in front of the number which represents a solution to the inequality : $2x - 5 > 1$

• 4 ☐ • 3 ☐ • 2 ☐ • 5 ☐ • 8 ☐

The degree of an equation (or an inequality) : It is determined by the highest power of the unknown (symbol) in the equation or inequality.

For example :

- $5x + 2 = 7$ is an equation of the first degree in one unknown x
- $x^2 + x - 3 = 0$ is an equation of the second degree in one unknown x
- $2x + 3y = 5$ is an equation of the first degree in two unknowns x and y
- $4x - 7 > 5$ is an inequality of the first degree in one unknown x

Example 4

Determine which of the following is equation or inequality "Give reasons" :

[a] $x - 3 = 5$ [b] $2x + 6$ [c] $4y - 7 < 1$

Solution

[a] $x - 3 = 5$ *is an equation* because it contains a variable " x " and contains the equality relation " = "

[b] $2x + 6$ *is neither equation nor inequality* because it doesn't contain the equality or the inequality relation.

[c] $4y - 7 < 1$ *is an inequality* because it contains a variable " y " and contains an inequality relation "<"

LESSON 1

Example (5)

Mention the degree and the unknown / unknowns of each of the following :

[a] $x + 13 = 2$
[b] $3x^2 - 7 = 4$
[c] $4y^3 - 4x^2 = 2$
[d] $y - 4 < 2$
[e] $4x^3 - 5x^4 = 8$
[f] $7x - 3y \geq 5$

Solution

[a] $x + 13 = 2$ an equation of 1st degree in one unknown (x)
[b] $3x^2 - 7 = 4$ an equation of 2nd degree in one unknown (x)
[c] $4y^3 - 4x^2 = 2$ an equation of 3rd degree in two unknowns (x and y)
[d] $y - 4 < 2$ an inequality of 1st degree in one unknown (y)
[e] $4x^3 - 5x^4 = 8$ an equation of 4th degree in one unknown (x)
[f] $7x - 3y \geq 5$ an inequality of 1st degree in two unknowns (x and y)

Example (6)

Express symbolically each of the following :

[a] x is less than -4

[b] x is less than or equal to 3 and greater than -1

Solution

[a] $x < -4$
[b] $-1 < x \leq 3$

Exercise 7

Equation and inequality of the first degree

Interactive test

From the school book

1 Determine which of the following represents an equation or an inequality and give reasons :

a $2x + 1 = 5$ | b $3x + 2 = 11$ | c $2x > 9$

d $x = 7 + 2$ | e $x > 7 - 5$ | f $x < -25$

g $2x = 24$ | h $2y + 3 \leq 5$ | i $5x \geq 30$

2 Determine the degree of each of the following :

a $x - 7 = 1$ | b $3x - 9 = 2$ | c $3x^2 - 6 = 14$

d $x - 2y = 5$ | e $3x - 2 < -2$ | f $3x^4 - 5 \leq 7$

g $x^3 - 4x^2 = 0$ | h $4x + 3y^2 > 2$ | i $x^4 + 2x = 3$

3 Express symbolically each of the following :

a x is less than -3

b x is less than or equal to 2

c x is greater than or equal to 3

d x is greater than -4 and less than 1

e x is less than or equal to 7 and greater than 1

f x is greater than or equal to -2 and less than or equal to 5

4 Find the solution set of each of the following equations :

a $x + 5 = 12$ if the substitution set is $\{3, 5, 7, 8\}$ (Suez 2015)

b $2x + 3 = 9$ if the substitution set is $\{2, 3, 4\}$ (Damietta 2016)

c $2x + 1 = 5$ if the substitution set is $\{-1, -2, 0, 2\}$

LESSON **1**

d $x + 4 = 0$ if the substitution set is $\{1 , 2 , 3 , 4\}$

e $4x - 3 = 9$ if the substitution set is $\{2 , 3 , 4\}$

f $2x - 5 = -1$ if the substitution set is $\{0 , 1 , 2 , 3\}$

g $-2 + 3x = 7$ if the substitution set is $\{0 , 1 , 2\}$

5 Find the solution set of each of the following inequalities :

a $x + 3 < 5$ if the substitution set is $\{4 , 3 , 2 , 1 , 0\}$

b $x - 4 > 1$ if the substitution set is $\{7 , 6 , 5 , 4\}$

c $3x - 1 > -2$ if the substitution set is $\{-2 , -1 , 0 , 1 , 2\}$

d $3x + 4 \leq -2$ if the substitution set is $\{-1 , 0 , 1 , 2 , 3\}$

e $-x + 1 < 4$ if the substitution set is $\{-3 , -2 , 0 , 2 , 3\}$

f $2x + 5 > 2$ if the substitution set is $\{-3 , -2 , -1 , 0 , 1\}$

g $5x - 1 > 4$ if the substitution set is $\{2 , 3 , 4 , 5 , 6\}$

6 Considering the set of substitution is $M = \{-1 , -2 , 0 , 2\}$, find the solution set of each of the following .

a $2x + 1 = 5$ b $x - 3 < -1$

7 Find the solution set of each of the following equations :

a $3(x - 2) = -6$ if the substitution set is $\{7 , 8 , 9\}$

b $2x + 1 = x - 3$ if the substitution set is $\{2 , 4 , -1 , -4\}$

c $2(x - 3) = x + 1$ if the substitution set is $\{4 , 5 , 6 , 7\}$

d $x \times 5 = x + 5$ if the substitution set is $\{1 , 2 , 3 , 4 , 5\}$

e $\frac{3x}{2} = x + 3$ if the substitution set is $\{4 , 6 , -2 , 0\}$

8 Choose the correct answer from those given :

a Which of the following represents an equation ?

(a) $x - 17$ (b) $22 - 7 = 15$ (c) $x > -11$ (d) $2x + 3 = 7$

b The equation $x^3 - 4x^2 = 0$ is an equation of .... degree.

(a) first (b) second (c) third (d) fourth

c The equation $x^2 + 3 = 4$ is of .... degree.

(a) first (b) second (c) third (d) fourth

d The set of substitution is $\{1, 2, 3, 4\}$, then the set of solution of the equation $x + 6 = 10$ is .... (Alexandria 2016)

(a) $\{1\}$ (b) $\{2\}$ (c) $\{3\}$ (d) $\{4\}$

e If the substitution set is $\{2, -1, 3, 4\}$, then the solution set of the equation : $2x + 3 = 3$ is ....

(a) $\{0\}$ (b) $\{-1\}$ (c) $\{3\}$ (d) $\varnothing$

f The number which satisfies the inequality : $x < -1$ is ....

(a) zero (b) 1 (c) 2 (d) -2

g All the following numbers satisfy the inequality : $x > -3$ except ....

(a) zero (b) -4 (c) -1 (d) -2

h The greatest integer that satisfies the inequality : $x < 6$ is ....

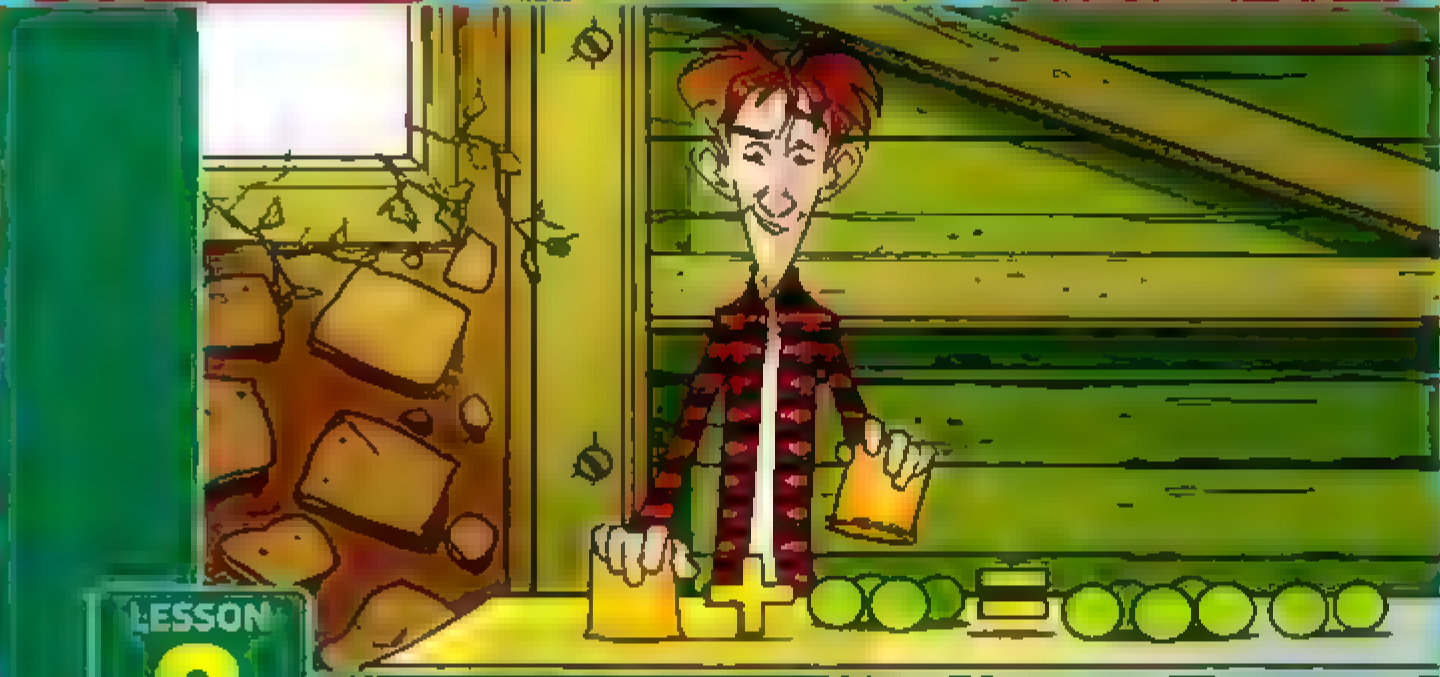
(a) 3 (b) 5 (c) 8 (d) 6

i The number that satisfies the inequality : $x - 2 > 3$ is ....

(a) 3 (b) 4 (c) 5 (d) 6

j If 3 is a solution to the equation : $2x - 4 = a$, then $a =$ ....

(a) 3 (b) 2 (c) -2 (d) -3



LESSON 2

Solving first degree equations in one unknown

Remember that :

$\bullet\ 2x+1=5$

$\bullet\ 3-x=-9$

1. The equation in which the highest exponent is 1 and has only 1 unknown is called **a first degree equation in one unknown , as ...**
2. Solving an equation means finding the value of the unknown that satisfies the equation , and this value is called **"solution"** of the equation.
3. In some cases , the equation can be solved easily by using the **substitution method**
4. It is impossible to solve an equation by using the substitution method , if the substitution set is **an infinite set.**

How to solve a first degree equation in one unknown without using the substitution method ?

The goal is to transform the equation so that the unknown is alone on one side of the equation and a constant term on the other. $(x = c)$

To do that, you need to know the following properties of the equations :

Properties of the equations

① An equation remains valid if the same number is added to both sides.

i.e. The same number can be added to each side of an equation without changing the solution of the equation.

For example :

If $x - 1 = 5$

, then $x - 1 + 1 = 5 + 1$

i.e. $x = 6$

② An equation remains valid if the same number is subtracted from both sides.

i.e. The same number can be subtracted from each side of an equation without changing the solution of the equation.

For example :

If $x + 3 = 2$

, then $x + 3 - 3 = 2 - 3$

i.e. $x = -1$

③ An equation remains valid if both sides are multiplied by the same number.

i.e. The same number can be multiplied by each side of an equation without changing the solution of the equation.

For example :

If $\frac{x}{5} = 2$

, then $\frac{x}{5} \times 5 = 2 \times 5$

i.e. $x = 10$

④ An equation remains valid if both sides are divided by the same non-zero number.

i.e. Each side of an equation can be divided by the same non-zero number without changing the solution of the equation.

For example :

If $7x = 14$

, then $\frac{7x}{7} = \frac{14}{7}$

i.e. $x = 2$

Generally

If a , b and c are three numbers, then we have the following properties :

① If $a = b$, then $a + c = b + c$

② If $a = b$, then $a - c = b - c$

③ If $a = b$, then $a \times c = b \times c$

④ If $a = b$, then $a \div c = b \div c$ where $c \neq 0$

LESSON 2

The following examples show how to use the previous equality properties to solve an equation of the first degree in one unknown :

Example 1

Find the solution set of each of the following equations in $\mathbb{Z}$:

[a] $x + 5 = 4$ [b] $5x = 30$ [c] $2x - 5 = 13$

Solution

[a] $\because x + 5 = 4$

(Subtracting 5 from each of the two sides)

$\therefore x + 5 - 5 = 4 - 5$

$\therefore x + 0 = -1$ $\therefore x = -1$

$\therefore$ The S.S. $= \{-1\}$

Check the solution :

Put $x = -1$ in the equation $x + 5 = 4$

L.H.S. $= -1 + 5 = 4 =$ R.H.S. ✓

Another method

You can imagine that 5 moved from L.H.S. to R.H.S. and became -5

$x + 5 = 4 \longrightarrow x = 4 - 5 = -1$

$x = -1$

[b] $\because 5x = 30$

(Dividing each of the two sides by 5)

$\therefore \frac{5x}{5} = \frac{30}{5}$ $\therefore x = 6$

$\therefore$ The S.S. $= \{6\}$

[c] $\because 2x - 5 = 13$

(Adding 5 to each of the two sides)

$\therefore 2x - 5 + 5 = 13 + 5$

$\therefore 2x = 18$

(Dividing each of the two sides by 2)

$\therefore \frac{2x}{2} = \frac{18}{2}$ $\therefore x = 9$

$\therefore$ The S.S. $= \{9\}$

Another method

You can imagine that 2 moved from L.H.S. to R.H.S. and became divisor.

$2x = 18 \longrightarrow x = \frac{18}{2} = 9$

$x = 9$

Example (2

Find in $\mathbb{Z}$ the solution set of the equation : $\frac{x}{2} - 5 = 3$

Solution

$\because \frac{x}{2} - 5 = 3$ (Adding 5 to each of the two sides) $\therefore \frac{x}{2} - 5 + 5 = 3 + 5$

$\therefore \frac{x}{2} = 8$ (Multiplying each of the two sides by 2) $\therefore \frac{x}{\cancel{2}} \times \cancel{2} = 8 \times 2$

$\therefore x = 16$ $\therefore$ The S.S. = $\{16\}$

Example (3

Find the solution set of the equation : $2x + 7 = 3$ in each of $\mathbb{N}$ and $\mathbb{Z}$

Solution

$\because 2x + 7 = 3$ (Subtracting 7 from each of the two sides)

$\therefore 2x + 7 - 7 = 3 - 7$

$\therefore 2x = -4$ (Dividing each of the two sides by 2)

$\therefore \frac{2x}{2} = \frac{-4}{2}$ $\therefore x = -2$

$\because x = -2 \notin \mathbb{N}$ $\therefore$ The S.S. in $\mathbb{N} = \varnothing$

, $\because x = -2 \in \mathbb{Z}$ $\therefore$ The S.S. in $\mathbb{Z} = \{-2\}$

Example (4

If we add a number to its double , we obtain 21 Find this number.

Solution

Let the number be x , then its double = $2x$

So, $x + 2x = 21$ $\therefore 3x = 21$ (Dividing each of the two sides by 3)

$\therefore \frac{3x}{3} = \frac{21}{3}$ $\therefore x = 7$ Then, the number is 7

Try by yourself

- Find the solution set of each of the following equations :

  [a] $x - 5 = 2$, where $x \in \mathbb{N}$ [b] $2x + 11 = 3$, where $x \in \mathbb{Z}$

- Two natural numbers, one of them is twice the other and their sum is 108 Find the two numbers.

Exercise 8

Solving first degree equations in one unknown

Interactive test

From the school book

1 Find the solution set of each of the following equations in $\mathbb{N}$:

a $x+3=7$

b $y+8=19$

c $m-7=4$

d $x-9=-5$

e $x+11=2$

f $8x=32$ (Luxor 2012)

g $4x=|-8|$ (Red Sea 2014)

h $\frac{x}{10}=2$

i $\frac{z}{12}=3$

2 Find the solution set of each of the following equations in $\mathbb{Z}$:

a $x+9=3$ (Giza 2017)

b $x-12=40$

c $x+8=0$

d $-4+x=-8$

e $n+17=|-13|$

f $9x=-18$ (Assiut 2015)

g $-4x=-24$

h $7-m=12$

i $y-(-5)=3$

3 Find the solution set of each of the following equations :

a $3x-2=7$, where $x\in\mathbb{Z}$ (Giza 2013)

b $4x+1=17$, where $x\in\mathbb{N}$ (Qena 2014)

c $6x+7=25$, where $x\in\mathbb{N}$

d $8x+12=4$, where $x\in\mathbb{Z}$

e $5x+2=-8$, where $x\in\mathbb{N}$

f $2x-5=-21$, where $x\in\mathbb{Z}$ (Beni Suef 2012)

g $5x+4=14$, where $x\in\mathbb{Z}$ (North Sinai 2017)

h $2x+9=-23$, where $x\in\mathbb{Z}$ (Kafr El-Sheikh 2016)

i $2y+16=2^4$, where $y\in\mathbb{N}$

j $3-2x=9$, where $x\in\mathbb{Z}$ (El-Fayoum 2015)

k $3x-2=-19$, where $x\in\mathbb{Z}$

l $-8-4x=16$, where $x\in\mathbb{N}$

m $\frac{x}{2}-4=7$, where $x\in\mathbb{N}$

4 Study the possibility of solving the following equations in $\mathbb{N}$ and $\mathbb{Z}$:

a $3x - 14 = |-16|$
b $2m + 12 = 6$
c $2x + 5 = -27$ (El Sharkia 2011)
d $2L - 15 = 8$
e $\frac{x}{7} = -5$
f $3x = 8$

5 Complete :

a If $x + 5 = 7$, then $x =$ ……
b If $5x = 10$, then $x =$ …… (Cairo 2013)
c If $3x - 3 = 12$, then $x =$ …… (Suez 2014)
d If $x + 8 = 11$, then $7x =$ ……
e If $2y = 8$, then $y + 3 =$ …… (El-Dakahlia 2016)
f If $3y = 6$, then $5y =$ ……
g If $4x = 24$, then $\frac{x}{3} =$ ……
h If $2a + 3 = 15$, then $\frac{1}{3}a =$ ……
i If $(x + 1)$ is the additive inverse of (-2) , then $x =$ …… (El-Dakahlia 2012)
j The S.S. of the equation $x - 5 = 2^4$ in $\mathbb{Z}^-$ is ……
k The S.S. of the equation $x - 3 = (6)^0$ in $\mathbb{Z}$ is ……

6 Choose the correct answer from those given :

a The solution set of the equation $x + 5 = 2$ in $\mathbb{Z}$ is {……} (Alexandria 2011)
(a) 7 (b) -7 (c) 3 (d) -3

b If $x + 3 = 5$, $x \in \mathbb{Z}^-$, then the solution set is …… (Damietta 2013)
(a) $\{-3\}$ (b) $\{5\}$ (c) $\{-5\}$ (d) $\varnothing$

c If zero $\in \{5 , x - 3\}$, then $x =$ …… (El Menofia 2017)
(a) zero (b) -5 (c) 3 (d) -3

d If $3x + 9 =$ zero , then the solution set of the equation in $\mathbb{Z}$ is …… (El-Fayoum 2012)
(a) $\{9\}$ (b) $\{-9\}$ (c) $\{3\}$ (d) $\{-3\}$

LESSON **2**

e The S.S. of the equation $4x = -16$ in $\mathbb{N}$ is ......

(a) $\varnothing$ (b) $\{-4\}$ (c) $\{0\}$ (d) $\{4\}$

f If $x + 2 = |-4|$, then $x =$ ......

(a) -2 (b) 2 (c) -6 (d) 6

g If $|-4| \times x = 64$, then $x =$ ......

(a) -16 (b) 16 (c) 6 (d) 8

h If $2x = 2$, then $3x - 1 =$ ......

(a) 2 (b) 3 (c) 4 (d) 5

i If $2x = 0$, then $x =$ ......

(a) 2 (b) 3 (c) 5 (d) zero

j If $\frac{x}{5} = 4$, then $x =$ ......

(a) 1 (b) 9 (c) 20 (d) -1

7 Number when added to its triple becomes 72 Find the number.

8 Three consecutive natural numbers whose sum is 213

What are these numbers ?

For Excellent Pupils

9 Find the solution set of the equation :

$3x - 1 = 2x + 5$, where $x \in \mathbb{N}$

10 Find the solution set of the equation :

$2(x + 3) = 4$, where $x \in \mathbb{Z}$

LESSON 3

Solving first degree inequality in one unknown

Properties of the inequalities

Inequalities have many properties in common with equations

The following properties will be used in solving first degree inequalities in one unknown .

1 The same number can be added to each side of an inequality without changing the solution of the inequality.

2 The same number can be subtracted from each side of an inequality without changing the solution of the inequality.

For example :

If $x - 1 > 3$

, then $x - 1 + 1 > 3 + 1$

(By adding 1 to each side)

i.e. $x > 4$ and the S.S. (in $\mathbb{Z}$) is $\{5 , 6 , 7 , 8 , ...\}$

If $x + 1 < 5$

, then $x + 1 - 1 < 5 - 1$

(By subtracting 1 from each side)

i.e. $x < 4$ and the S.S (in $\mathbb{Z}$) is $\{3 , 2 , 1 , 0 , -1 , -2 , ...\}$

LESSON 3

3. The same positive number can be multiplied by each side of an inequality without changing the solution of the inequality.

4. Each side of an inequality can be divided by the same positive number without changing the solution of the inequality.

For example :

If $\frac{x}{2} > -2$

, then $\frac{x}{2} \times 2 > -2 \times 2$

(By multiplying each side by 2)

i.e. $x > -4$ and the S.S. (in $\mathbb{Z}$) is $\{-3, -2, -1, 0, 1, 2, ...\}$

If $3x < -15$

, then $3x \div 3 < -15 \div 3$

(By dividing each side by 3)

i.e. $x < -5$ and the S.S. (in $\mathbb{Z}$) is $\{-6, -7, -8, -9, -10, ...\}$

5. If we multiply or divide each side of an inequality by a negative number we must revers the inequality.

For example :

$-2x > 6$ $(\div -2)$

revers

$x < -3$

If $-2x > 6$, then $-2x \div -2 < 6 \div -2$

(By dividing each side by -2)

i.e. $x < -3$ and the S.S. (in $\mathbb{Z}$) is $\{-4, -5, -6, -7, -8, ...\}$

Generally

Assuming that a, b and c are three integers, then :

1. If $a < b$, then $a + c < b + c$
2. If $a < b$, then $a - c < b - c$
3. If $a < b$, c is a positive number, then $ac < bc$
4. If $a < b$, c is a positive number, then $\frac{a}{c} < \frac{b}{c}$
5. If $a < b$, c is a negative number, then $ac > bc$
6. If $a < b$, c is a negative number, then $\frac{a}{c} > \frac{b}{c}$

Example (1)

Find in $\mathbb{Z}$ the solution set of each of the following inequalities :

[a] $x + 7 < 9$ [b] $2x - 5 \geq 5$

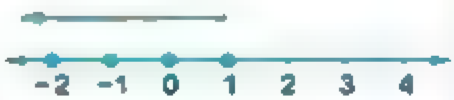
then represent the solution set on the number line.

Solution

[a] $\because x + 7 < 9$ (Subtracting 7 from each of the two sides)

$\therefore x + 7 - 7 < 9 - 7$ $\therefore x < 2$

$\therefore$ The S.S. $= \{1 , 0 , -1 , -2 , -3 , \cdots\}$



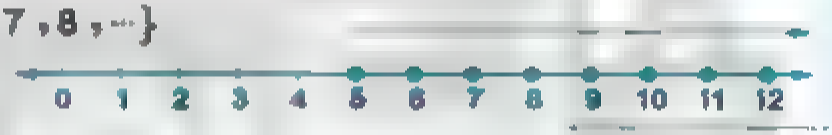
[b] $\because 2x - 5 \geq 5$ (Adding 5 to each of the two sides)

$\therefore 2x - 5 + 5 \geq 5 + 5$

$\therefore 2x \geq 10$ (Dividing each of the two sides by 2)

$\therefore \frac{2x}{2} \geq \frac{10}{2}$ $\therefore x \geq 5$

$\therefore$ The S.S. $= \{5 , 6 , 7 , 8 , \cdots\}$



Example (2)

Find the solution set of the inequality : $2x + 5 < 11$, where :

[a] $x \in \mathbb{Z}$ [b] $x \in \mathbb{N}$

then represent the S.S. on the number line in each case.

Solution

$\because 2x + 5 < 11$ (Subtracting 5 from each of the two sides)

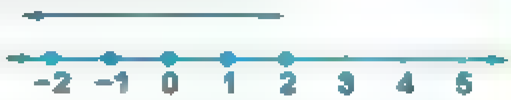
$\therefore 2x + 5 - 5 < 11 - 5$ $\therefore 2x < 6$ (Dividing each of the two sides by 2)

$\therefore \frac{2x}{2} < \frac{6}{2}$ $\therefore x < 3$

[a] When $x \in \mathbb{Z}$:

The solution set is all the integers which are less than 3

i.e. The S.S. $= \{2 , 1 , 0 , -1 , -2 , \cdots\}$



[b] When $x \in \mathbb{N}$:

The solution set is all the natural numbers which are less than 3

i.e. The S.S. $= \{2 , 1 , 0\}$

LESSON 3

Example 3

Find the S.S. of the inequality : $4 - 2x \geq 10$, where :

[a] $x \in \mathbb{N}$ [b] $x \in \mathbb{Z}$

Solution

$\because 4 - 2x \geq 10$ (Subtracting 4 from each of the two sides)

$\therefore 4 - 2x - 4 \geq 10 - 4$

$\therefore -2x \geq 6$ (Dividing each of the two sides by -2)

$\therefore \frac{-2x}{-2} \leq \frac{6}{-2}$

$\therefore x \leq -3$

Notice that : The change of the direction of the inequality sign.

[a] When $x \in \mathbb{N}$:

The S.S. is all the natural numbers which are less than or equal to -3

i.e. The S.S. $= \varnothing$

[b] When $x \in \mathbb{Z}$:

The S.S. is all the integers which are less than or equal to -3

i.e. The S.S. $= \{-3, -4, -5, \dots\}$

Try

- Find the solution set of each of the following inequalities :

[a] $2x - 3 \geq 5$, where $x \in \mathbb{Z}$

[b] $1 - 2x > 7$, where $x \in \mathbb{Z}$

Remark

The value of x can be between two integers , so the inequality can be written on one of the following forms :

1. $1 < x < 5$

, that means : $x > 1$ and $x < 5$

Therefore : $x = 2$ or 3 or 4

2. $-2 \leq x \leq 3$

, that means : $x \geq -2$ and $x \leq 3$

Therefore : $x = -2$ or -1 or 0 or 1 or 2 or 3

3. $-3 < x \leq 1$

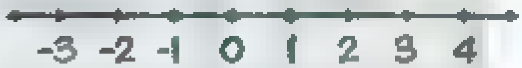
, that means : $x > -3$ and $x \leq 1$

Therefore : $x = -2$ or -1 or 0 or 1

4. $-1 \leq x < 2$

, that means : $x \geq -1$ and $x < 2$

Therefore : $x = -1$ or 0 or 1



Example (4)

Find in $\mathbb{Z}$ the solution set of the inequality : $-7 \leq 2x - 5 < 1$

Solution

$\because -7 \leq 2x - 5 < 1$ (Adding 5 to each of the three sides)

$\therefore -7 + 5 \leq 2x - 5 + 5 < 1 + 5$

$\therefore -2 \leq 2x < 6$ (Dividing each of the three sides by 2)

$\therefore \frac{-2}{2} \leq \frac{2x}{2} < \frac{6}{2}$ $\qquad \therefore -1 \leq x < 3$

$\therefore$ The S.S. $= \{-1 , 0 , 1 , 2\}$

Exercise 9

Solving first degree inequality in one unknown

Interactive test

From the school book

1 Find in the solution set of each of the following inequalities , then represent the solution set on the number line :

a $x - 3 < 1$ (El-Fayoum 2015)
b $x + 2 > 5$
c $x + 4 > 1$
d $x + 4 < 7$ (Port Said 2016)
e $x + 3 \geq 6$
f $x - 4 \leq -1$
g $19 < a + 14$
h $m - 5 \geq |-7|$
i $-1 \geq x + 3$
j $3x < 12$
k $4k \geq -16$
l $-2y < -14$

2 Find the solution set of the inequality : $x + 3 \leq 6$, where :

a $x \in \mathbb{Z}$
b $x \in \mathbb{N}$

Then represent the solution set on the number line.

3 Find the S.S. of each of the following inequalities , then represent the S.S. on the number line :

a $2x + 1 < 7$, where $x \in \mathbb{N}$
b $2x - 3 < 5$, where $x \in \mathbb{Z}$ (Qena 2013)
c $3x - 2 < 1$, where $x \in \mathbb{N}$
d $2x + 9 < 1$, where $x \in \mathbb{Z}$ (El-Sharkia 2016)
e $4x + 2 \geq -10$, where $x \in \mathbb{Z}$
f $3x + 9 > 0$, where $x \in \mathbb{N}$
g $3x - 5 \leq 4$, where $x \in \mathbb{Z}$ (South Sinai 2017)
h $2x - 5 \leq -7$, where $x \in \mathbb{Z}$
i $4x + 1 < 13$, where $x \in \mathbb{Z}$ (El-Kalyoubia 2017)
j $3x + 2 \leq 11$, where $x \in \mathbb{N}$ (Red Sea 2016)

k $9 - 6x < 15$, where $x \in \mathbb{Z}$

l $1 + 2x \leq -3$, where $x \in \mathbb{N}$

m $3x + 2 \geq 12$, where $x \in \mathbb{N}$

n $3x - 5 \leq 7$, where $x \in \mathbb{Z}^{+}$ (Cairo 2017)

o $1 - 8x < 33$, where $x \in \mathbb{Z}$ (Suez 2015)

4 Find the solution set of the inequality : $3x + 5 < 2$ and represent it on the number line if :

a $x \in \mathbb{N}$ b $x \in \mathbb{Z}$ (South Sinai 2012)

5 Find the S.S. of each of the following inequalities , then represent the S.S. on the number line :

a $3 < x + 2 \leq 6$, where $x \in \mathbb{N}$

b $-3 \leq x - 1 < 3$, where $x \in \mathbb{Z}$

c $3 < 2x + 1 \leq 9$, where $x \in \mathbb{N}$

d $5 \leq 1 - 2x \leq 11$, where $x \in \mathbb{Z}$

6 Complete :

a If $x + 5 > 2$, then $x >$ .... (Alexandria 2015)

b If $3x - 1 \leq 8$, then : $3x \leq$ .... , $x \leq$ ....

c The S.S. of the inequality : $4x < 8$ in $\mathbb{N}$ is ....

d The S.S. of the inequality : $2x - 3 < 5$ in $\mathbb{Z}$ is ....

e The S.S. of the inequality : $1 - x > 4$ in $\mathbb{N}$ is ....

f The S.S. of the inequality : $-2 < x \leq 0$ in $\mathbb{N}$ is ....

g If $b < 0$, then $b + 3$ 3

7 Choose the correct answer :

a The solution set of the inequality : $x > 0$ in $\mathbb{Z}$ is .... (Ismailia 2012)

(a) $\mathbb{Z}$ (b) $\mathbb{Z}^{+}$ (c) $\mathbb{Z}^{-}$ (d) $\mathbb{N}$

LESSON 3

b The S.S. of the inequality : $-2x < 0$ in $\mathbb{Z}$ is .....

(a) $\varnothing$ (b) $\mathbb{N}$ (c) $\mathbb{Z}^-$ (d) $\mathbb{Z}^+$

c If $x \in \mathbb{N}$, then the S.S. of the inequality : $-x > 3$ is .....

(a) $\{4, 5, 6, \cdots\}$ (b) $\{-4, -5, -6, \cdots\}$

(c) $\{-3\}$ (d) $\varnothing$

d If $2x + 5 > 3$ and $x \in \mathbb{Z}$, then the solution set = ... (Damietta 2012)

(a) $\mathbb{N}$ (b) $\mathbb{N} - \{0\}$ (c) $\mathbb{Z}^-$ (d) $\mathbb{Z}^+$

e The S.S. of the inequality : $4 - x > 3$ in $\mathbb{Z}^+$ is .....

(a) $\{0, -1, -2, -3, \cdots\}$ (b) $\{0, 1, 2, 3, \cdots\}$

(c) $\{0\}$ (d) $\varnothing$

f The S.S. of the inequality : $-1 \leq x < 1$ in $\mathbb{Z}$ is ..... (South Sinai 2013)

(a) $\{-1, 0\}$ (b) $\{0, 1\}$ (c) $\{0\}$ (d) $\{1\}$

g The solution set of the inequality : $2 \leq x < 3$ where $x \in \mathbb{N}$ is (Damietta 2017)

(a) {zero} (b) $\{2\}$ (c) $\{3\}$ (d) $\{2, 3\}$

h If $x > 5$, then : $-x$ ....

(a) < -9 (b) ≥ -5 (c) < -5 (d) > -5

i The greatest integer that satisfies the inequality : $3 \leq x < 6$ is .... (Aswan 2015)

(a) 3 (b) 4 (c) 5 (d) 6

j 2 belongs to the S.S. of the inequality : .... , where $x \in \mathbb{Z}$

(a) $x > 2$ (b) $x < 2$ (c) $-x > -3$ (d) $-x > 3$

For Excellent Pupils

8 Find the solution set of the inequality :

$2x - 1 \leq x + 3$, where $x \in \mathbb{Z}$

Test on Unit Two

Answer the following questions :

1 Choose the correct answer :

a Which of the following represents an equation ?

($x + 8$ *or* $10 - 4 = 6$ *or* $x > 9$ *or* $x + 1 = 5$)

b The equation $x^2 + x = -2$ is an equation of .... degree.

(first *or* second *or* third *or* fourth)

c All the following numbers satisfy the inequality : $x > -2$ except ....

(-1 *or* 0 *or* 1 *or* -3)

d If $x + 4 = 7$, then $2x =$ .... (3 *or* 4 *or* 6 *or* -3)

e If $a < b$, then $-2a$ .... $-2b$ ($>$ *or* $<$ *or* $=$ *or* $\leq$)

2 a Determine the degree of each of the following :

(1) $x^3 + 8 = 0$ (2) $2x - 1 > 7$

b Express symbolically each of the following :

(1) x is less than or equal to -4

(2) x is greater than -3 and smaller than 7

3 a Find the solution set of the equation :

$3x + 2 = 8$ if the substitution set is $\{-1 , 0 , 1 , 2\}$

b Find the solution set of each of the following equations :

(1) $4x - 3 = 5$, where $x \in \mathbb{N}$

(2) $2x + 7 = 1$, where $x \in \mathbb{Z}$

TEST

4 a Find the solution set of the inequality :

$3x + 2 \leq 2$ if the substitution set is $\{-2, -1, 0, 1\}$

b Find the solution set of each of the following inequalities , then represent the solution set on the number line :

(1) $5x - 1 > -11$ where $x \in \mathbb{Z}$

(2) $-1 \leq 2x + 3 < |-7|$ where $x \in \mathbb{Z}$

5 a Find the solution set of each of the following in $\mathbb{N}$ and in $\mathbb{Z}$:

(1) $3x + 1 = -5$

(2) $4x - 9 < -1$

b A number when added to its double becomes 15 Find the number.

Activity of Unit Two

① Use Excel program to find the solution set of the equation : $4x - 3 = 17$ if the substitution set is $\{2, 5, 6, 7\}$

② Below each balance, express the suitable mathematical statement, then solve it in $\mathbb{N}$:

[a]

- The mathematical statement ....

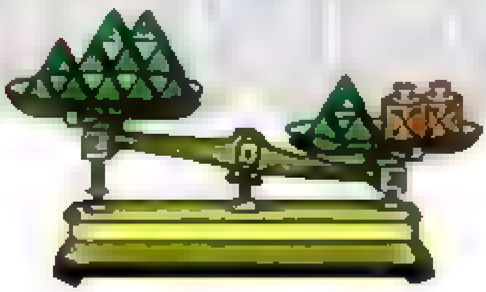
- The solution set ....

[b]

- The mathematical statement ....

- The solution set ....

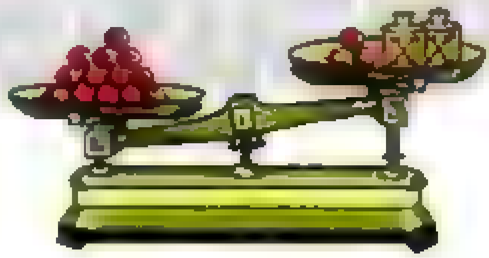
[c]



- The mathematical statement ....

- The solution set ....

[d]



- The mathematical statement ....

- The solution set ....

A research project

on unit two

Project aims

- Write a mathematical relation which relates two variables.
- Solve an equation by finding the value of the unknown which satisfies the equation.
- Form an equation from a word sentence and solve it.
- Linking mathematics with history.

Do a research project on the following topic

"Mustafa Kamel is one of the most important leaders of the nationalist movement in Egypt in the 19th century".

Discuss the following points using available resources

- Write, in Arabic, a short essay about Mustafa Kamel's fight against the occupation.
- A school trip was planned to Mustafa Kamel Museum. Find the mathematical relation between (y), which is the total cost of trip and (x), which is the number of pupils taking part, supposing one admission ticket costs 50 pounds and the transport cost is 500 pounds for all pupils.

  If the total cost of this trip is 1500 pounds, find the number of pupils participating in this trip

UNIT
3

Geometry and measurement

Lessons of the unit :

Test on unit three

Scan the QR code to solve an interactive test on each lesson.

UNIT AIMS

By the end of this unit, student should be able to:

- find the distance between two points on a ray and on a straight line.
- graph the points in the coordinate plane.
- find the distance between two points in the coordinate plane.
- remember the geometric transformations (Reflection – Translation – Rotation)
- recognize the definition of the translation.
- find the image of each of a point, a line segment and geometric shape by a given translation in the plane.
- find the image of each of a point, a line segment and a geometric shape by a given translation in the coordinate plane.
- find the area of the circle.
- find the lateral area and the total area for each of the cube and the cuboid.
- solve word problems on the area of the circle and the lateral area and the total area for each of the cube and the cuboid.

LESSON 1

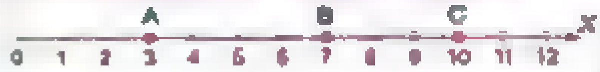
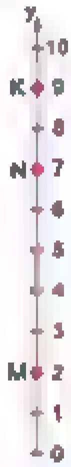
Distance between two points in the coordinates plane

A The distance between two points on a ray

The distance between the two points A and B on a horizontal ray or vertical ray = AB where :

The length of $\overline{AB}$ = number of the ending point – number of the starting point
= B – A

For example :

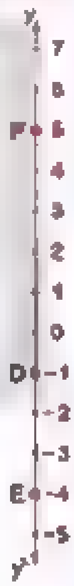
[1] In the following figure :	[2] In the following figure :
 If A represents the number 3 , B represents the number 7 and C represents the number 10 , then AB = B – A = 7 – 3 = 4 units. , BC = C – B = 10 – 7 = 3 units. and AC = C – A = 10 – 3 = 7 units.	If M represents the number 2 , N represents the number 7 and K represents the number 9 , then MN = N – M = 7 – 2 = 5 units. , NK = K – N = 9 – 7 = 2 units. and MK = K – M = 9 – 2 = 7 units. 

B The distance between two points on a straight line

- The straight line which mentioned here is the integers number line whether horizontal or vertical. As you know it is an enlargement of the ray of the natural numbers by adding $\mathbb{Z}^{-}$
- The distance between the two points A and B on a horizontal line or a vertical line = AB where :

The length of $\overline{AB}$ = | number of the ending point – number of the starting point |
= | B – A |

For example :

[1] In the following figure :	[2] In the following figure :
 If A represents the number – 3 , B represents the number 4 and C represents the number 7 , then AB = | 4 – (– 3) | = | 4 + 3 | = 7 units. , BC = | 7 – 4 | = 3 units. and AC = | 7 – (– 3) | = | 7 + 3 | = 10 units.	 If E represents the number – 4 , D represents the number – 1 and F represents the number 5 , then ED = | – 1 – (– 4) | = | – 1 + 4 | = 3 units. , DF = | 5 – (– 1) | = | 5 + 1 | = 6 units. and EF = | 5 – (– 4) | = | 5 + 4 | = 9 units.

LESSON **1**

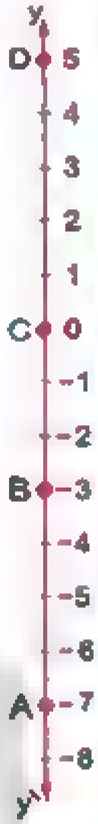
Example (1

In the opposite figure :

If the points A , B , C and D represent

the numbers -7 , -3 , 0 and 5 respectively.

Find : AB , AC , BD , CD , AD and BC



Solution

$AB = |B - A| = |-3 - (-7)| = |-3 + 7| = |4| = 4$ units.

$, AC = |C - A| = |0 - (-7)| = |0 + 7| = |7| = 7$ units.

$, BD = |D - B| = |5 - (-3)| = |5 + 3| = |8| = 8$ units.

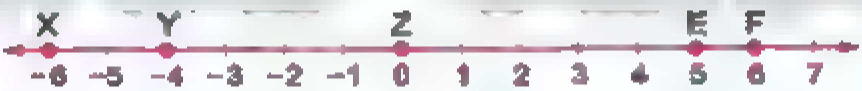
$, CD = |D - C| = |5 - 0| = |5| = 5$ units.

$, AD = |D - A| = |5 - (-7)| = |5 + 7| = |12| = 12$ units.

and $BC = |C - B| = |0 - (-3)| = |0 + 3| = |3| = 3$ units.

Try

- **From the following figure , complete :**



[a] XY = ...

[b] XZ = ...

[c] YE = ...

[d] XF = ...

[e] ZE = ...

[f] YF = ...

Graphing points in the coordinate plane

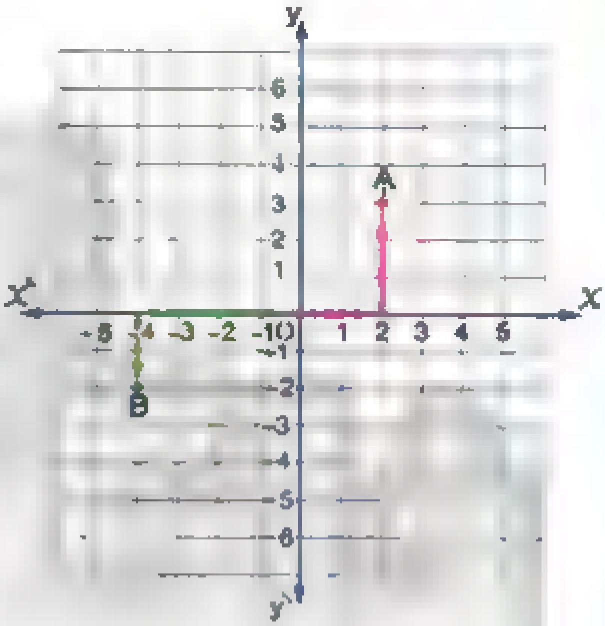
The position of any point in the coordinate plane is determined by a unique ordered pair.

For example :

[1] To graph the point A (2 , 3) ,

follow the following steps :

- Start at 0
- Move 2 units to the right.
- Then , move 3 units up.



[2] To graph the point B (– 4 , – 2) ,

follow the following steps :

- Start at 0
- Move 4 units to the left.
- Then , move 2 units down.

Remarks

1. The horizontal line $\overleftrightarrow{xx'}$ is called the "x-axis"
2. The vertical line $\overleftrightarrow{yy'}$ is called the "y-axis"

The distance between two points in the coordinate plane

To calculate the distance between two points in the coordinate plane , do as follows :

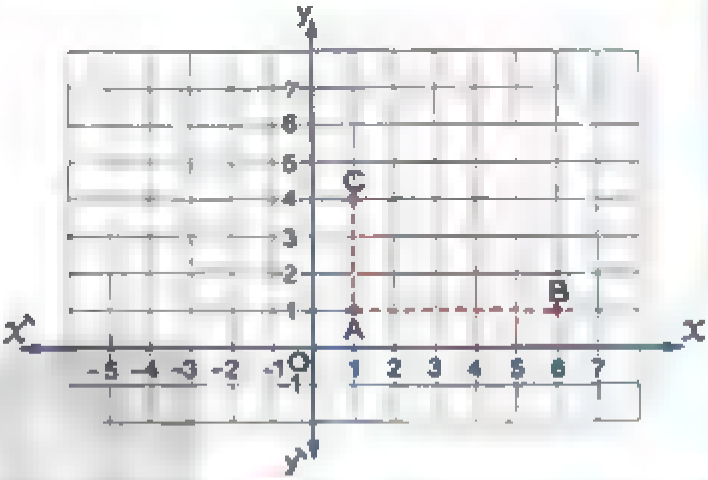
[1] Determine the line segment joining between them.

[2] If it is parallel to $\overrightarrow{OX}$ (or $\overleftrightarrow{xx'}$) , calculate the distance as on a horizontal ray (or a straight line) and if it is parallel to $\overrightarrow{OY}$ (or $\overleftrightarrow{yy'}$) , calculate the distance as on a vertical ray (or a straight line).

LESSON 1

For example :

[1] **In the opposite figure :**



(1) $\overline{AB} \parallel \overrightarrow{OX}$

$\therefore AB = B - A$

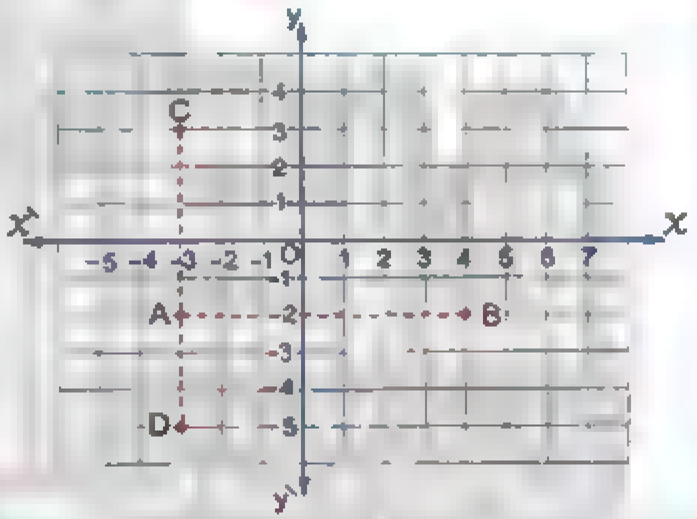
$= 6 - 1 = 5$ units.

(2) $\overline{AC} \parallel \overrightarrow{OY}$

$\therefore AC = C - A$

$= 4 - 1 = 3$ units.

[2] **In the opposite figure :**



(1) $\overline{AB} \parallel \overleftrightarrow{xx}$

$\therefore AB = |B - A| = |4 - (-3)|$

$= |4 + 3| = 7$ units.

(2) $\overline{DC} \parallel \overleftrightarrow{yy}$

$\therefore DC = |C - D| = |3 - (-5)|$

$= |3 + 5| = 8$ units.

Example 2

In the coordinate plane :

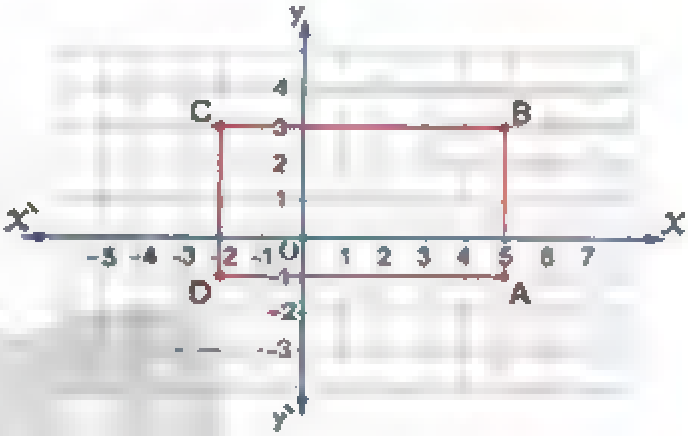
[1] Determine the position of the following points : A (5 , – 1) , B (5 , 3) , C (– 2 , 3) , D (– 2 , – 1) and mention the name of the figure ABCD

[2] Find the area and the perimeter of the figure.

[3] Determine whether the shape is symmetric or not ?

Solution

[1] The figure ABCD is a rectangle.



[2] AB = | B – A | = | 3 – (– 1) |

= | 3 + 1 | = | 4 | = 4 units.

∴ AB = CD = 4 units.

, DA = | A – D | = | 5 – (– 2) |

= | 5 + 2 | = | 7 | = 7 units.

∴ DA = BC = 7 units.

∴ The area of the figure = L × W = 7 × 4 = 28 square units.

, the perimeter of the figure = (L + W) × 2 = (7 + 4) × 2 = 22 units.

[3] The shape is symmetric.

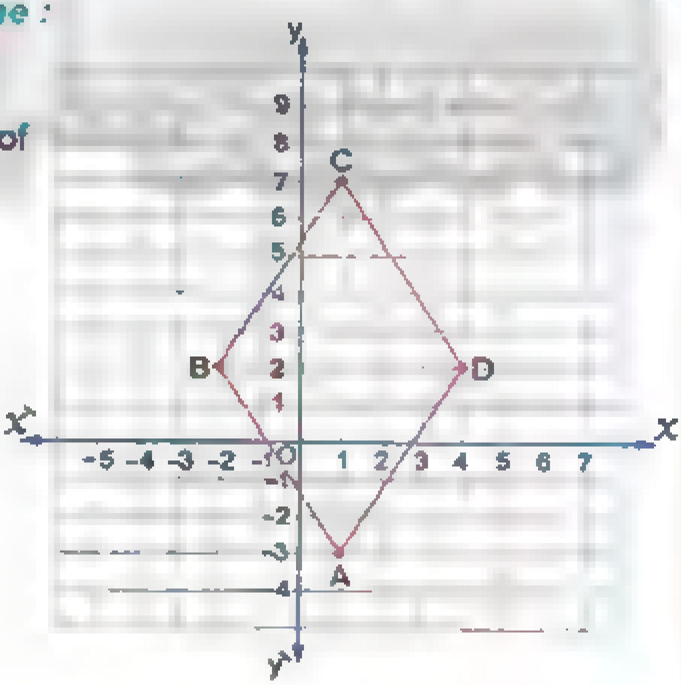
Example (3

In the opposite coordinate plane :

ABCD is a rhombus.

[1] Find the coordinates of each of the points A , B , C and D

[2] Find the area of the rhombus ABCD



Solution

[1] A (1 , – 3) , B (– 2 , 2) , C (1 , 7) and D (4 , 2)

LESSON 1

[2] • The length of $\overline{AC} = |C - A| = |7 - (-3)| = |7 + 3| = |10| = 10$ units.

• The length of $\overline{BD} = |D - B| = |4 - (-2)| = |4 + 2| = |6| = 6$ units.

• The area of the rhombus ABCD $= \frac{1}{2} \times AC \times BD$

$= \frac{1}{2} \times 10 \times 6 = 30$ square units.

Try

• **In the coordinate plane , determine the position of each of the following points : A (– 1 , – 3) , B (4 , – 3) and C (– 1 , 2) , then complete :**

[a] AB = ... units. , AC = ... units.

[b] The type of the triangle ABC with respect to its side lengths is .... , the type of the triangle with respect to its angles is ....

[c] The area of $\triangle$ ABC = ... square units.

Exercise 10

Distance between two points in the coordinates plane

Interactive test

From the school book

1 From the following figure, complete :

a EF = ...

b EG = ...

c EH = ...

d EK = ...

e FG = ...

f FH = ...

g FK = ...

h GK = ...

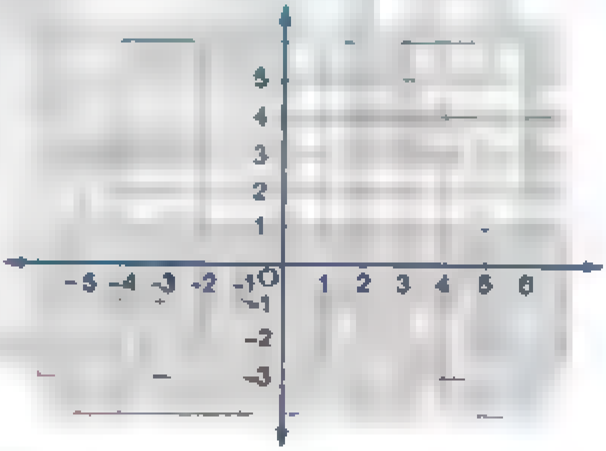
i HK = ...

2 In the opposite coordinate plane :

Locate the points A (0 , 4) ,

B (2 , 1) and C (– 2 , 1) , then

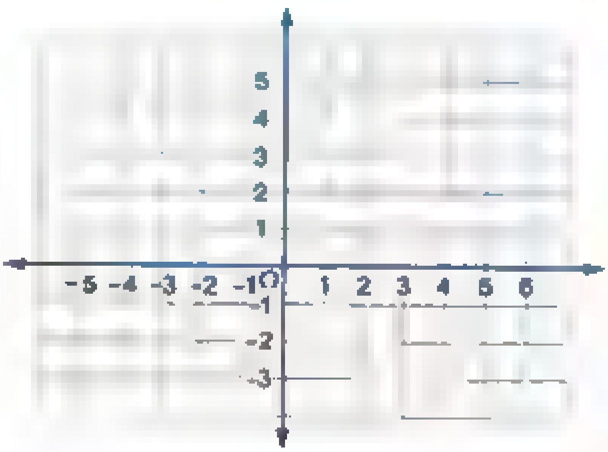
Find the length of $\overline{BC}$ *(Giza 2017)*



3 In the opposite coordinate plane :

a Determine the position of the following points : A (– 3 , – 3) , B (– 3 , 2) , C (5 , 2) and D (5 , – 3) and mention the name of the shape ABCD

b Find the perimeter and the area of the shape ABCD



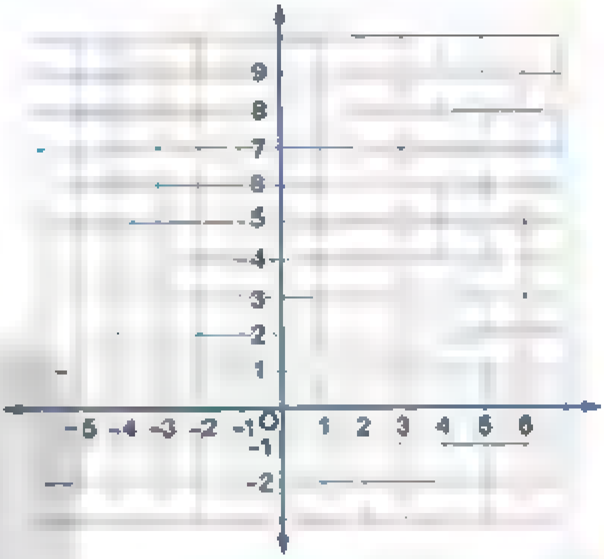
LESSON 1

4 In the opposite coordinate plane :

a Determine the position of the following points :
L (– 1 , 1) , M (1 , 1) , N (1 , 8) and E (– 1 , 8)

b Find the perimeter and the area of the shape LMNE

c Determine whether the shape is symmetric or not ? Why ?



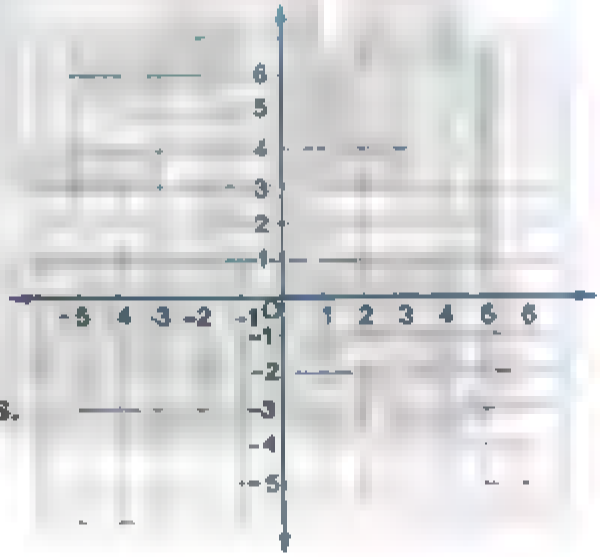
5 In the opposite coordinate plane

Determine the position of the following points :
A (– 1 , – 4) , B (– 1 , 3) and C (5 , – 4) , then find :

a The length of each of $\overline{AB}$ and $\overline{AC}$

b The type of the triangle ABC with respect to its sides and angles.

c The area of Δ ABC



5 In the opposite coordinate plane ,

ABCD is a rhombus.

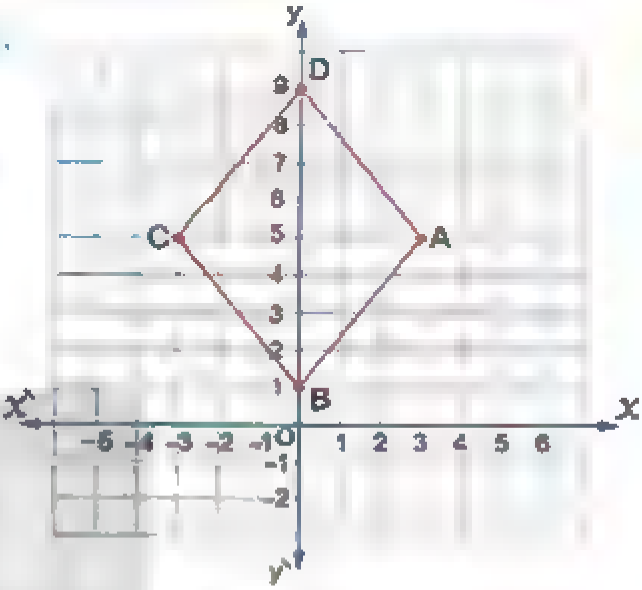
a Complete the coordinates of the following points :

A (...... , ...)

, B (...... , ...)

, C (...... , ...)

and D (...... , ...) (Suez 2015)



b The area of the rhombus ABCD can be calculated by using the length of its perpendicular diagonals , where :

The length of $\overline{AC}$ = ...

, the length of $\overline{BD}$ = ...

The area of the rhombus = ...

6 In the opposite coordinate plane :

ABCD is a square , then complete :

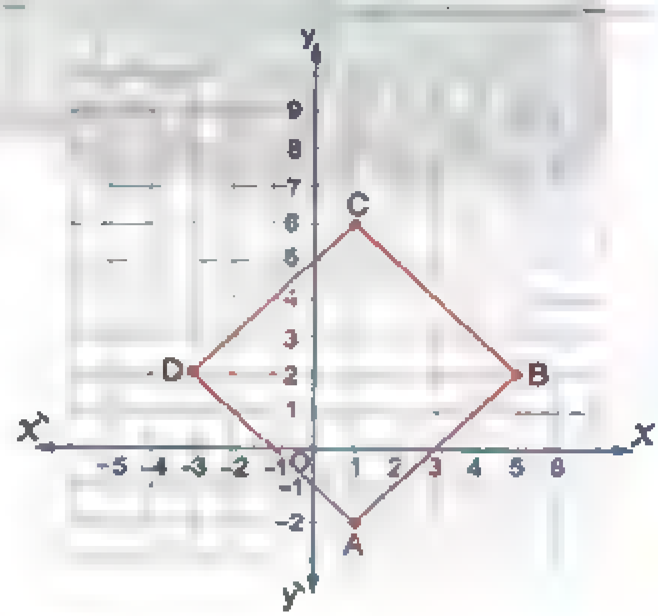
a The coordinates of the points :

A (...... , ...)

, B (...... , ...)

, C (...... , ...)

and D (...... , ...)



b The length of $\overline{AC}$ = ...

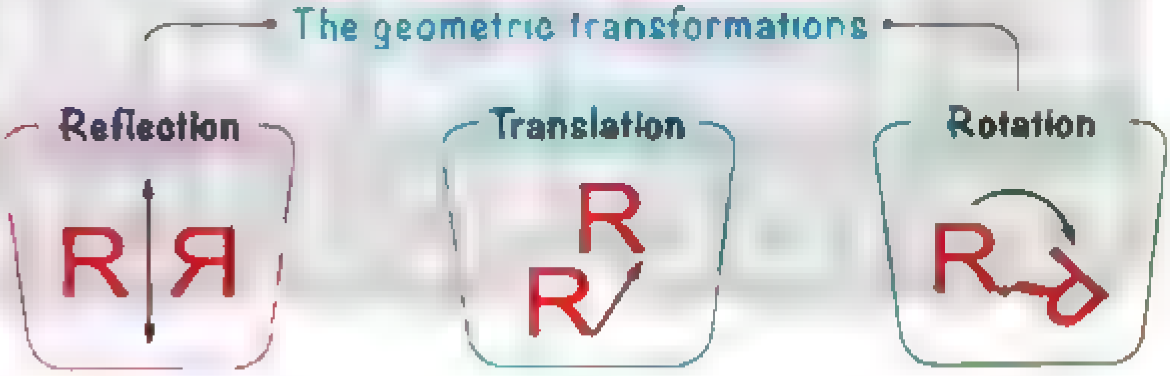
, the length of $\overline{DB}$ = ...

c The area of the square ABCD = ...

LESSON **2**

Geometric transformations (Translation)

- You have known that , there are three types of the geometric transformations which are shown in the following diagram :



Translation

- You have studied last year "The reflection" and a general idea about geometric transformations.
- Now , you will study with more details "**The translation**".

Preface

If a car moved in a straight line a distance of 25 metres forward. In this case , we can describe the movement of the car as a translation of 25 metres forward. This movement changed the place of the car without changing its orientation.

i.e. **To determine the new position of the car after its movement , we should know two important elements which are :**

1. The magnitude of the translation (25 metres).
2. The direction of the translation (forward in a straight line).

Definition : The translation is a geometric transformation which slides a shape from a place to another place (image) such as every point of the original shape moves the same distance in the same direction to form the image.

First

A Finding the image of a point by a given translation

- **To find À which is the image of A by translation MN in the direction of $\overline{MN}$, we do as follows :**

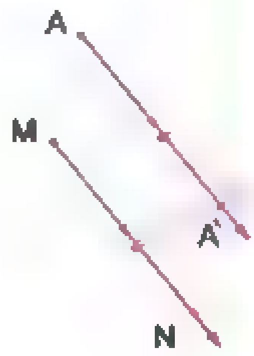
STEP 1

Draw from A a ray parallel to $\overline{MN}$ and in the same direction.

LESSON 2

STEP 2

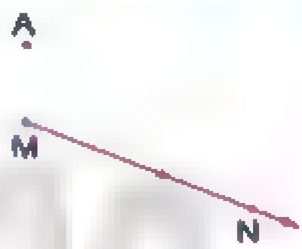
By the compasses in A as a centre with radius length = MN , draw an arc to intersect the ray drawn from A at the point À ($A\grave{A}$ = MN and $\overline{A\grave{A}} \parallel \overline{MN}$)



Then : À is the image of A by translation of magnitude MN in the direction of $\overrightarrow{MN}$

Example (1)

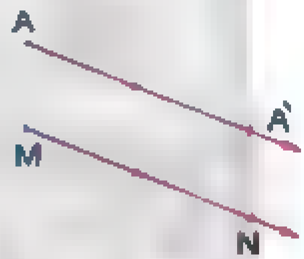
Find the image of the point A by translation of magnitude 3 cm. in the direction of $\overrightarrow{MN}$



Solution

[1] Draw from A a ray parallel to $\overrightarrow{MN}$ and in the same direction of $\overrightarrow{MN}$

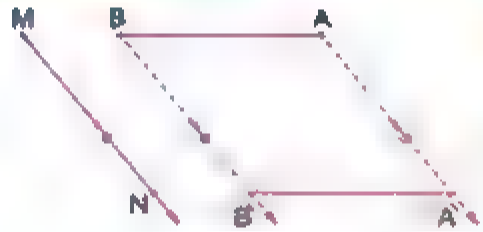
[2] By the compasses in A as a centre with radius length = 3 cm. , draw an arc to intersect the ray drawn from A at the point À



Then : À is the image of A by translation of magnitude 3 cm. in the direction of $\overrightarrow{MN}$

B Finding the image of a line segment by a given translation

- To find the image of $\overline{AB}$ by translation MN in the direction of $\overrightarrow{MN}$, we do as follows :



STEP 1
Find the image of the point A by translation MN in the direction of $\overrightarrow{MN}$ as we mentioned before , say A'

STEP 2
Similarly , we find the image of the point B by translation MN in the direction of $\overrightarrow{MN}$, say B'

STEP 3
Draw $\overline{A'B'}$ to be the image of $\overline{AB}$ by translation MN in the direction of $\overrightarrow{MN}$

Check that : $AB = A'B'$ and $\overline{AB} \parallel \overline{A'B'}$

C Finding the image of a geometric shape by a given translation

- **To find the image of a geometric shape by a given translation, translate each vertex by the same given translation as shown before in (A).**

The opposite figure shows the image of a triangle by a certain translation.

Every point of the shape must move :

- The same distance.
- In the same direction.

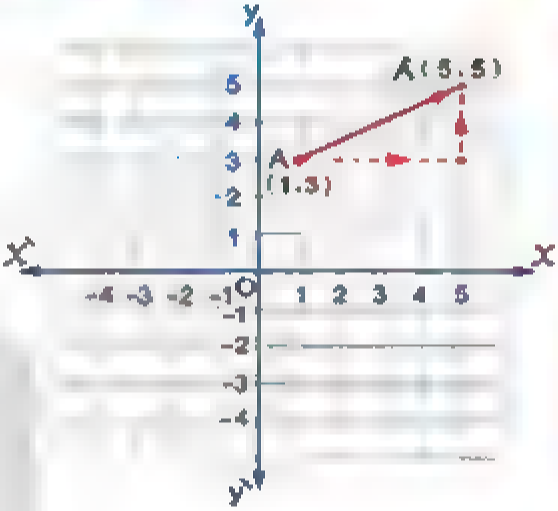
LESSON 2

Second Translation in the coordinates plane

In the opposite graph :

If A (1 , 3) is a point in the orthogonal coordinates plane and to find its image À by translation with magnitude 4 length units in the direction of $\overrightarrow{OX}$ followed by a translation with magnitude 2 length units in the direction of $\overrightarrow{OY}$, we get À to be the point (5 , 5)

i.e. À (1 + 4 , 3 + 2)



According to this :

The image of the point A (x , y) $\xrightarrow{\text{By translation } (a\ ,\ b)}$ the point À (x + a , y + b)

i.e. Translation in the orthogonal coordinates plane transforms each point A in the plane into its image À in the same plane by a displacement (a) in the direction of the x-axis followed by a displacement (b) in the direction of the y-axis.

Example 2

Find the image of the line segment $\overline{AB}$ where A (– 4 , 3) , B (2 , 0) by the translation (x , y) ⟶ (x + 2 , y – 3)

Solution

∵ (x , y) ⟶ (x + 2 , y – 3) , then :

- The image of A (– 4 , 3)

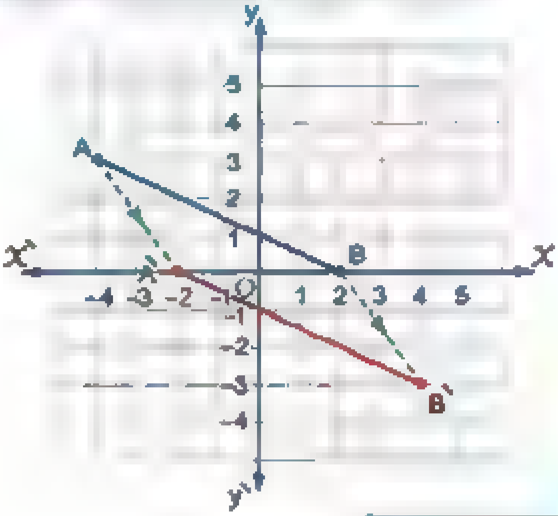
  is À (– 4 + 2 , 3 – 3)

  i.e. À (– 2 , 0)

- The image of B (2 , 0)

  is B̀ (2 + 2 , 0 – 3)

  i.e. B̀ (4 , – 3)



Notice that :

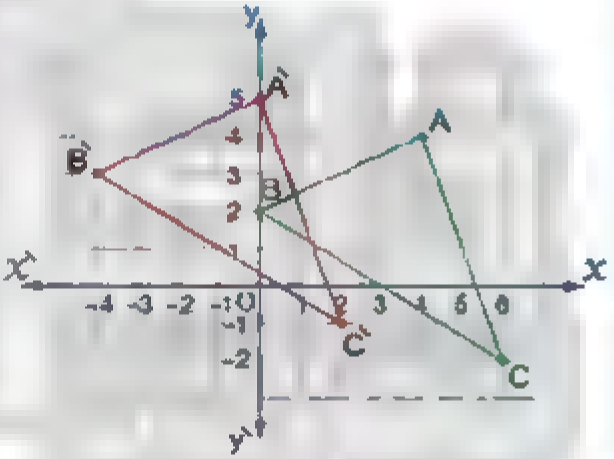
The translation $(x , y) \longrightarrow (x + 2 , y - 3)$ transforms each point to another point by a horizontal displacement of 2 units to the right and a vertical displacement of 3 units downwards.

Example (3

Draw on a square lattice $\triangle ABC$ where $A (4 , 4)$, $B (0 , 2)$ and $C (6 , -2)$, then find its image by the translation $(x , y) \longrightarrow (x - 4 , y + 1)$

Solution

The point	Its image by the translation
(x , y)	$(x - 4 , y + 1)$
$A (4 , 4)$	$\grave{A} (0 , 5)$
$B (0 , 2)$	$\grave{B} (-4 , 3)$
$C (6 , -2)$	$\grave{C} (2 , -1)$



$\triangle \grave{A}\grave{B}\grave{C}$ is the image of $\triangle ABC$ by the translation $(x , y) \longrightarrow (x - 4 , y + 1)$

Notice that :

- $\overline{AB} \parallel \overline{\grave{A}\grave{B}}$, $\overline{BC} \parallel \overline{\grave{B}\grave{C}}$ and $\overline{AC} \parallel \overline{\grave{A}\grave{C}}$
- $AB = \grave{A}\grave{B}$, $BC = \grave{B}\grave{C}$ and $AC = \grave{A}\grave{C}$
- $m (\angle A) = m (\angle \grave{A})$, $m (\angle B) = m (\angle \grave{B})$ and $m (\angle C) = m (\angle \grave{C})$

LESSON 2

Remark

The translation $(x , y) \longrightarrow (x + a , y + b)$ can be written simply :

"The translation (a , b)"

For example :

The translation $(x , y) \longrightarrow (x + 2 , y - 1)$ can be written simply :

"The translation (2 , − 1)"

Try

On a square lattice , draw Δ ABC where A (− 3 , 2) , B (− 1 , 1) and C (− 2 , 0) , then find its image by the translation :

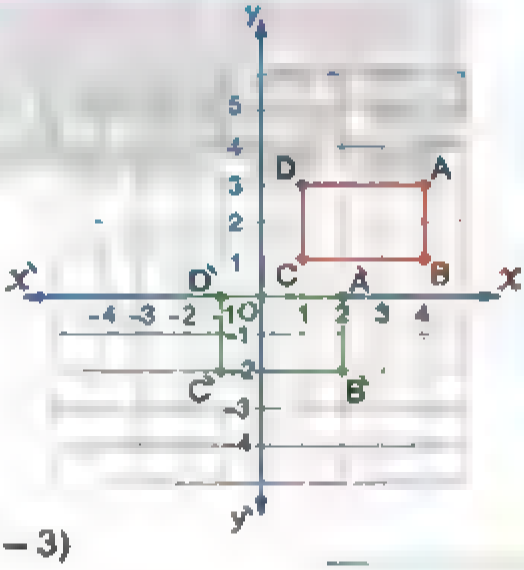
$(x , y) \longrightarrow (x + 2 , y + 1)$

Example 4

On a square lattice , draw the rectangle ABCD where A (4 , 3) , B (4 , 1) , C (1 , 1) and D (1 , 3) , then find its image by the translation (− 2 , − 3)

Solution

The point	Its image by the translation (− 2 , − 3)
A (4 , 3)	A' (2 , 0)
B (4 , 1)	B' (2 , − 2)
C (1 , 1)	C' (− 1 , − 2)
D (1 , 3)	D' (− 1 , 0)



∴ The rectangle $A'B'C'D'$ is the image of the rectangle ABCD by translation (− 2 , − 3)

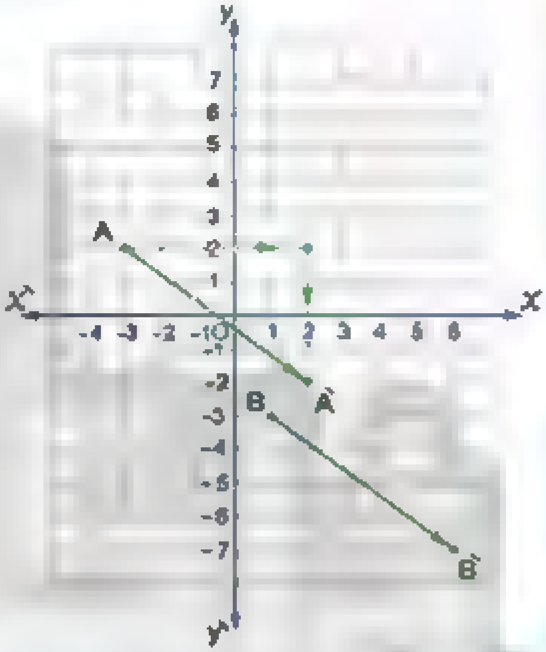
Example (5)

If the image of the point A $(-3, 2)$ by translation in the Cartesian coordinates plane is A' $(2, -2)$:

[1] Find the rule of translation.

[2] Find the image of B $(1, -3)$ by the same translation.

Solution



[1] By noticing the opposite graph, we find that the translation which makes A' $(2, -2)$ the image of A $(-3, 2)$ is equivalent to :

- Horizontal displacement of 5 units to the right (5)
- Vertical displacement of 4 units downwards (-4)

$\therefore$ The rule of translation is

$(x, y) \longrightarrow (x+5, y-4)$

[2] B $(1, -3) \longrightarrow B'$ $(1+5, -3-4)$

i.e. B' $(6, -7)$

Example (6)

If A' $(7, -2)$ is the image of A by the translation whose rule is $(x, y) \longrightarrow (x-3, y+1)$, find A

Solution

Let A be (x, y)

$\therefore A(x, y) \longrightarrow A'(x-3, y+1)$

$, \because A'(7, -2)$

$\therefore (x-3, y+1) = (7, -2)$

$\therefore x - 3 = 7$ $\qquad \therefore x = 10$

$, y + 1 = -2$ $\qquad \therefore y = -3$ $\qquad \therefore A(10, -3)$

Remember that :

If $(x, y) = (a, b)$, then $x = a, y = b$

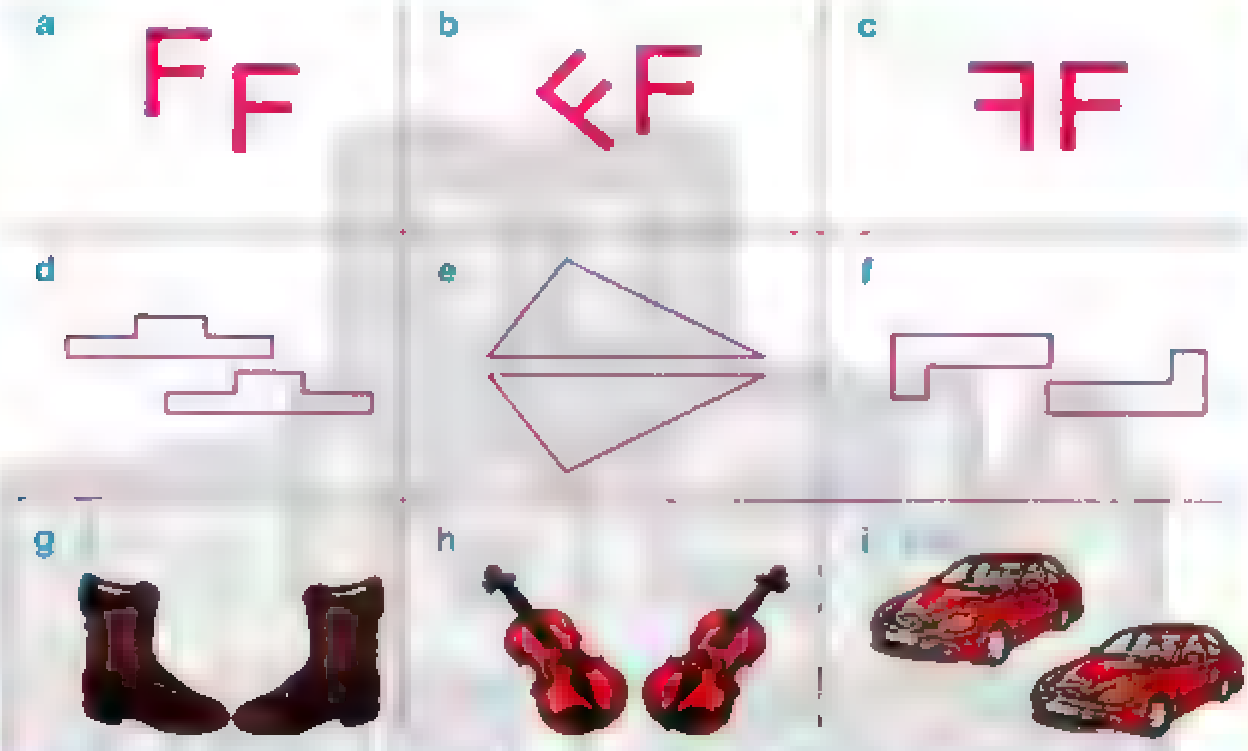
Exercise 11

Geometric transformations (Translation)

Interactive test

From the school book

In the following shapes determine the type of the geometric transformation (reflection , translation or rotation) :



Determine which of the following shapes is symmetric and which is not symmetric , then draw the axes of symmetry .

3 Complete each of the following :

a The image of the point (2 , 5) by translation
$(x , y) \longrightarrow (x + 2 , y + 1)$ is ....

b The image of the point (– 5 , 4) by translation
$(x , y) \longrightarrow (x + 4 , y – 5)$ is ....

c The image of the point (3 , 2) by translation $(x + 3 , y – 2)$ is ....
(Giza 2013)

d The image of the point A (2 , – 1) by translation $(x – 1 , y + 3)$ is ....
(Suez 2016)

e The image of the point A (– 5 , 2) by translation (– 1 , – 3)
is À (.......... , ...) *(El-Sharkia 2011)*

f The image of the point (2 , – 1) by translation (– 3 , 5) is ....
(Assiut 2016)

g The image of the point (0 , 2) by translation
$(x + 1 , y + 3)$ is (.......... , ...) *(Alexandria 2011)*

h The image of the point (– 2 , – 5) by translation
$(x , y) \longrightarrow (x – 2 , y)$ is ....

i The image of the point (3 , – 2) by translation
$(x , y) \longrightarrow (x , y + 3)$ is ....

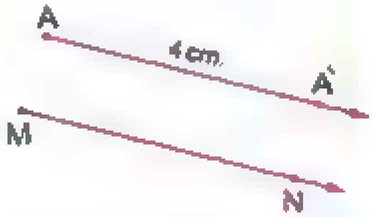
j The image of the point .... by translation
$(x , y) \longrightarrow (x – 2 , y + 3)$ is (7 , 4)

k If the image of the point (3 , 2) is the point (6 , 1) , then the translation rule is $(x , y) \longrightarrow$ (.......... , ...)

l If À is the image of A by translation of magnitude (MN) in the direction of $\overrightarrow{MN}$, then AÀ = ... *(El-Monofia 2011)*

LESSON **2**

m In the opposite figure :



$\grave{A}$ is the image of A by translation.

Its magnitude is .... cm.

in the direction of .... (Red Sea 2011)

Choose the correct answer :

a The image of the point A (– 4 , 3) by translation (– 1 , – 4) is .... (Damietta 2017)

(a) (– 3 , – 1) (b) (– 7 , 3) (c) (– 5 , – 7) (d) (– 5 , – 1)

b The image of the point (3 , – 2) by translation (– 3 , 2) is .... (El-Beheira 2016)

(a) (0 , 0) (b) (2 , 0) (c) (3 , 0) (d) (6 , 4)

c The image of the point A (3 , – 4) by translation (x + 1 , y + 4) is .... (Giza 2012)

(a) (4 , 0) (b) (4 , 8) (c) (3 , 8) (d) (3 , 0)

d The image of the point A (5 , 1) by translation (x – 1 , y – 1) is .... (El-Kalyoubia 2012)

(a) (4 , 0) (b) (6 , 2) (c) (4 , 2) (d) (– 4 , – 2)

e The image of the point (3 , 5) by the translation (x + 2 , y – 1) is ....

(a) (5 , 6) (b) (5 , 4) (c) (1 , 4) (d) (1 , 6)

f The image of the point (– 1 , 2) by translation of magnitude of 3 units in the positive direction of the x-axis is ....

(a) (– 1 , 5) (b) (2 , 2) (c) (– 2 , 2) (d) (– 1 , 3)

g The image of the point (– 3 , 4) by translation of magnitude of 4 units in the negative direction of the y-axis is .... (El-Beheira 2015)

(a) (– 3 , 0) (b) (– 7 , 4) (c) (– 3 , 8) (d) (– 1 , 4)

h The image of the point (3 , 0) by translation of magnitude 3 units in the negative direction of x-axis is ....

(a) (0 , 0) (b) (3 , 3) (c) (3 , – 3) (d) (0 , – 3)

i The image of the point (2 , – 1) by translation of magnitude 3 units in the positive direction of y-axis is ....

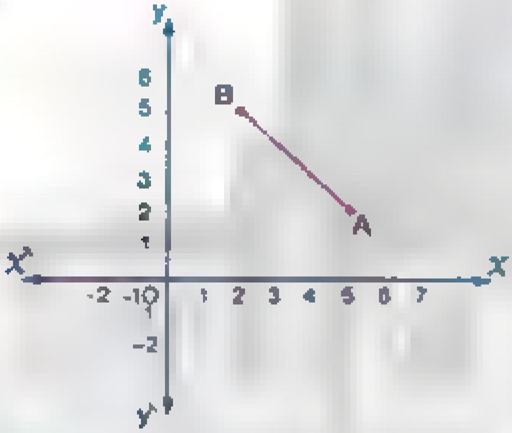
(a) (2 , 2) (b) (5 , – 1) (c) (5 , 2) (d) (2 , – 4)

j If $\grave{A}$ (3 , – 3) is the image of A by translation $(x , y) \longrightarrow (x - 1 , y - 4)$, then the point A is ....

(a) (2 , – 7) (b) (4 , 1) (c) (– 4 , – 1) (d) (2 , 1)

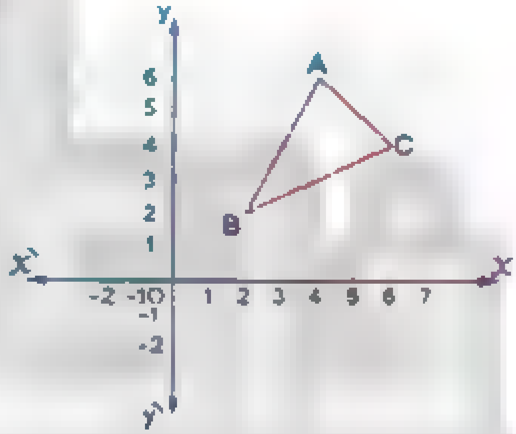
Find the image of each of the following figures by the indicated translation :

a



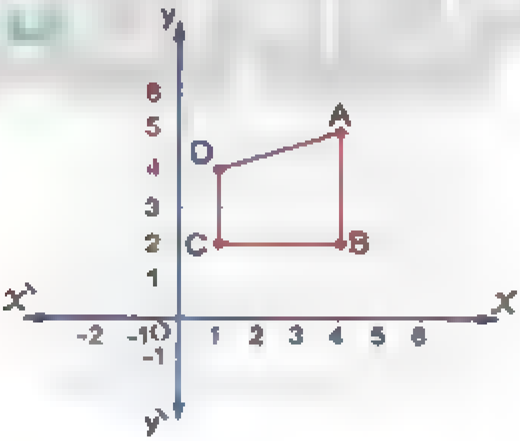
$(x , y) \longrightarrow (x - 3 , y - 4)$

b



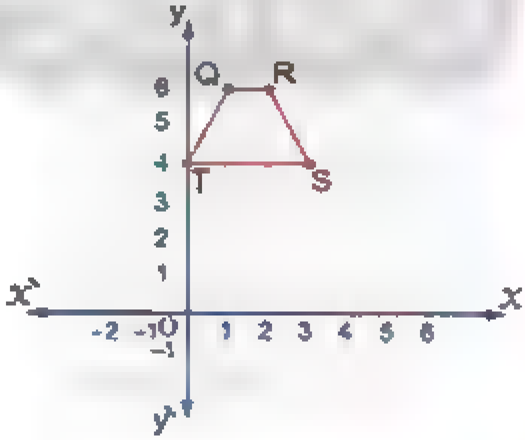
$(x , y) \longrightarrow (x + 2 , y + 3)$

c



by translation (3 , – 4)

d



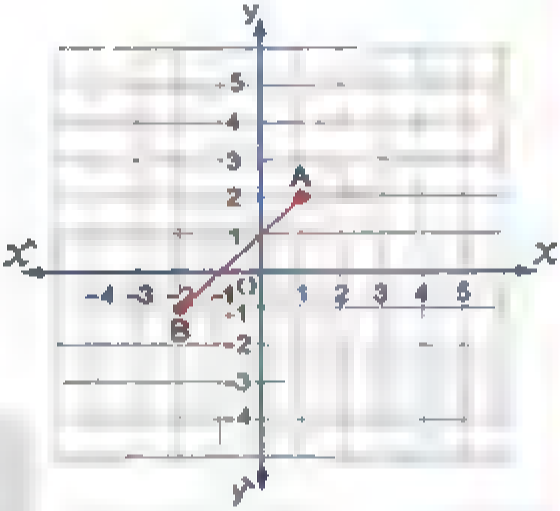
translate 1 unit to the right and 4 units downwards.

LESSON 2

6 In the opposite figure :

Find the image of the line segment $\overline{AB}$ where A (1 , 2), B (– 2 , – 1) by translation (x + 2 , y – 2)

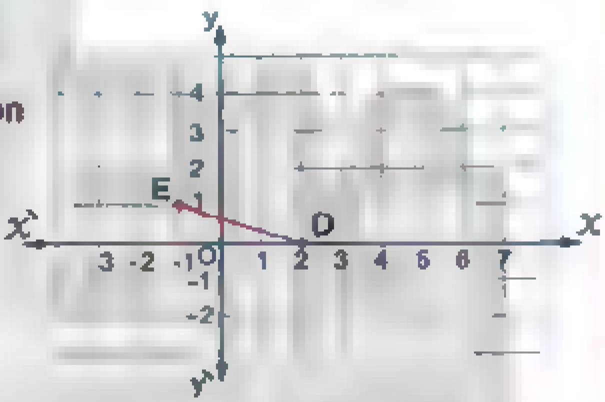
(South Sinai 2011)



In the opposite coordinate plane

a Determine the image of $\overline{DE}$ where D (2 , 0) and E (– 1 , 1) by translation $(x , y) \longrightarrow (x + 3 , y + 2)$

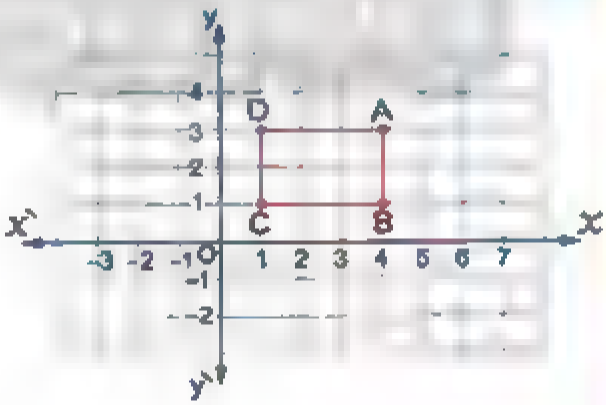
b What is the name of the shape DD̀ÈE ? Why ?



In the opposite figure :

ABCD is a rectangle where A (4 , 3) , B (4 , 1) , C (1 , 1) and D (1 , 3)

Find its image by translation (x – 2 , y – 5)



9 In the opposite figure :

a Determine the coordinates of the following points :

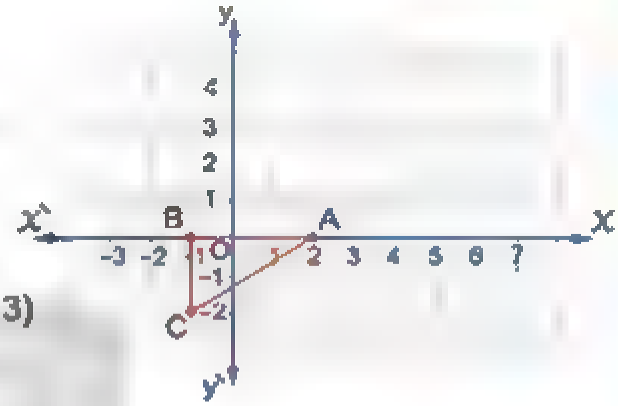
A (…… , ……) , B (…… , ……) and C (…… , ……)

b Find the image of the Δ ABC by translation $(x , y) \longrightarrow (x + 2 , y + 3)$

c The length of $\overline{BC}$ = ……

The length of $\overline{AB}$ = ……

d Is Δ ABC symmetric or not ? Why ?



10 In the opposite figure :

Find the image of the point A by the translation $\overline{ME}$ in the direction of $\overrightarrow{MN}$

11 Using the geometric tools , draw the image of each of the following by translation MN in the direction of $\overrightarrow{MN}$ as shown in each case .

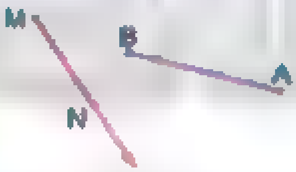


Fig. (1)

Fig. (2)

12 Determine in the coordinates plane the image of the line segment $\overline{AB}$ where A (2 , 3) and B (– 2 , 0) by translation $(x + 3 , y – 2)$

13 In the coordinates plane :

Draw the Δ ABC where A (0 , 1) , B (2 , 3) and C (– 1 , 4) , then find its image by translation $(x + 2 , y + 3)$

14 Draw Δ ABC where A (1 , 1) , B (– 3 , – 1) and C (0 , – 5) , then determine graphically its image by translation (5 , 0)

LESSON 2

15 Determine in the coordinate plane the following points A $(-3, 4)$, B $(1, 4)$ and C $(1, 2)$, then find :

a AB = ... , BC = ...

b The image of Δ ABC by the translation $(0, -3)$ [illegible]

16 Represent the points A $(2, 3)$, B $(4, 3)$ and C $(4, 7)$ in the lattice, then find :

a BC = ... length unit , AB = ... length unit

b The image of Δ ABC by translation $(0, -4)$ [illegible]

c The area of Δ ABC

17 In the coordinates plane :

Draw the rectangle ABCD where A $(4, 2)$, B $(4, 4)$, C $(1, 4)$ and D $(1, 2)$

a Draw its image by translation $(x + 2, y + 2)$

b Calculate the perimeter of the image of rectangle ABCD [illegible]

18 Find the image of each of the following points by the translation $(x, y) \longrightarrow (x + 2, y - 3)$ followed by the translation $(x, y) \longrightarrow (x - 3, y + 1)$

a $(4, -2)$ b $(-1, 3)$ c $(0, 2)$

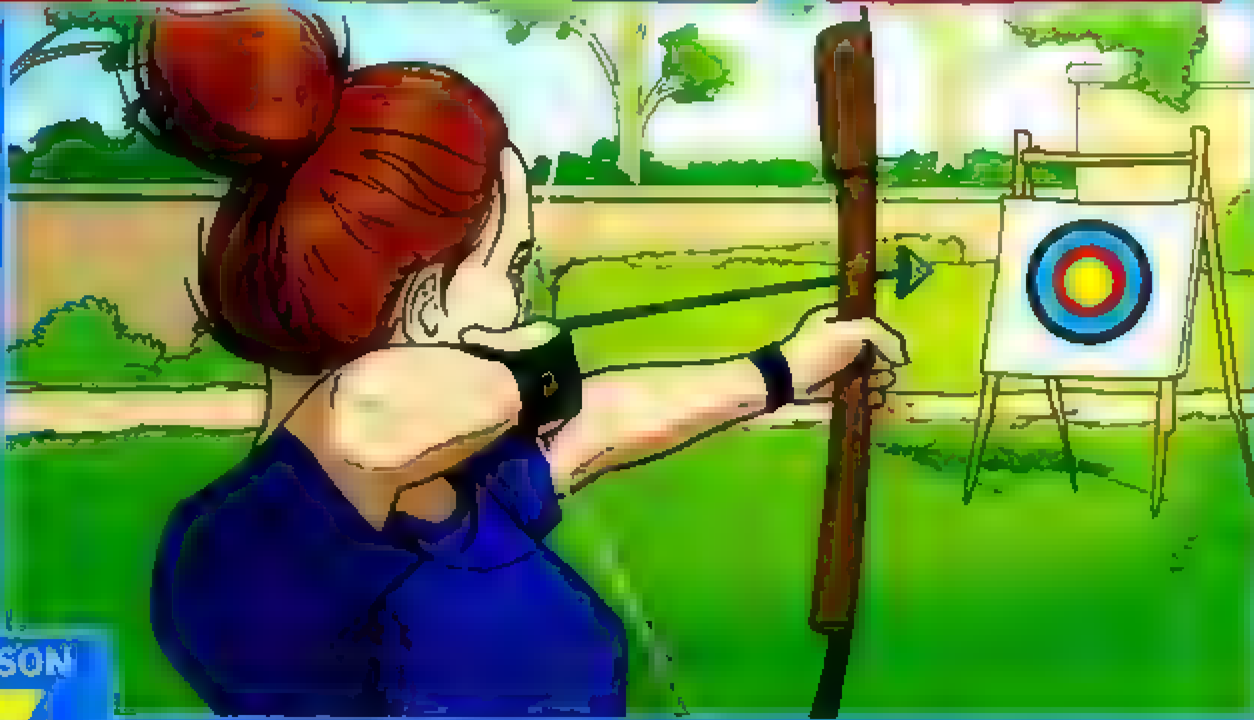
19 Use the translation $(x, y) \longrightarrow (x + 2, y + 3)$ to locate the point whose image is $(2, 3)$

20 The image of (a, b) by translation $(2, -3)$ is $(5, -4)$, find (a, b)

(Matrouh 2017)

21 If the image of the point A $(1, 1)$ by translation in the plane is $\grave{A}$ $(2, 2)$, find the images of the points O $(0, 0)$, B $(-1, 3)$ and C $(-3, 5)$ by the same translation.

22 If A $(-3, 1)$ and B $(1, -2)$, write the rule of the translation that makes B the image of A



LESSON 3

Area of the circle

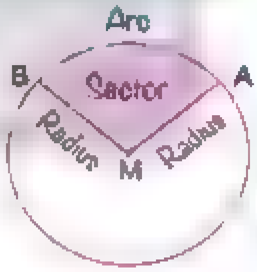
- You have studied before what is a circle, and what we mean by a circular sector.

You have known that :

> A circular sector is a part of a surface of a circle bounded by an arc and two radii passing through the ends of the arc.

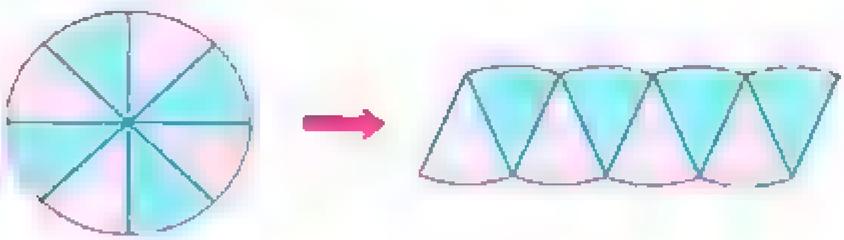
For example :

The coloured part in the opposite circle represents the circular sector AMB



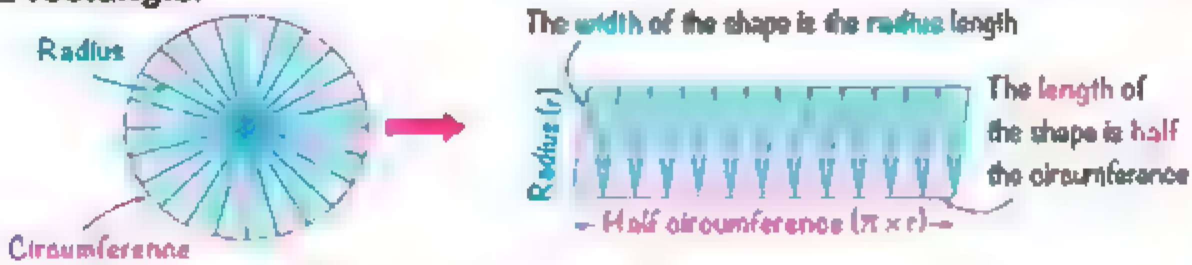
Finding the area of the circle

- The figure below shows the surface of a circle divided into equal circular sectors. These sectors were arranged in a certain way to form a shape similar to a parallelogram.



LESSON 3

- If the number of circular sectors increases, then the shape will be closer to a rectangle.



The area of the circle = the area of the resulted rectangle

= Length × width

= half the circumference × radius length

$= \frac{1}{2}(2\pi r) \times r = \pi r \times r = \pi r^2$

Where

$\pi = \frac{22}{7} \simeq 3.14$

So, The area of the circle = πr^2

Example (1)

Find the area of a circle whose radius length is 14 cm. (Consider $\pi = \frac{22}{7}$)

Solution :

$A = \pi r^2 = \frac{22}{7} \times (14)^2 = 616 \text{ cm}^2$

Example (2)

Find the area of each of the following circles (Consider $\pi = 3.14$) :

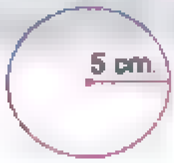


Fig. (1)

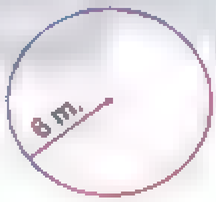


Fig. (2)

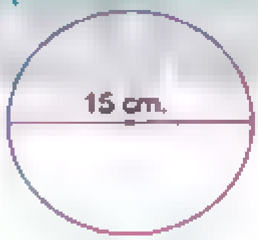


Fig. (3)

Solution

Fig. (1) : $A = \pi r^2 = 3.14 \times 5^2 = 78.5 \text{ cm}^2$

Fig. (2) : $A = \pi r^2 = 3.14 \times 6^2 = 113.04 \text{ m}^2$

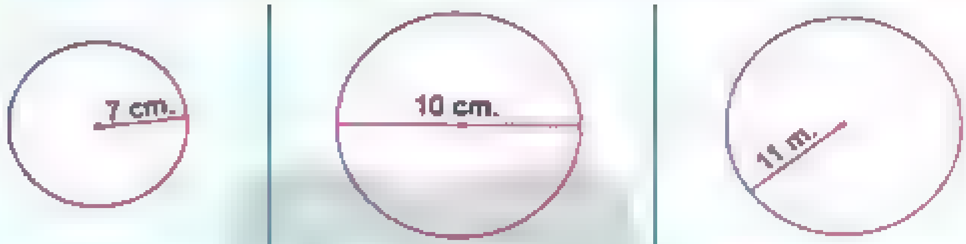
Fig. (3) : $\because d = 15$ cm.

$\therefore r = \frac{1}{2} d = \frac{1}{2} \times 15 = 7.5$ cm.

$\therefore A = \pi r^2 = 3.14 \times (7.5)^2 = 176.625 \text{ cm}^2$

Try

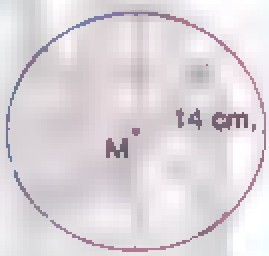
- Find the area of the following circles (Consider $\pi = 3.14$) :



Area = ... cm² | Area = ... cm² | Area = ... m²

Example (3)

In the opposite figure , a circle M of a radius length 14 cm. , is divided into eight equal circular sectors. Calculate the surface area of one sector.
(Consider $\pi = \frac{22}{7}$)



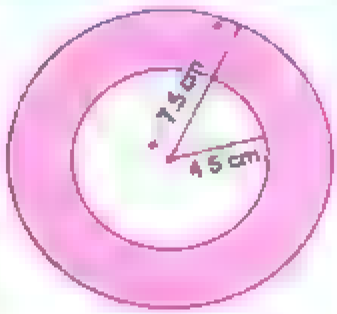
Solution

$\because A = \pi r^2 = \frac{22}{7} \times (14)^2 = 616 \text{ cm}^2$

$\therefore$ The area of one sector $= 616 \div 8 = 77 \text{ cm}^2$

Example (4)

In the opposite figure , find the area of the coloured part. (Consider $\pi = 3.14$)



Solution

$\because$ The area of the greater circle $= \pi r^2 = 3.14 \times (7.5)^2$
$= 176.625 \text{ cm}^2$

, the area of the smaller circle $= \pi r^2 = 3.14 \times (4.5)^2$
$= 63.585 \text{ cm}^2$

$\therefore$ The area of the coloured part $= 176.625 - 63.585 = 113.04 \text{ cm}^2$

LESSON 3

Example 5

Calculate the area of the opposite figure.
(Consider $\pi = \frac{22}{7}$)

Solution

$\because$ The area of the rectangle = L × W = 10 × 7 = 70 cm^2

, the area of the semicircle = $\frac{1}{2}\pi r^2 = \frac{1}{2} \times \frac{22}{7} \times (3.5)^2 = 19.25\ cm^2$

$\therefore$ The area of the whole figure = 70 + 19.25 = 89.25 cm^2

Try

- **In the opposite figure :**
  ABCD is a rectangle its length is 14 cm. and its width is 10 cm. A circle is drawn to touch the sides $\overline{AB}$ and $\overline{CD}$
  Calculate the area of the coloured part.
  (Consider $\pi \simeq 3.14$)

Example 6

Calculate the area of the circle whose circumference is 44 cm.
(Consider $\pi = \frac{22}{7}$)

Solution

$\because$ The circumference = $2\pi r$

$\therefore 44 = 2 \times \frac{22}{7} \times r$

$\therefore 44 = \frac{44}{7} \times r$

$\therefore r = \frac{44 \times 7}{44} = 7$ cm.

$\therefore A = \pi r^2 = \frac{22}{7} \times 7^2 = 154\ cm^2$

Exercise 12

Area of the circle

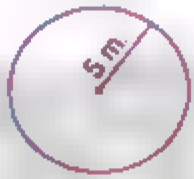
Interactive

▲ From the school book

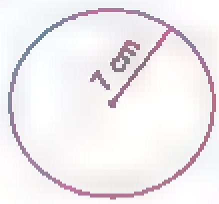
1 Find the area of each of the following circles (Consider $\pi = 3.14$) :

a


Area = ...

b


Area = ...

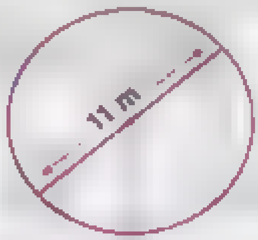
c


Area = ...

(Suez 2014)

d


Area = ...

e


Area = ...

f


Area = ...

2 Find the area of each of the following circles for the given radius.
Round your answer to the nearest hundredth (Consider $\pi = 3.14$) .

a $r = 8$ cm. b $r = 3.6$ m. c $r = 5$ km.

3 Find the area of each of the following circles for the given diameter.
Round your answer to the nearest hundredth (Consider $\pi = 3.14$)

a $d = 16$ cm. b $d = 21$ m. c $d = 1.4$ cm.

4 Calculate the area of a circle of radius length 7 cm. (Consider $\pi = \frac{22}{7}$)

(Souhag 2015)

5 A circle with radius length 4 cm. Calculate its area. (Consider $\pi = 3.14$)

(Luxor 2011)

LESSON 3

6 A circle , its diameter is 12 cm. Calculate its area.
(Consider $\pi = \frac{22}{7}$ or 3.14) (El-Sharkia 2016)

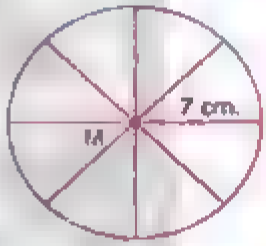
7 A circle , its diameter length is 20 cm. Find its surface area.
(Consider $\pi = 3.14$) (Souhag 2017)

8 A circle , its diameter is 14 cm. Calculate its surface area and its circumference. (Consider $\pi = \frac{22}{7}$) (El-Monofia 2015)

9 In the opposite figure :

A circle M of radius 4 cm , is divided into five equal circular sectors.
Calculate the surface area of one sector. (Consider $\pi = 3.14$)

10 In the opposite figure :



A circle M of radius length 7 cm. , is divided into eight equal circular sectors
Calculate the area of one sector. (Consider $\pi = \frac{22}{7}$)
(Matrouh 2015)

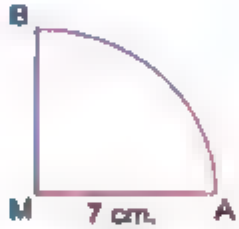
11 A circular birthday tart the diameter of its upper base equals 25 cm. is divided into eight equal circular sectors , then find the area of one sector "Approximating the result to the nearest integer"
(Consider $\pi = \frac{22}{7}$ or 3.14)

12 Find the area of each of the following figures (Consider $\pi = \frac{22}{7}$) :

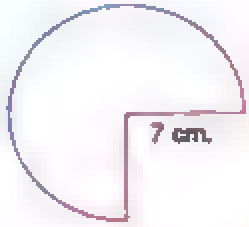
a

(Souhag 2014)

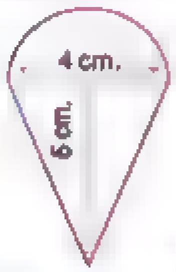
b



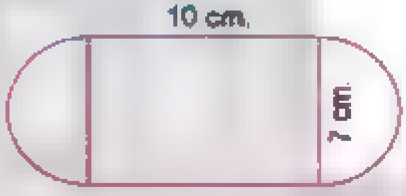
c



d

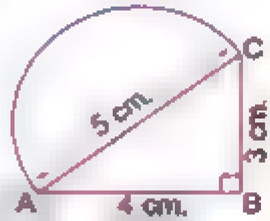


e



(El-Dakahlia 2016)

f

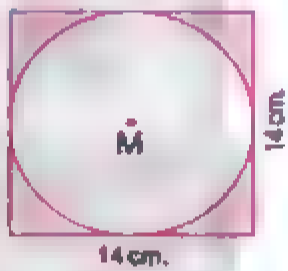


13 In the opposite figure :

Circle M is drawn inside a square of side length 14 cm. and touches its sides.

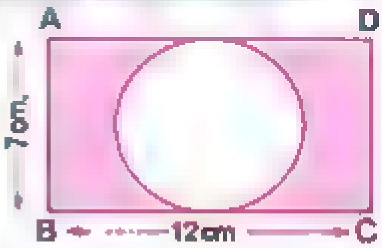
Calculate the area of the coloured part.

(Consider $\pi = \frac{22}{7}$) (Damietta 2012)



14 In the opposite figure :

ABCD is a rectangle its length 12 cm. and its width 7 cm. Calculate the area of the coloured part. (Consider $\pi = \frac{22}{7}$) (El-Kalyoubia 2017)

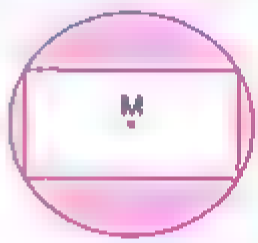


15 In the opposite figure :

M is a circle of radius length 5 cm. , a rectangle is drawn inside it , its length is 8 cm. and width is 6 cm.

Calculate the area of the shaded part.

(Consider $\pi = \frac{22}{7}$ or 3.14)



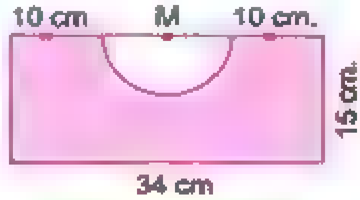
LESSON 3

16 In the opposite figure :

A rectangle where its length = 34 cm. and its width = 15 cm. , and a semicircle of centre M

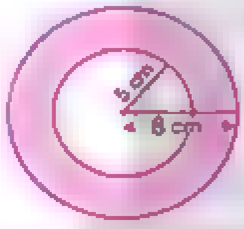
Calculate the area of the coloured part.

(Consider $\pi = \frac{22}{7}$)

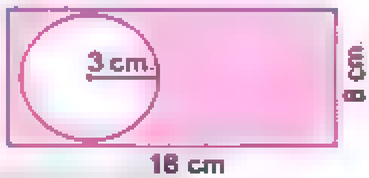


17 Find the area of the coloured part of each of the following figures (Consider $\pi = 3.14$) :

a



b



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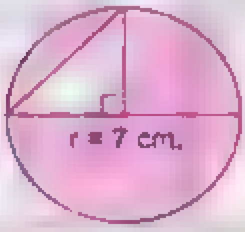

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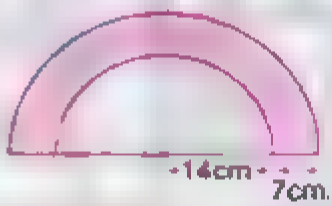

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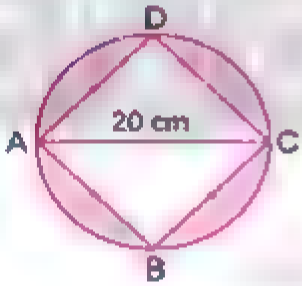
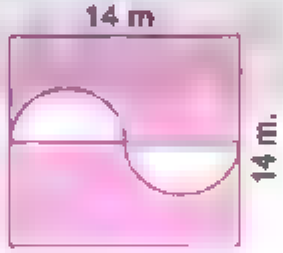


f



18 In the opposite figure :

Find the area of the coloured part. (Consider $\pi = 3.14$)

19 Complete :

a The surface area of the circle = ... ([illegible] 2017 – [illegible] 2015)

b The radius length of a circle is 14 cm. , then its circumference = ... cm. and its area = ... cm^2 (Consider $\pi = \frac{22}{7}$)

c The diameter of a circle is 20 cm. , then its circumference = ... cm. and its area = ... cm^2 (Consider $\pi = 3.14$)

d The surface area of the circle of radius length 7 cm. = ... $\pi\ cm^2$ (Beni Suef 2016)

e A circle, its area is $25\pi\ cm^2$, then the length of its radius is .... cm. (Damietta 2016)

f If the circumference of a circle is 30π mm. , then the area of this circle equals ....

20 Choose the correct answer :

a The area of a circle = ... (Qena 2017)
(πr or πr^2 or $2\pi r$ or $2\pi r^2$)

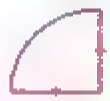
b The circumference of a circle = ...
(πr or $2\pi r$ or πr^2 or $2\pi r^2$)

c A circle, its radius length is 3.5 cm. , then the surface area = ... cm^2 (Consider $\pi = \frac{22}{7}$) ([illegible] 2012) (11 or 22 or 38.5 or $38\frac{1}{8}$)

d A circle with radius length = 1 cm. , then its area = ... cm^2
([illegible] 2011) (π or 2π or $\frac{1}{2}\pi$ or π^2)

e The area of the circle whose diameter length is 8 cm. = ... $\pi\ cm^2$
(El-Menia 2015) (4 or 8 or 16 or 64)

f The perimeter of the opposite figure = ... cm. (Souhag 2017)
(2π or 5π or $\pi + 4$ or $4\pi + 4$)



2 cm.

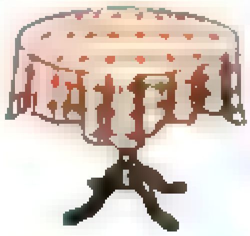
21 A circle its circumference is 14π m. Calculate its area.

22 A circle its circumference is 88 cm. Find its radius length and its area. (Consider $\pi = \frac{22}{7}$) (Alexandria 2015)

LESSON 3

23 A circle its circumference is 62.8 cm. Calculate its area. (Consider $\pi = 3.14$)

24 A table its surface in the form of a circle , its diameter is 1.5 m. Its surface is wanted to be covered by a sheet of glass equal to its surface. Calculate the cost price if the square meter of the glass costs L.E. 60 (Consider $\pi = \frac{22}{7}$ or 3.14)

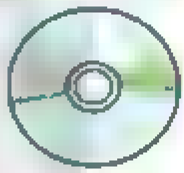


25 A garden , which is circular in shape , its circumference is 132 metres , find :

a The length of diameter of the garden in metre.

b The area of the garden in square metre. (Consider $\pi = \frac{22}{7}$)

26 If the length of the outer diameter of a computer CD is 12 cm. , and the length of the inner diameter is 1.5 cm. Find the area of this CD. (Consider $\pi = 3.14$)



For Excellent Pupils

27 A circle its area is 616 cm^2. Calculate its radius length and its circumference. (Consider $\pi = \frac{22}{7}$)

LESSON 4

Lateral area and total area for each of the cube and the cuboid

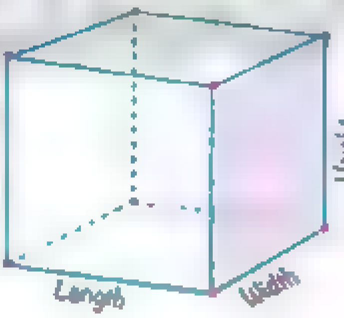
Definitions

- The lateral area of a solid is the sum of the areas of all its faces that are not bases.
- The total area of a solid is the sum of the areas of all its faces included the bases.

First : lateral area and total area of the cube

Remember that :

6 faces
8 vertices
12 edges

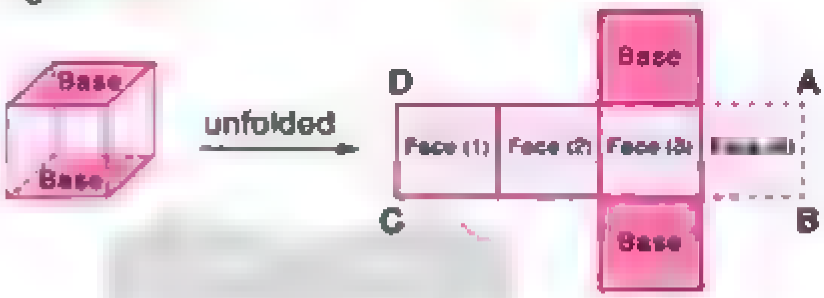


1. The cube has 6 faces , each face is a square and all faces are equal in area.
   So , the area of one face = the area of the square
   = the edge length × itself
2. The cube has 8 vertices and 12 edges.
3. The cube has 3 dimensions : "length , width and height" and they are equal in length.
4. The volume of the cube = The edge length × itself × itself

LESSON 4

Finding the lateral area and the total area of a cube

If we have a carton box in the form of a cube, and we want to see how to find the lateral area and the total area of a cube, we can unfold the box as shown in the figure below :



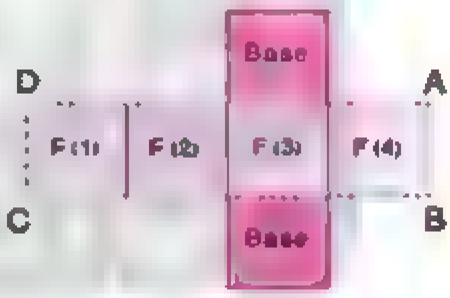
- The lateral area of a cube is the sum of the areas of its four lateral faces (1) , (2) , (3) and (4) which are perpendicular to the base of the cube.

So, The lateral area of a cube = The area of one face × 4

= Edge length × Itself × 4

Deducing another law

When the faces of the cube were unfolded , the rectangle ABCD was formed from the lateral faces.



- The length of this rectangle = The sum of the edge lengths of the four lateral faces (1) , (2) , (3) and (4) which represents the perimeter of the base of the cube.
- The width of this rectangle = The height of the cube

So, The lateral area of the cube = Perimeter of the base × Height

- The total area of a cube is the sum of the areas of all the faces of the cube.

So, The total area of a cube = The area of one face × 6

= Edge length × Itself × 6

Remarks

1. The area of one face $= \frac{\text{The lateral area}}{4} = \frac{1}{4} \times$ (the lateral area)
2. The area of one face $= \frac{\text{The total area}}{6} = \frac{1}{6} \times$ (the total area)

Example (1)

A cube-shaped box , whose edge length is 3 cm. Find :

[1] The lateral area.

[2] The total area.

Solution

The area of one face $= 3 \times 3 = 9$ cm^2

[1] The lateral area = the area of one face $\times 4 = 9 \times 4 = 36$ cm^2

[2] The total area = the area of one face $\times 6 = 9 \times 6 = 54$ cm^2

Example (2)

The lateral area of a cube is 28 cm^2 Find its total area.

Solution

The area of one face $= \frac{\text{the lateral area}}{4} = \frac{28}{4} = 7$ cm^2

The total area = the area of one face $\times 6 = 7 \times 6 = 42$ cm^2

Example (3)

If the perimeter of a face of a cube is 20 cm.

Find its lateral area and its total area.

Solution

The side length of a face = the perimeter $\div 4 = 20 \div 4 = 5$ cm.

The area of one face $= 5 \times 5 = 25$ cm^2

The lateral area $= 25 \times 4 = 100$ cm^2

The total area $= 25 \times 6 = 150$ cm^2

LESSON 4

Example (4

If the sum of the edge lengths of a cube is 108 cm. , find :

[1] The lateral area.

[2] The total area.

Solution

The edge length of a cube = 108 ÷ 12 = 9 cm.

The area of one face = 9 × 9 = 81 cm^2

[1] The lateral area = 81 × 4 = 324 cm^2

[2] The total area = 81 × 6 = 486 cm^2

Try

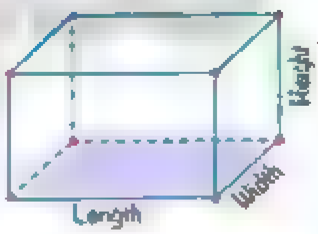
[1] A cube of edge length 7 cm.
Find its lateral area and its total area.

[2] If the sum of the edge lengths of a cube is 96 cm.
Find its total area.

Second : Lateral area and total area of a cuboid

Remember that :

6 faces
8 vertices
12 edges



① The cuboid has 6 faces ,
each face is a rectangle and each
two opposite faces are equal in area.
So, the area of one face = The area of the rectangle = Length × Width

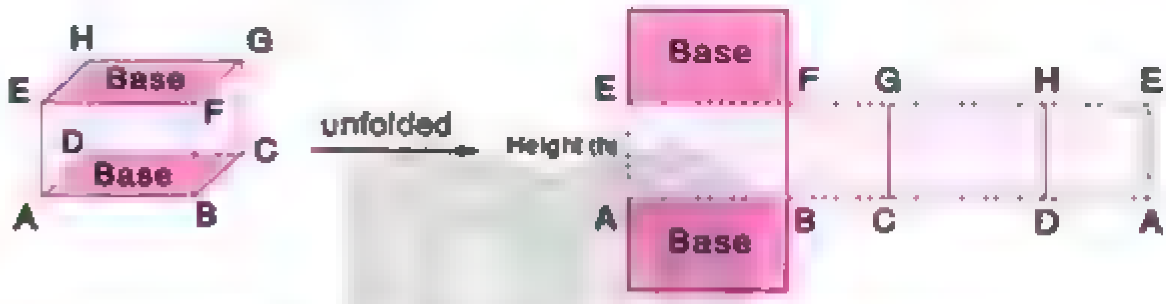
② The cuboid has 8 vertices and 12 edges.

③ The cuboid has 3 dimensions : "Length , Width and Height"

④ The volume of the cuboid = Length × Width × Height
= Base area × Height

Finding the lateral area and the total area of a cuboid

If we have a carton box in the form of a cuboid, and we want to see how to find the lateral area and the total area of a cuboid, we can unfold the box as shown in the figure below :



- The lateral area of a cuboid is the sum of the areas of its four lateral faces which are perpendicular to the base of the cuboid.

i.e. The lateral area of the cuboid (in the diagram) whose base is the rectangle ABCD

= The area of ABFE + The area of BCGF + The area of CDHG + The area of DAEH

= AB × h + BC × h + CD × h + DA × h

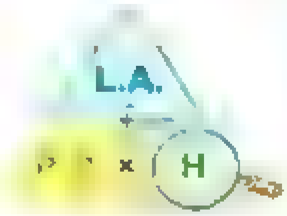
= (AB + BC + CD + DA) × h

= The perimeter of the base × The height

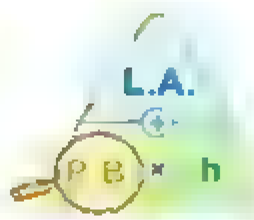
So, The lateral area of the cuboid = The perimeter of the base × The height

and we deduce that :

- The height = $\frac{\text{The lateral area of the cuboid}}{\text{The perimeter of the base}}$



- The perimeter of the base = $\frac{\text{The lateral area of the cuboid}}{\text{The height}}$



LESSON 4

- The total area of the cuboid is the sum of its lateral area and the areas of its two parallel bases.

So, The total area of the cuboid = The lateral area + 2 × (The area of the base)

Notice that :

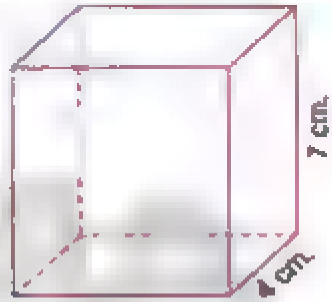
If a cuboid without a lid , then :
the total area = The lateral area + The area of one base

Example (5)

A cuboid-shaped box is 6 cm. long , 4 cm. wide and 7 cm. high , find :

[1] The lateral area.

[2] The total area.



Solution

[1] The perimeter of the base = (L + W) × 2 = (6 + 4) × 2 = 20 cm

The lateral area = The perimeter of the base × The height

$= 20 \times 7 = 140$ cm^2

[2] The area of the base = 6 × 4 = 24 cm^2

The total area = The lateral area + 2 × (The area of the base)

$= 140 + 2 \times 24 = 188$ cm^2

Example (6)

The inner dimensions of a swimming pool are 24 m. , 16 m. and 3.5 m. , it is necessary to cover its inner floor and sides with square-shaped ceramic tiles of side length 20 cm.

How many tiles are needed ?

Solution

The perimeter of the base = (L + W) × 2 = (24 + 16) × 2 = 80 m.

The lateral area = the perimeter of the base × the height

$$= 80 \times 3.5 = 280 \text{ m}^2$$

The area of the base = L × W = 24 × 16 = 384 m^2

Since , the swimming pool without a lid ,

Then , the total area = the lateral area + the area of the base

$$= 280 + 384 = 664 \text{ m}^2 = 6\,640\,000 \text{ cm}^2$$

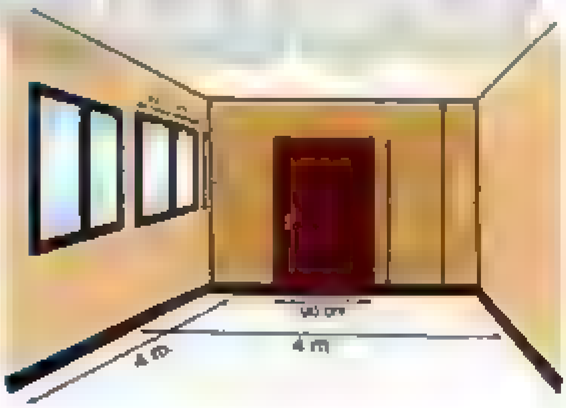
The area of one tile = 20 × 20 = 400 cm^2

$$\text{Then the number of tiles} = \frac{\text{total area of swimming pool}}{\text{the area of one tile}}$$

$$= \frac{6\,640\,000}{400} = 16\,600 \text{ tiles.}$$

Example (7)

A room has a square floor of side length 4 m. and height 2.8 m. It has a door of width 90 cm. and height 2.2 m. and two windows each of dimensions 1 m. and 60 cm. The walls and ceiling of the room are painted. If the cost of one square metre to be painted is P.T. 475 Find the cost of painting the room , approximated to the nearest pound.



LESSON 4

Solution

The perimeter of the base $= 4 \times 4 = 16$ m.

The lateral area $= 16 \times 2.8 = 44.8$ m^2.

The area of the ceiling $= 4 \times 4 = 16$ m^2.

The total area $= 44.8 + 16 = 60.8$ m^2.

The area of the door $= 0.9 \times 2.2 = 1.98$ m^2.

The area of the two windows $= 2 (1 \times 0.6) = 1.2$ m^2.

The total area of the painted part of the room $= 60.8 - (1.98 + 1.2)$
$= 57.62$ m^2.

The cost $= 57.62 \times 4.75 = 273.695 \simeq$ L.E. 274

Try

[1] A cuboid of length 8 cm. , width 6 cm. and height 7 cm.
Find its lateral and total area.

[2] A cuboid-shaped box with a square base whose side length is 5 cm. and its height is 10 cm.
Calculate the lateral area and total area.

Exercise 13

Lateral area and total area for each of the cube and the cuboid

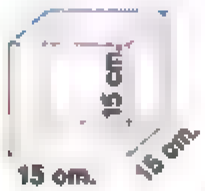
Interactive test

... From the school book

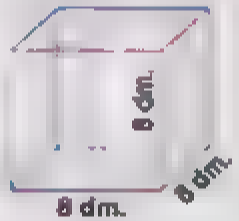
The cube

1 Calculate the lateral area and total area for each of the following cubes :

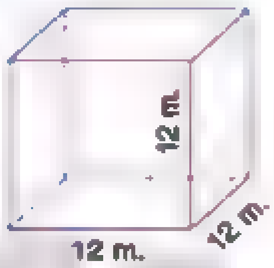
a



b



c



2 A cube of edge length 6 cm.

Find its lateral area and its total area. (El-Fayoum 2017)

3 If the edge length of a cube is 1.5 cm. Find its total area.

4 If the lateral area of a cube is 36 cm^2. Find its total area. (Aswan 2012)

5 Find the lateral area of a cube whose total area is 48 m^2.

6 The total area of a cube is 384 cm^2. Find :

a Area of one face of this cube.

b Its edge length.

c Its lateral area. (El-Dakahlia 2011)

7 If the total area of a cube is 216 m^2. Find its lateral area and its volume.

8 The sum of edge lengths of a cube is 120 cm. Find :

a The lateral area of the cube.

b The total area of the cube.

c The volume of the cube. (Cairo 2013)

LESSON 4

9 The perimeter of one face of a cube is 24 cm. Find :

a The edge length of the cube. b The lateral area of the cube.

c The total area of the cube.

10 The perimeter of the base of a cube is 28 cm.
Calculate its lateral area and total area.

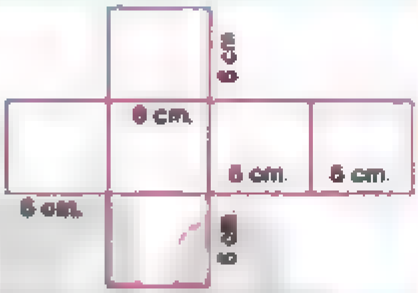
11 A cube is of edge length 8 cm. Calculate the ratio between its lateral area and its total area. (El-Gharbia 2017)

12 When folding the opposite shape

a The formed solid is ·

b The lateral area of this solid is ·

c The total area of this solid is ······ ··



13 Complete :

a The lateral area of a cube = · × ······

b The total area of a cube = · ×

c A cube of edge length 6 cm. , then its lateral area = ······ cm^2 (El-Gharbia 2014)

d The total area of a cube of edge length 4 cm. = cm^2

e The edge length of a cube is 50 mm. , then its total area is · · cm^2

f If the area of one face of a cube = 5 cm^2 , then the total area of this cube = ········ cm^2 (Alexandria 2011)

g The sum of the edge lengths of a cube = 24 cm., then the area of one face = ········ cm^2 (Assiut 2016)

h The face area of a cube is 4 cm^2 , then its volume = ·· · · cm^3 (El-Monofia 2016)

i The lateral surface area of a cube is 100 cm^2 , then its volume equals ········ cm^3 (Cairo 2012)

j If the volume of a cube is 1000 cm³ , then its total area = ... cm²

(El-Dakahlia 2015)

k The ratio between the area of one face of a cube and its lateral area = $\frac{\cdots}{\cdots}$

l The ratio between the area of one face of a cube and its total area = $\frac{\cdots}{\cdots}$

m The ratio between the lateral area and the total area of a cube = $\frac{\cdots}{\cdots}$

14 Choose the correct answer from the given ones :

a The lateral area of the cube = Area of one face × ...... (South Sinai 2013)

(a) 2 (b) 4 (c) 6 (d) 8

b If the perimeter of one face of a cube = 4 cm. , then its total area = ... cm² (El-Dakahlia 2011)

(a) 3 (b) 4 (c) 5 (d) 6

c The area of base of a cube is 49 cm² , then its lateral area equals .... cm² (Alexandria 2016)

(a) 392 (b) 294 (c) 196 (d) 98

d A cube of total area 150 cm² , then the length of its edge is ...... cm. (Luxor 2015)

(a) 5 (b) 6 (c) 15 (d) 10

e If the total area of a cube is 24 cm² , then its volume = ... cm³ (El-Fayoum 2012)

(a) 8 (b) 2 (c) 4 (d) 16

f A cube , its volume is 1000 cm³ , then its lateral area = ... cm² (Damietta 2016)

(a) 600 (b) 500 (c) 400 (d) 200

g A cube-shaped box , without a lid , has ...... faces.

(a) 4 (b) 5 (c) 6 (d) 8

h A cube without a lid of edge length 3 cm. , then its total area = ...

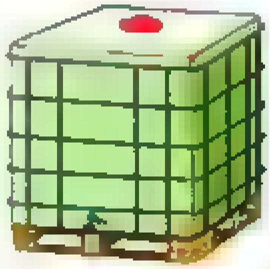
(a) 54 (b) 45 (c) 36 (d) 9

i The area of one face of the cube = ... its total area. (Kafr El-Sheikh 2013)

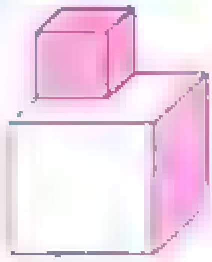
(a) $\frac{1}{2}$ (b) $\frac{1}{8}$ (c) $\frac{1}{6}$ (d) $\frac{1}{4}$

LESSON 4

15 A container water tank is in the form of a cube whose inner length is 1.5 m.
It is wanted to paint it to prevent the rust.
The cost price of one square metre is L.E. 15
Calculate the cost of painting. (Giza 2017)



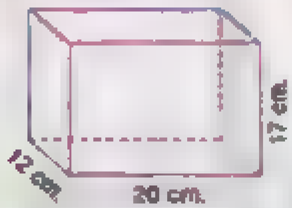
16 In the opposite figure :
A solid consists of two sticking cubes, the length of the edge of one of them is 2 cm. and that of the other is 1 cm. , then find the total area of the solid.



The cuboid

1 Calculate the lateral area and total area for each of the following cuboids :

a



b

c

2 The perimeter of the base of a cuboid = 24 cm. and its height = 10 cm.
Calculate the lateral area. (Giza 2014)

3 A cuboid , its length is 6 cm. , its width is 4 cm. and its height is 8 cm.
Find its lateral area and its total area. (Alexandria 2017)

4 A cuboid of a square base with side length 8 cm and its height equals 10 cm
Find :

a Its lateral area. b Its total area. (El-Dakahlia 2012)

5 A cuboid-shaped box with a square base whose side length is 9 cm. and its height is 20 cm. Calculate the lateral area and the total area. (El-Fayoum 2012)

6 A cuboid whose total area = 132 cm^2 and its lateral area = 112 cm^2. Find the area of its base. *(Giza 2016)*

7 A cuboid whose lateral area is 160 cm^2 and the dimensions of its base are 7 cm. and 3 cm. Find its height. *([illegible])*

8 A cuboid with a square base whose perimeter is 20 cm. and its height is 8 cm
Find :

a The lateral area.
b The length of its base side.
c The total area. *(Ismailia 2012)*

9 The perimeter of the base of a cuboid = 32 cm. and its height = 10 cm. , if the length of the base = 9 cm. Find the lateral area and the total area of the cuboid. *(El-Gharbia 2015)*

10 A cube is of edge length 10 cm. and a cuboid whose length is 8 cm. , its width is 5 cm. , and its height is 17 cm.
Calculate the difference between their lateral areas.

11 A box without a lid whose length is 16 cm. , its width is 7 cm. and its height is 19 cm. Calculate its lateral area and total area *([illegible] 2016)*

12 A cuboid base is a square of side length 32 cm. and its height is $\frac{3}{8}$ the side length of its base. Find its total area.

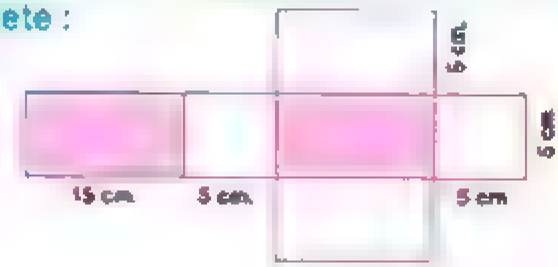
13 The volume of a cuboid is 180 $cm.^3$ and the dimensions of its base are 5 cm. and 1.2 dm. Find its total area.

14 When folding the opposite figure , complete :

a The formed solid is ·

b The lateral area of the solid = ...

c The total area of the solid = ...



(Souhag 2013)

LESSON 4

15 Choose the correct answer from the given ones :

a The lateral area of the cuboid = the perimeter of the base × .... (Suez 2016)

(a) height (b) width (c) length (d) volume

b The lateral area of the cuboid with length is 3 cm. , width is 2 cm. and height is 4 cm. = ... cm^2 (El-Beheira 2013)

(a) 20 (b) 24 (c) 40 (d) 52

c The lateral area of a cuboid with base in the shape of a square with side length 8 cm. and the height of the cuboid is 5 cm. = ... cm^2

(a) 40 (b) 80 (c) 160 (d) 240

d The total area of the cuboid with length is 12 cm. , width is 6 cm. and height is 4 cm. = ... cm^2

(a) 216 (b) 36 (c) 360 (d) 288

e The height of the cuboid whose lateral area is 120 cm^2 and the dimensions of its base are 6 cm. and 4 cm. = ... cm. ([illegible] 2014)

(a) 5 (b) 6 (c) 12 (d) 2.5

f If the total area of a cuboid = 32 cm^2 and its lateral area = 12 cm^2 , then the area of one of its bases = ... cm^2

(a) 32 (b) 20 (c) 18 (d) 10

g The dimensions of a base of a cuboid are 4 cm. and 3 cm. and its lateral area = 140 cm^2 , then its volume = ... cm^3

(a) 1680 (b) 120 (c) 168 (d) 60

16 A swimming pool whose base is of dimensions 40 m. , 10 m. and its height equals 2.5 m. Calculate :

a Its lateral area. b Its total area. ([illegible])

17 The cuboid-shaped box of a truck with inner dimensions 4 , 2.5 and 1 metres, as shown in the figure, is painted internally. If each square metre of the box to be painted costs L.E. 8

Calculate the paint cost.

18 A truck box is in the form of a cuboid, whose inner dimensions are 5 m. , 2 5 m. and 1.6 m. It is wanted to paint the inner box with paint , the cost price of one square metre is L.E. 12 Calculate the cost of paint.

19 A truck box for carrying goods is in the form of cuboid whose inner dimensions are 4 m. , 2.5 m and 1.8 m. It is wanted to cover its sides and ceiling with a sheet iron, the cost price of square metre is L.E. 15 Calculate the cost of required sheet iron.

20 A water tank is in the form of a cuboid of inner dimensions 3 m. , 2 m. and $1\frac{1}{2}$ m. The required is to paint it internally. If the cost price of each square metre is L.E. 10 , then calculate the cost of painting of all the inner surface of the tank.

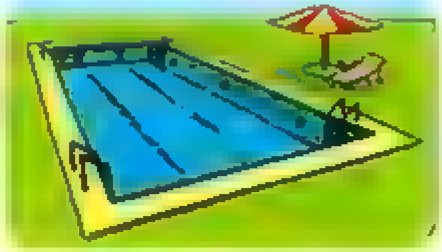
21 A room whose length is 5 m. , its width is 4 m. , and its height is 3 2 m. It is wanted to paint its lateral walls and ceiling The cost price of one square metre to be painted is L.E. 8 Calculate the required cost knowing that the room has 2 windows and a door whose areas are 8 m^2

22 A room of a squared floor whose length is 4 m. and its height is 3 m. It has a door whose width is 90 cm. , and 2 m. high. It has two rectangular equal windows of length 100 cm. and width 61 cm. Calculate the cost of painting of the walls , given that the cost price of painting one square metre is L.E. 9

23 A room has a square floor of side length 5 m. and height 2.8 m. It has a door of width 90 cm. and height 2.20 m. and two windows each of dimensions 1 m. and 60 cm. The walls and ceiling of the room are painted. If the cost of one square metre to be painted is L.E. 10 Find the cost of painting the room.

LESSON 4

24 The inner dimensions of a swimming pool are 25 m. , 16 m. and 3.5 m. It is necessary to cover its inner floor and sides with square-shaped ceramic tiles of side length 20 cm. How many tiles are needed ?



25 A swimming pool whose base is with dimensions 40 m. , 10 m. and its height equals 205 cm. It is needed to be covered by ceramic with square shape of side length = 25 cm. Find :

a The number of boxes of ceramic is needed if each box contains 25 units of ceramic.

b The total cost if the price of one square metre of ceramic = 45 pounds and the cost for covering 1 square metre by ceramic = 5 pounds.

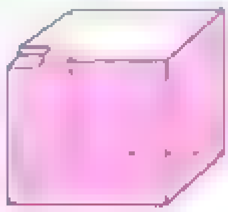
26 Youssef used a piece of cardboard in the form of a rectangle , its length is 1.2 m. and its width is 80 cm. to form a cube-shaped box whose edge length is 30 cm. Calculate the remained paper area after forming the box.

27 If the total area of the cuboid is 400 cm^2. and the side length of its square-shaped base is 10 cm. , then find its height.

For Excellent Pupils

28 The total area of a cube-shaped piece of metal is 384 cm^2 It is melted and shaped like a cuboid whose base dimensions are 16 cm. and 2 cm. Find the total area of the cuboid.

29 A cube with edge length 12 cm. , a part of it is cut to form a cuboid whose side lengths are 3 cm. , 2 cm. and 1 cm. , find the total area of the remained part of the cube.



Test on Unit Three

Answer the following questions :

1 Choose the correct answer :

a The total area of the cube = area of one face × ....

(4 or 6 or 8 or 12)

b The image of the point (3 , – 4) by translation (x – 1 , y + 4) is ....

((4 , 8) or (2 , 0) or (2 , 8) or (3 , 0))

c If A (6 , 1) and B (6 , 4) , then the length of the line segment $\overline{AB}$ = ... units. (– 4 or 4 or 5 or 3)

d The area of the circle whose radius length is 7 cm. = ... cm^2

(Consider $\pi = \frac{22}{7}$) (88 or 44 or 49 or 154)

e The lateral area of the cuboid = the perimeter of the base × ....

(height or width or length or volume)

2 Complete each of the following :

a The area of the circle = ... × ....

b A cuboid of length 6 cm. , width 4 cm. and height 10 cm. , then its lateral area = ... cm^2

c The total area of a cube of edge length 7 cm. = ... cm^2

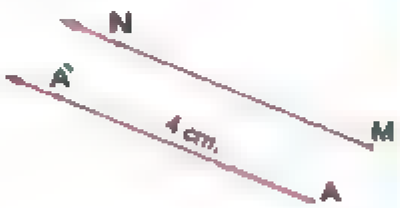
d The image of the point (2 , 3) by translation (– 1 , 1) is ....

e **In the opposite figure :**

À is the image of A by a translation ,

its magnitude is .... cm. in the

direction of ....



TEST ?

a A circle, its circumference is 88 cm. , calculate its surface area. (Consider $\pi = 3.14$)

b In the coordinates plane, draw the triangle ABC where A $(0 , 1)$, B $(2 , 3)$ and C $(-1 , 4)$, then find the image of the triangle ABC by translation $(2 , 3)$

---

a A case in the shape of a cuboid its base is a square of side length 8 cm. and the height of the case is 22 cm. Calculate its lateral area and its total area.

b In the coordinates plane , draw the rectangle ABCD where A $(4 , 2)$, B $(4 , 4)$, C $(1 , 4)$ and D $(1 , 2)$, then :

(1) Draw its image by translation $(x + 2 , y + 2)$

(2) Calculate the perimeter of the image of the rectangle ABCD

---

a The sum of edge lengths of a cube is 60 cm. Find :

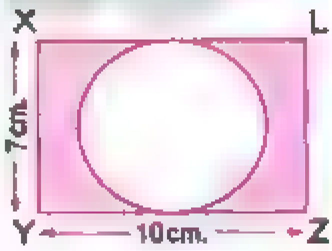
(1) The lateral area of the cube.

(2) The total area of the cube.

(3) The volume of the cube.

b **In the opposite figure :**

XYZL is a rectangle, its length is 10 cm. , its width is 7 cm. Calculate the area of the shaded part. (Consider $\pi = \frac{22}{7}$)

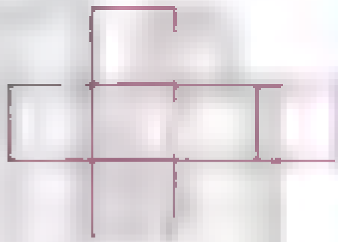


Activity of Unit Three

① Bring a sheet of cardboard (Bristol) , then cut a square from each corner whose side length is 15 cm. to form the opposite figure. Fold the shape and use the glue to form a cuboid without a lid. Use your instruments to calculate its lateral area and its total area.

② The opposite figure represents an unfolded cube. Copy the shape with the same given dimensions on a lattice. Show how you can fold it to get a cube , then calculate :

- a Its total area.
- b Its volume.



A research project

on unit three

Project aims

- Drawing a circle given its radius length.
- Finding the circumference and the area of the circle
- Applying geometry in decorative designs.
- Linking mathematics with arts.
- Linking mathematics with history.

Do a research project on the following topic

"Gamal Abdel Nasser is one of the most important leaders of nationalist movement against occupation in Egypt".

Discuss the following points using available resources

- Write, in Arabic, a short essay about Gamal Abdel Nasser and his role in 23 July 1952 revolution, explaining the reasons, the principles, and achievements of this revolution.
- Design a 23 July 1952 revolution memorial circular coin with a picture of Gamal Abdel Nasser in the middle. Calculate its circumference and its area.

UNIT 4

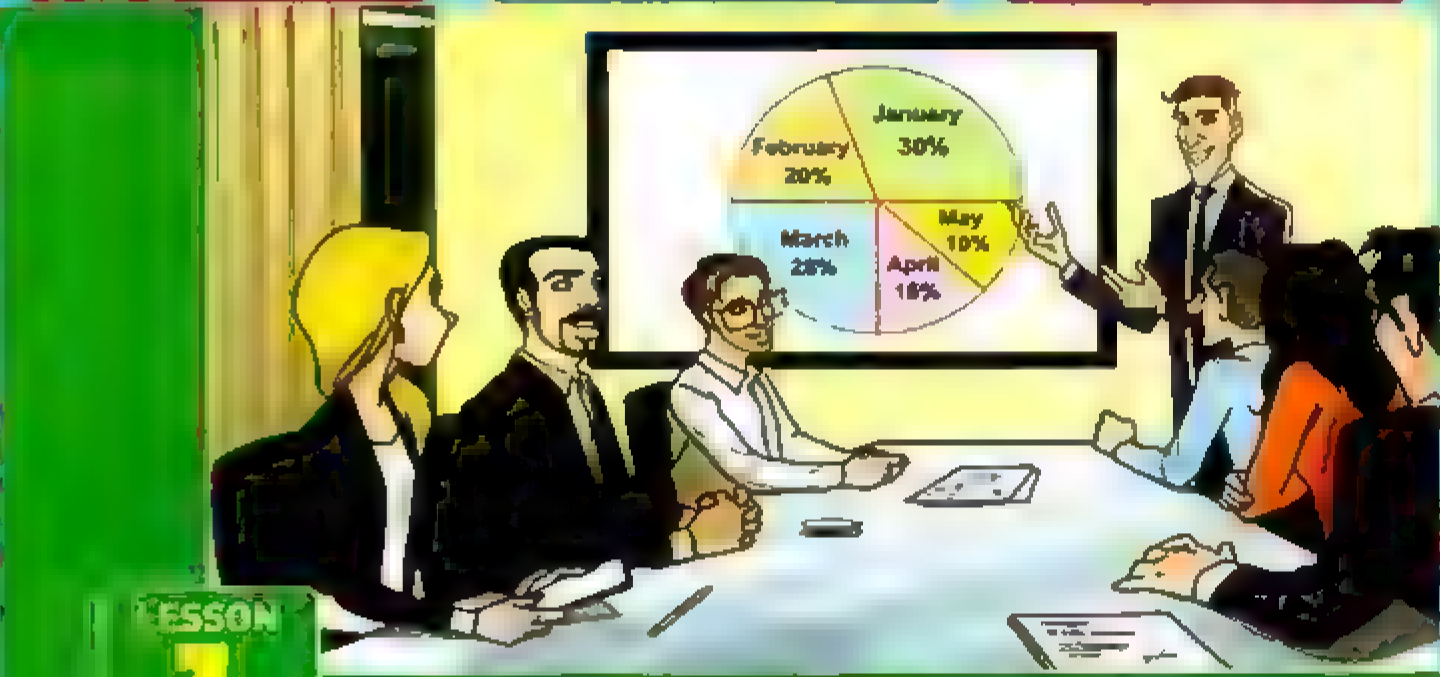
Statistics and probability

Scan the QR code to solve an interactive test on each lesson

UNIT AIMS

By the end of this unit, student should be able to :

- recognize the circular sector (minor - major).
- know that the sum of measures of the accumulative angles at the centre of the circle is 360°
- represent the statistical data by using the circular sectors (a pie chart)
- recognize the random experiment and the sample space
- write the sample space of a random experiment
- recognize the event in a random experiment
- calculate the probability of occurrence of an event



LESSON 1

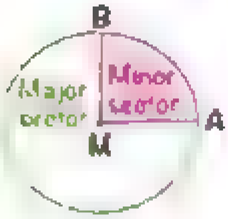
Representing the statistical data by using the circular sectors

- You have studied before how to represent data by **bar graph** , **broken line graph or double bars graph**.
- Now, we are going to present another type of graph called "pie chart" in which a circle is divided into sectors that each represent a proportion of the whole. It is a good way to show relative size of data
- When different items of data are presented on a pie chart, you can easily do a quick comparison between these items, and also between any item and the total.

For example

In the opposite figure :

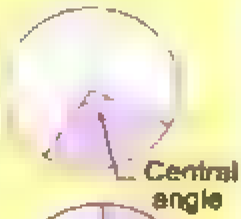
Each of the coloured part and the uncoloured part of the circle M represents a circular sector , where :



- The **coloured part AMB** is called "minor sector" because its area is less than $\frac{1}{2}$ the area of the circle.
- The **uncoloured part AMB** is called "major sector" because its area is more than $\frac{1}{2}$ the area of the circle.

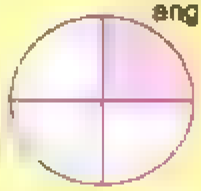
Notice that :

(1) Each circular sector has an angle whose vertex is the centre of the circle which is called a "**central angle**".

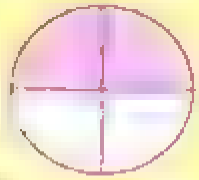


(2) The sum of the measures of angles accumulating around at a point as the centre of the circle is equal to $360°$

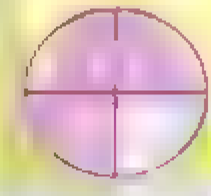
(3) • A quarter ($\frac{1}{4}$) of the area of a circle represents 25 % of the whole data.



• A half ($\frac{1}{2}$) of the area of a circle represents 50 % of the whole data.

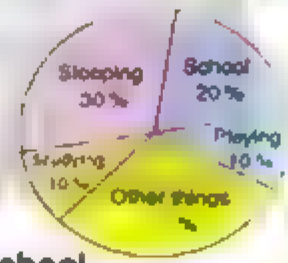


• Three quarters ($\frac{3}{4}$) of the area of a circle represent 75 % of the whole data.



Example (1)

The opposite figure represents the different activities which Sally does during a day. Study the figure , then answer the following questions :



[a] Find the percentage of the time that Sally spends at school.

[b] Find the percentage of the time that Sally spends in sleeping.

[c] Find the percentage of the time that Sally spends in other things.

[d] Complete : Sally spends the same percentage of the time in .... and .... (or : .... and ....)

Solution

[a] 20 % [b] 30 %

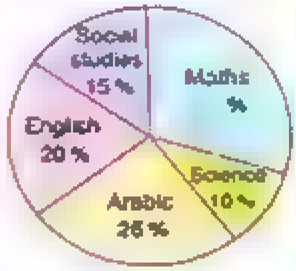
[c] 100 % – (30 % + 20 % + 10 % + 10 %) = 30 %

[d] Studying , playing (or : sleeping , other things)

LESSON 1

Example (2

The opposite figure represents the percentages of the favourite subjects of 200 pupils in a school



Answer the following questions :

[a] What is the ratio of the pupils who prefer English ?

[b] What is the ratio of the pupils who prefer Science ?

[c] What is the ratio of the pupils who prefer Mathematics ?

[d] Which sector represents the greatest ratio ?

[e] Which sector represents the smallest ratio ?

[f] Find the measure of the central angle of Maths in degree.

[g] How many pupils prefer studying English ?

Solution

[a] 20 % [b] 10 %

[c] 100 % – (15 % + 20 % + 25 % + 10 %) = 30 %

[d] Maths [e] Science

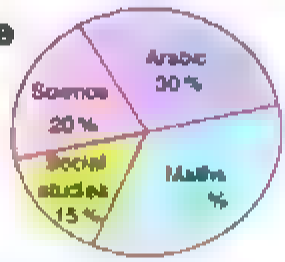
[f] The measure of the central angle of maths = 30 % of 360°

$= \frac{30}{100} \times 360° = 108°$

[g] The number of pupils = 20 % of 200 = $\frac{20}{100} \times 200$ = 40 pupils.

Try

- The opposite figure shows the percentages of time that Enas spends in studying different subjects.



Complete :

[a] The ratio of the time that Enas spends in studying maths is ....

[b] The measure of the central angle of science in degree is ....°

[c] The subject that needs more time is ....

Example (3

The following table shows the percentages of the number of hours that Marwa studied in different subjects in a week :

Subject	Arabic	Maths	Science	English
Percentage	10 %	40 %	20 %	30 %

Represent these data by a pie chart.

Solution

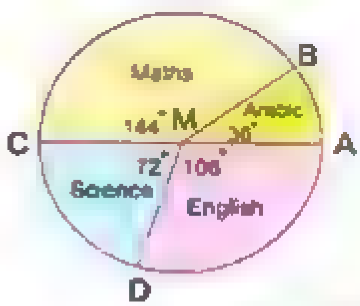
First : Find the measure of the central angle which represents the percentage of each sector as the following :

- The measure of the central angle for Arabic = $\frac{10}{100} \times 360° = 36°$
- The measure of the central angle for **Maths** = $\frac{40}{100} \times 360° = 144°$
- The measure of the central angle for **Science** = $\frac{20}{100} \times 360° = 72°$
- The measure of the central angle for **English** = $\frac{30}{100} \times 360° = 108°$

Second : Draw a circle of a suitable radius , with centre M

Third : Draw the radius $\overline{MA}$, use your protractor to draw the central angle AMB with the measure of 36° The sector AMB represents Arabic.

Fourth : Similarly , draw ∠ BMC of measure 144° The sector BMC represents Maths , using the same method , draw the other two sectors , then you will have the opposite figure.



LESSON **1**

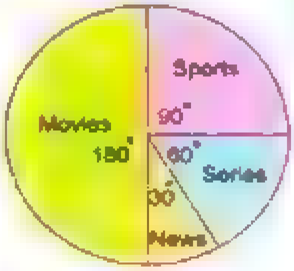
Example (4

The following table shows the favourite TV programs for some pupils :

TV program	Sports	News	Series	Movies
Number of pupils	15	5	10	30

Represent this data by a pie chart.

Solution



The sum of pupils = 15 + 5 + 10 + 30 = 60 , then :

- The measure of the central angle for **sports**

  $= \frac{15}{60} \times 360° = 90°$

- The measure of the central angle for **news** $= \frac{5}{60} \times 360° = 30°$
- The measure of the central angle for **series** $= \frac{10}{60} \times 360° = 60°$
- The measure of the central angle for **movies** $= \frac{30}{60} \times 360° = 180°$

Try

- **The following table shows the percentage of the production of egg in three farms monthly :**

Farm	First	Second	Third
Percentage of egg production	50 %	20 %	30 %

Represent these data by a pie chart.

Exercise 14

Representing the statistical data by using the circular sectors

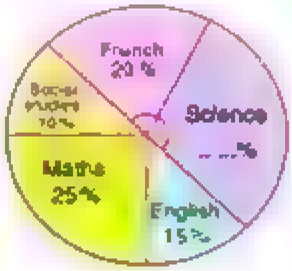
From the school book

1 The opposite figure shows the percentages of sales of different types of books. Complete :

a The sales percentage of science books is ....

b The least sales percentage is in ....

c The ascending order of books types according to the percentage of sales is : ...., ..., ..., ... and ....



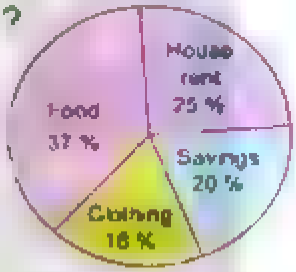
2 The opposite figure shows the percentages of family spend in different purposes. Study the figure , then answer the following questions

a What is the ratio (in fraction) of clothing to house rent ?

b What is the ratio (in fraction) of clothing to food ?

c What is the ratio (in fraction) of clothing to savings ?

d Find the measure of the central angle of clothing in degrees.



3 The opposite figure shows the favourite hobbies for the pupils of one of the classes in the sixth primary , study the figure , then answer

a What is the ratio of the theatre with respect to all hobbies ?

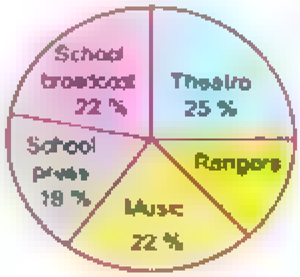
b What is the ratio of the broadcast with respect to all hobbies ?

c What is the ratio of the rangers with respect to all hobbies ?

d What is the measure of the central angle of the sector of the music ?

e What is the hobbies that the least pupils prefer ?

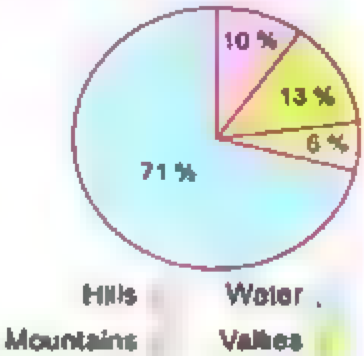
f What is the hobbies that the most pupils prefer ?



LESSON 1

4 The opposite figure shows the distribution of the natural components of the earth's surface , study the figure , then complete the following table

The components of the earth's surface	Water natural supplies	Vallies	Hills	Mountains
The percentage of the forming	....	....	....	....

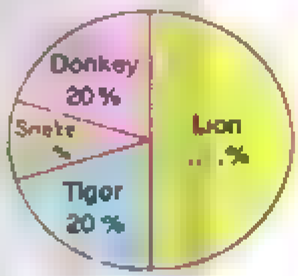


a What is the component which represents the smallest ratio of the earth's surface ?

b What is the component which represents the greatest ratio of the earth's surface ?

c What is the measure of the central angle of the sector of the vallies ?

5 The opposite figure shows the percentages of the favourite animals sites at the zoo. Study the figure , then answer the questions :



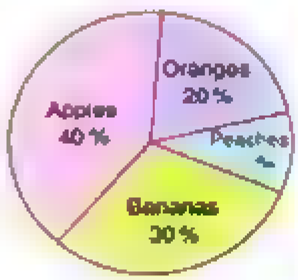
a Which animal site is favoured by almost half of the people ?

b Which two animals are favoured by almost the same number of people ?

c What is the percentage of lion site ?

d What is the percentage of donkey and snake sites ?

6 Forty students were surveyed about their favourite fruit. The opposite figure represents the outcome of the survey. Study the figure , then answer the question :



a How many students like apples ?

b How many students like bananas and peaches ?

c What is the ratio (in fractions) of students who like apples and oranges ?

d Arrange the percentage of students in a descending order according to their favourite fruit.

7 The following table shows the percentage of the production of meat in 3 slaughter houses during a month :

Slaughter house	1^{st}	2^{nd}	3^{rd}
The percentage	20 %	30 %	50 %

Represent these data by circular sectors. (Red Sea 2016)

8 The following table shows the percentage of the egg production in three farms , a merchant collected these eggs to distribute them on the grocery stores , represent these data by using the circular sectors :

The farm	First	Second	Third
The percentage of the production	25 %	35 %	40 %

(Alex. 2016)

9 The following table shows the ratio for producing electronic sets :

Set kind	1^{st}	2^{nd}	3^{rd}	4^{th}
The ratio of production	30 %	15 %	40 %	15 %

Represent these data by a pie chart.

10 The following table shows the percentage of the production of one factory of 4 kinds of electric sets :

Type of the set	TV	Washing machine	Refrigerator	Cooker
Amount of the production	35 %	20 %	15 %	30 %

Represent these data by using the circular sectors. (Damietta 2013)

11 The following table shows the percentage of the favourte sports of students in your class :

The favourite sport	Football	Basketball	Volleyball	Swimming	Ping-Pong
Percentage	45 %	9 %	24 %	10 %	12 %

Represent the previous data by using the circular sectors.

LESSON 1

12 The following table shows the product of 4 farms in a month :

The farm	1^{st}	2^{nd}	3^{rd}	4^{th}
Percentage	40 %	25 %	20 %	15 %

a Represent these data by using the circular sectors.

b If the total product of these farms in a month was 1200 chicken.

Find the product of the 1^{st} farm in this month

13 The following table shows the rate of the score of 200 students in one school of Cairo governorate :

Rate	Excellent	Good	Pass	Weak
Percentage	15 %	50 %	25 %	10 %

a Represent these data by a pie chart.

b Find the number of excellent students.

14 The following table shows the percentage of chicken production for three farms during October :

The farm	First	Second	Third
The percentage of production	25 %	50 %	....

a Complete the previous table.

b Represent these data by the circular sectors.

15 The following table shows the ratio for producing chickens in four farms in a month :

Farm	1^{st}	2^{nd}	3^{rd}	4^{th}
The ratio of production	10 %	35 %	30 %	....

a Complete the table.

b Represent these data by a pie chart.

16 Nahid is a clerk in an institution. She contributes with her husband by her salary as follows :

25 % for house rent , 50 % for food and expenses and 25 % for savings.

Represent those data by using the circular sectors. (The New Valley 201[illegible])

17 One of the families spends its salary as the following : 40 % for food , 20 % for house rent , 30 % for expenses and saves the remainder , represent these data by using the circular sectors , then answer the following :

- If the family monthly income is L.E. 900, so how much does the family save in the year ?
- Another family spends its monthly salary by the same way and saves L.E. 70 monthly , so what is the monthly salary of that family ?

18 The following table shows the number of studying hours that Mohamed has done in a week :

Subject	Arabic	Maths	Science	English	Social studies	Total
Number of hours	9	10	6	7	4	36

Represent these data by a pie chart. ([illegible] Sohag 201[illegible])

19 The following table shows the favourite TV programmes which the pupils of one of the classes in the primary six watch as the following :

Kind of programme	Entertaining	Cultural	News	Drama	Sport
Number of hours	9	5	4	7	11

Represent the data by using the circular sectors, then answer the following questions :

What is the programme that the most of pupils prefer, also the least of pupils prefer ?

LESSON 1

20 Complete each of the following .

a The sum of the measures of the accumulative angles around the centre of the circle = ...° (El-Kalyoubia 2016)

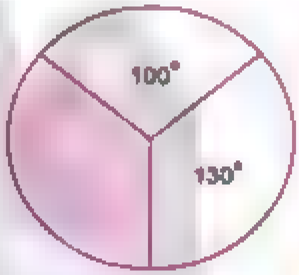
b The measure of the angle of the circular sector whose area represents $\frac{1}{2}$ from the area of the circle = ...° (Beni Suef 2013)

c The measure of the angle of the circular sector whose area represents $\frac{1}{4}$ from the area of the circle = ...° (Qena 2017)

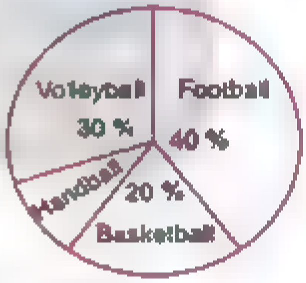
d The measure of the angle of the circular sector whose area represents $\frac{1}{6}$ from the area of the circle = ...° (Ismailia 2016)

e The measure of the angle of the circular sector whose area represents $\frac{1}{8}$ from the area of the circle = ...° (Assiut 2012)

f In the opposite figure , the measure of the central angle of the shaded circular sector equals ....° (Cairo 2016)



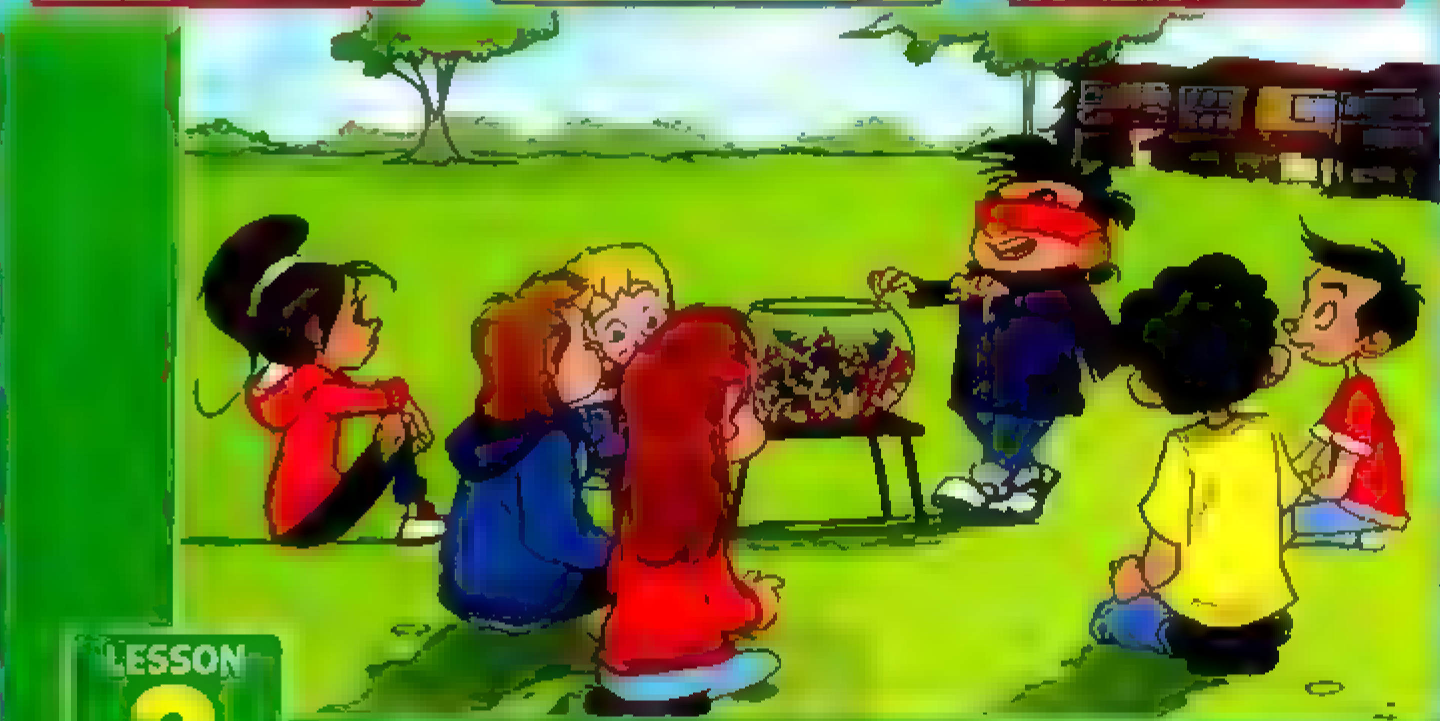
g The opposite figure represents the percentages of distribution of the sports activities for the pupils of a school , their number is 960 pupils.



(1) The percentage of the pupils participated in handball = ... %

(2) The number of pupils who participated in football activity = ... pupils.

(3) The measure of the central angle of the sector representing the pupils who are participating in volleyball activity = ...° (Aswan 2015)



LESSON 2

Random experiment

A box contains 3 balls (red, green, black)

I'm not sure if the selected ball will be red or green or black

This is a random experiment

A box contains 3 red balls

I'm sure that the selected ball will be red

This is NOT a random experiment

Definitions

- **The random experiment**

  It is an experiment in which we can determine all its possible outcomes before carrying it out , but we can't predict in certainty which of these outcomes will occur when the experiment is carried out.

- **Sample space (outcomes space)**

  It is the set of all possible outcomes for a random experiment.
  It is usually denoted by the symbol (S) and the number of all elements of the sample space is denoted by n (S)

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Examples of random experiments :

1. Tossing a coin is a random experiment, because before tossing the coin you do not know the result.

2. Rolling a die is a random experiment, because before rolling the die you do not know the result.

3. Drawing a marble from a bag containing marbles different in colour (or size) is also a random experiment, because before drawing the marble you do not know the result.

Example (1)

Write the sample space of each of the following random experiments and give the number of its elements :

[a] Tossing a coin once and observing its apparent face.

[b] Rolling a die once and observing the number appearing on the upper face.

[c] Drawing one card from five cards numbered from 10 to 14 and observing the written number on the card.

[d] Drawing a ball from a bag containing : one black ball , one red ball , one yellow ball and one white ball and observing the colour of the drawn ball.

[e] Playing a football match between the team of your school and the team of another school , and observing the possible results of the team of your school.

[f] Choosing a prime number less than 18

Solution

[a] The possible outcomes are : head (H) or tail (T)

$\therefore S = \{H , T\}$, $n(S) = 2$

[b] The possible outcomes are : 1 , 2 , 3 , 4 , 5 , 6

$\therefore S = \{1 , 2 , 3 , 4 , 5 , 6\}$, $n(S) = 6$

[c] $S = \{10, 11, 12, 13, 14\}$, $n(S) = 5$

[d] $S = \{\text{black}, \text{red}, \text{yellow}, \text{white}\}$, $n(S) = 4$

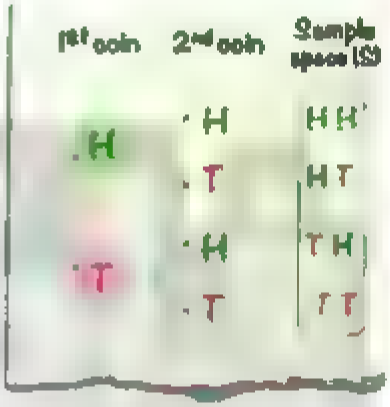
[e] $S = \{\text{win}, \text{loss}, \text{draw}\}$, $n(S) = 3$

[f] $S = \{2, 3, 5, 7, 11, 13, 17\}$, $n(S) = 7$

Example 2

Write the sample space of tossing two distinct coins once and give the number of its elements.

Solution



We can get the elements of the sample space by using the tree diagram as shown in the opposite diagram.

$\therefore S = \{HH, HT, TH, TT\}$, $n(S) = 4$

Where :

HH means that the result of tossing the coins in the first coin is head and the second is head,

HT means that the result of tossing the coins in the first coin is head and the second is tail and so on.

Remarks

1. The sample space of tossing two distinct coins (different in colour or size or shape, ...) simultaneously is the same as the sample space of tossing one coin twice one after the other.
2. The sample space of rolling two distinct dice is the same as the sample space of rolling a die two consecutive times.

LESSON 2

Try by yourself

- **Write the sample space of each of the following random experiments and give the number of its elements :**

  [a] Drawing a ball from a bag containing : one white ball , one green ball and one red ball and observing the colour of the drawn ball.

  [b] Tossing a coin twice.

Example 3

A box contains three balls. One of them is white , the second is red and the third is black.

The experiment is drawing two balls one after the other with replacement and observing their colours.

State the sample space and give the number of its elements.

Solution

Let W denote the white ball , B denote the black one and R denote the red one.

Using the opposite tree diagram , we get :

S = {WW , WR , WB , BW , BR , BB , RW , RR , RB} , n (S) = 9

1st drawing	2nd drawing	Sample space (S)
W	W	WW
	R	WR
	B	WB
B	W	BW
	R	BR
	B	BB
R	W	RW
	R	RR
	B	RB

Example 4

From the set of digits {4 , 5} , a number is formed from two digits.

Determine the sample space of this experiment and give the number of its elements.

Solution

From the opposite tree diagram :

S = {44 , 45 , 54 , 55} , n (S) = 4

Tens digit	Units digit	Sample space (S)
4	4	44
	5	45
5	4	54
	5	55

Exercise 15

Random experiment

Interactive test

📖 From the school book

1 Determine the sample space of each of the following random experiments , and give the number of its elements :

a Choosing a card from 5 cards numbered from 3 to 7 and observing the written number on the card.

b Choosing one of the digits of the number 23791

c Choosing an even number included between 21 and 29

d Choosing a prime number included between 10 and 20

e Drawing a ball from a bag containing : one green ball , one yellow ball and one black ball and observing the colour of the drawn ball.

2 A box contains 9 identical cards having the same colour and numbered from 1 to 9

Write the sample space for this experiment.

3 A bag contains 4 identical cards having the same colour and numbered from 30 to 33 Write the sample space for this experiment.

4 📖 If the random experiment of visiting one of your relatives to know the gender of his newly-born child.

Write the sample space of this experiment.

5 Write the sample space of tossing a coin twice in succession and observing the sequence of heads and tails showing the number of its elements.

6 In the experiment of selecting a ball from a box containing 3 red balls, 4 yellow balls all of them are equal in volume, observing the colour of the selected ball , write the sample space of this experiment.

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7 In an experiment of getting a 2-digit number using the digits 1 and 2
Write the sample space of this experiment.

8 From the set of digits $\{3, 4, 9\}$, a number is formed from two digits.
Determine the sample space of this experiment and give the number of its elements.

9 A family has three children (there are no twins among them).
Write down the sample space (S) of the gender (boy or girl) of each of them ordering them according to their ages.

10 Determine the sample space of tossing three distinct coins once and observing the sequence of appearance of heads and tails.

11 A die is designed such that two faces have the number 1, two faces have the number 2 and two faces have the number 3, this die is thrown once and the apparent number on the upper face is observed.
Write down the sample space and give the number of its elements.

12 A coin is tossed, then a die is thrown and the upper faces of the coin and the die are observed.
Write down the sample space.

13 Complete each of the following

a The random experiment is ....

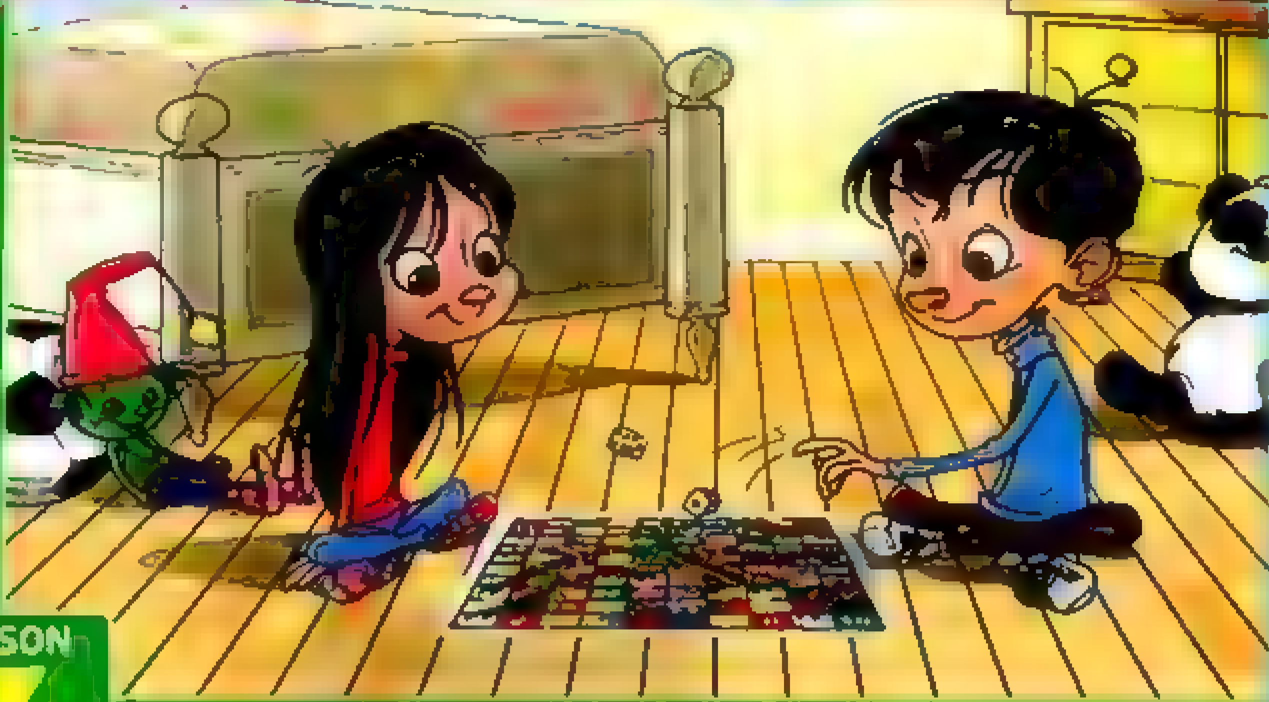
b .... is the set of all possible outcomes for a random experiment.

(Suez 2015)

c In the experiment of tossing a regular coin and observing the appearing face, set of sample space S = ..., n (S) = ... (Giza 2012)

d The sample space for tossing a coin twice = ...

e The sample space for rolling a dice once = ...



LESSON 3

Probability

The event In a random experiment, an event is a subset of the sample space.

Example

In the random experiment of rolling a die once and observing the apparent number on the upper face, we have :

$$S = \{1, 2, 3, 4, 5, 6\}$$

Any subset of S can be considered as an event as :

- $\{5\}$ is the event of getting 5 on the upper face of the die.
- $\{2, 4, 6\}$ is the event of getting an even number on the upper face of the die.
- $\{1, 3, 5\}$ is the event of getting an odd number on the upper face of the die.

Probability of occurrence of an event

In a random experiment with a sample space **S**, if each element of **S** has the same chance to occur, and if $A \subset S$, then the probability that the event **A** will occur is :

$$P(A) = \frac{\text{The number of elements of } A}{\text{The number of elements of } S} = \frac{n(A)}{n(S)}$$

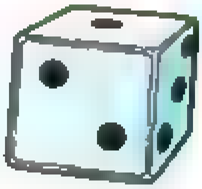
LESSON 3

Example (1)

If a fair die is thrown once and we observe the number on the upper face , find the probabilities of each of the following events :

[a] A is the event of appearance of a number greater than 4

[b] B is the event of appearance of an even number.



[c] C is the event of appearance of the number 5

[d] D is the event of appearance of the number 7

[e] E is the event of appearance of a number less than 7

Solution

$S = \{1 , 2 , 3 , 4 , 5 , 6\}$, $n(S) = 6$

[a] $A = \{5 , 6\}$, $n(A) = 2$ $\quad \therefore P(A) = \frac{2}{6} = \frac{1}{3}$

[b] $B = \{2 , 4 , 6\}$, $n(B) = 3$ $\quad \therefore P(B) = \frac{3}{6} = \frac{1}{2}$

[c] $C = \{5\}$, $n(C) = 1$ $\quad \therefore P(C) = \frac{1}{6}$

[d] $D = \{\ \}$ or $\varnothing$, $n(D) =$ zero

$\therefore P(D) = \frac{0}{6} =$ zero (The impossible event)

[e] $E = \{1 , 2 , 3 , 4 , 5 , 6\}$, $n(E) = 6$

$\therefore P(E) = \frac{6}{6} = 1$ (The certain event)

Try

- **A fair die is thrown once. Find the probability of each of the following events :**

[a] A : The event of getting the number 6

[b] B : The event of getting a number less than 3

[c] C : The event of getting an odd number.

[d] D : The event of getting the number 8

[e] E : The event of getting a prime even number.

[f] F : The event of getting a number greater than 1

Remarks

1. The impossible event is the event which cannot occur.
   - The impossible event = $\varnothing$ while the probability of the impossible event = zero

     $\therefore P(\varnothing) = 0$
2. The certain event (sure event) is the event whose outcomes are all the possible outcomes.
   - The certain event = S while the probability of the certain event = 1

     $\therefore P(S) = 1$
3. The possible event : some of outcomes of the experiment.

   i.e. The probability of the possible event = proper fraction.

So, the probability of any event is not less than zero and it is not more than 1

i.e. For any event A , we found that : $0 \le P(A) \le 1$

Example 2

A jar contains 9 similar balls "4 of them are white , 3 are red and 2 are black"

A ball is drawn randomly.

Calculate the probability of each of the following events :

[a] A : The drawn ball is white.

[b] B : The drawn ball is red.

[c] C : The drawn ball is green.

[d] D : The drawn ball is white or black

[e] E : The drawn ball is not black.

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Solution

[a] $P(A) = \frac{\text{The number of white balls}}{\text{The number of all balls}} = \frac{4}{9}$

[b] $P(B) = \frac{\text{The number of red balls}}{\text{The number of all balls}} = \frac{3}{9} = \frac{1}{3}$

[c] $P(C) = \frac{\text{The number of green balls}}{\text{The number of all balls}} = \frac{0}{9} = 0$

[d] $P(D) = \frac{\text{The number of white balls and black balls}}{\text{The number of all balls}} = \frac{4+2}{9} = \frac{6}{9} = \frac{2}{3}$

[e] $P(E) = \frac{\text{The number of balls which are not black}}{\text{The number of all balls}} = \frac{9-2}{9} = \frac{7}{9}$

Example 3

From the set of digits {3 , 4 , 5} , form all possible two-digit numbers , then find the probability of each of the following events :

[a] A "the event that the units digit is odd"

[b] B "the event that the tens digit is even"

[c] C "the event that the two digits are odd numbers"

[d] D "the event that the sum of the two digits = 8"

[e] E "the event that the product of the two digits = 20"

Solution

$S = \{33 , 43 , 53 , 34 , 44 , 54 , 35 , 45 , 55\}$, $n(S) = 9$

[a] $A = \{33 , 43 , 53 , 35 , 45 , 55\}$, $n(A) = 6$ $\therefore P(A) = \frac{6}{9} = \frac{2}{3}$

[b] $B = \{43 , 44 , 45\}$, $n(B) = 3$ $\therefore P(B) = \frac{3}{9} = \frac{1}{3}$

[c] $C = \{33 , 53 , 35 , 55\}$, $n(C) = 4$ $\therefore P(C) = \frac{4}{9}$

[d] $D = \{53 , 44 , 35\}$, $n(D) = 3$ $\therefore P(D) = \frac{3}{9} = \frac{1}{3}$

[e] $E = \{54 , 45\}$, $n(E) = 2$ $\therefore P(E) = \frac{2}{9}$

Remarks

1. The probability can be written as a fractional form , decimal form or in the form of percentage.

   i.e. if the probability of an event = $\frac{2}{5}$, it can be written as 0.4 or 40%

2. The sum of probabilities of all outcomes of a random experiment = 1

3. If the probability of occurrence of an event A is P (A)

   , then the probability that it doesn't occur = 1 – P (A)

   For example :

   If the probability of success of a student is $\frac{7}{10}$,

   then the probability of his failure = $1 - \frac{7}{10} = \frac{3}{10}$

Example (4

A card is selected randomly from 30 cards numbered from 1 to 30

Find the probability of each of the following events :

[a] A : The selected card carries a number divisible by 5

[b] B : The selected card carries a number divisible by 9

[c] C : The selected card carries a number satisfying the equation : $2x + 3 = 15$

[d] D : The selected card carries a number satisfying the inequality : $x - 5 \geq 22$

[e] E : The selected card carries a number satisfying the inequality : $6 < x \leq 12$

Solution

S = {1 , 2 , 3 , 4 , ... , 30} $\therefore$ n (S) = 30

[a] A = {5 , 10 , 15 , 20 , 25 , 30} , n (A) = 6 $\therefore P(A) = \frac{6}{30} = \frac{1}{5}$

[b] B = {9 , 18 , 27} , n (B) = 3 $\therefore P(B) = \frac{3}{30} = \frac{1}{10}$

LESSON 3

[c] $\because 2x + 3 = 15$ $\therefore 2x = 12$

$\therefore x = 6$ $\therefore$ The S.S. $= \{6\}$

$\therefore C = \{6\}$, $n(C) = 1$ $\therefore P(C) = \frac{1}{30}$

[d] $\because x - 5 \geq 22$ $\therefore x \geq 27$

$\therefore$ The S.S. $= \{27, 28, 29, 30\}$

$\therefore D = \{27, 28, 29, 30\}$, $n(D) = 4$

$\therefore P(D) = \frac{4}{30} = \frac{2}{15}$

[e] $\because 6 < x \leq 12$

$\therefore$ The S.S. $= \{7, 8, 9, 10, 11, 12\}$

$\therefore E = \{7, 8, 9, 10, 11, 12\}$, $n(E) = 6$

$\therefore P(E) = \frac{6}{30} = \frac{1}{5}$

Example (5)

A bag contains 45 similar marbles. Wael drew one of them randomly and found it green. If the probability of drawing a green marble = $\frac{3}{5}$ Find the number of green marbles in the bag.

Solution

The number of all marbles = 45

Let A be the event of drawing a green marble , then $P(A) = \frac{3}{5}$

$$\therefore \frac{\text{The number of green marbles}}{\text{The number of all marbles}} = \frac{3}{5}$$

$$\therefore \frac{\text{The number of green marbles}}{45} = \frac{3}{5}$$

$\therefore$ The number of green marbles $= \frac{3}{5} \times 45 = 27$ marbles.

Example (6)

Two players play in a football team. During training , one of them kicked 20 penalty kicks and he scored 14 goals , and the other kicked 25 penalty kicks and he scored 18 goals. Which of them should you choose to kick a penalty kick in the game? why?

Solution

$\because$ The probability that the first player scores a goal $= \frac{14}{20} = 0.7$

The probability that the second player scores a goal $= \frac{18}{25} = 0.72$

$\because 0.72 > 0.7$

$\therefore$ The second player should kick the penalty because his probability is greater.

Exercise 16

Probability

From the school book

1 While throwing a fair die and observing the upper face , complete the following

a The probability of appearance of a number greater than 2 = ……

b The probability of appearance of a number less than 3 = ……

c The probability of appearance of an odd number = ……

d The probability of appearance of the number 5 = ……

e The probability of appearance of the number 6 = ……

f The probability of appearance of the number 7 = ……

g The probability of appearance of a number less than or equal to 6 = ……

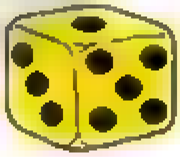
h The probability of appearance of a prime number = ……

i The probability of appearance of a prime even number = ……

j The probability of appearance of a number divisible by 5 = ……

k The probability of appearance of the number 5 or the number 6 = ……

2 A fair die is rolled once and the number of dots on the upper face is observed , write down the sample space , then find the probability of the following events :



a Getting a number greater than 6

b Getting a number satisfying the inequality : $1 \leq x \leq 6$

c Getting a number satisfying the inequality : $2 < x < 4$

3 In the experiment of tossing a regular die once and observing the number of dots on the upper face , find the probability of :

a The event A , where A is the event of appearance of a number less than 5

b The event B , where B is the event of appearance of a number satisfying the inequality : $x \geq 3$

4 A box contains 8 white balls and 12 red balls all of them are symmetric , a ball is selected without looking inside the box , find the following probabilities :

a The selected ball is white.
b The selected ball is red.
c The selected ball is blue

(South Sinai 2014)

5 A box contains 8 white balls , 5 red balls and 7 blue balls , all balls identical , if a ball is chosen randomly. Find the probability of :

a The chosen ball is white.
b The chosen ball is not red.
c The chosen ball is white or red or blue.

(Cairo 2013)

6 A bag contains 25 balls (4 balls are yellow , 7 balls are red and the remainder is black). If a ball is drawn randomly , find the probability that the drawn ball is :

a black.
b yellow or black.
c not yellow.
d green.
e neither black nor yellow.

7 In the experiment of selecting a card at random from 7 equal cards numbered from 1 to 7 , write the sample space , then find the probability of :

a The event A , where A is the appearance of a number less than 4
b The event B , where B is the appearance of an odd number.
c The event C , where C is the appearance of a number more than 5

(Matrouh 2015)

8 A basket contains 15 balls numbered from 1 to 15 , if one of the balls is chosen randomly , write the sample space for this experiment , then find the probability that the chosen ball :

a carried an odd number.
b carried a prime number.
c carried a number divisible by 3

(El-Beheira 2017)

LESSON **3**

9 A box contains 10 cards numbered by the even numbers from (2 to 20), one of the cards is selected at random. Calculate the probability of :

a The event A : appearance of a multiple of the number 4

b The event B : appearance of an even number.

c The event C : appearance of a number that is divisible by 3

10 A number is chosen randomly from the numbers 1, 2, 3, ..., 40

Find the probability of the following events :

a A : the chosen number is a multiple of 3

b B : the chosen number is divisible by 7

c C : the chosen number is greater than 16 and less than 25

d D : the chosen number is a prime number less than 16

e E : the chosen number is divisible by 2 and 3

f F : the chosen number has 7 as a units digit.

11 By using cardboard, cut 10 squared or rectangular equal cards and have the same colour, then write a number in each one of them from the numbers (1 to 10), then put them in a bag that is not transparent and mix them carefully, choose one of them at random

Calculate the probability of the following events :

a The event A : appearance of a number more than 7

b The event C : appearance of an odd number.

c The event B : appearance of a number that satisfies the inequality :

$x \leq 10$

d The event D : appearance of a number that satisfies the equation :

$x - 4 = 2$

12 If the experiment of a student is chosen at random from a class of 40 students , 32 students succeeded in Maths test , 35 students succeeded in Arabic test , find the probability of :

a The event A , where A is the event that he succeeded in Arabic.

b The event C , where C is the event that he failed in Maths.

13 In the ideal student competition of one of the schools 63 students applied for the competition , if the probability that one of the girls is an ideal student is $\frac{4}{9}$ Find the number of girls who participate in the competition.

14 A box contains 80 similar balls Some of them are red and the rest is blue. If the probability of drawing a red ball is $\frac{1}{4}$, find the number of blue balls.

15 In a meeting for discussing the problems of the workers in a factory , 100 workers were attending from men and women. If the probability of a man standing to show the problems of the workers is $\frac{3}{5}$

Calculate the number of the men and women in this meeting.

16 Complete the following :

a The event is a subset of the ....

b In an experiment of throwing a fair die once , the event of getting a number less than 2 is { .... }

c The probability of the impossible event = ...

and the probability of the certain event = ...

d For every event A , we find that .... $\leq$ P (A) $\leq$ ....

e If S is the sample space of a random experiment , then P (S) = ...

(Matrouh 2012)

f If n (S) = 12 , n (A) = 4 where A $\subset$ S , then P (A) = $\frac{....}{....}$ (2012)

g If a fair coin is tossed once , then the probability of appearance of a tail = ...

LESSON 3

h In an experiment of forming a number from the two digits $\{2, 3\}$, the probability of getting an even number = ... (El-Monofia 2017)

i If the probability of occurrence of the event A is $\frac{5}{7}$, then the probability of non-occurrence of it = ... (El-Fayoum 2017)

j If the probability that a pupil solve a problem is 0.7, then the number of problems expected to be solved from the same kind from 20 problems = ... (El-Menia 2016)

17 Choose the correct answer from those given :

a If A is an event of the sample space of random experiment, then : $0 \leq P(A) \leq$ .... (Giza 2012)

(a) $\varnothing$ (b) -1 (c) 2 (d) 1

b If P (A) = 1, then A is called .... event. (Ismailia 2012)

(a) a random (b) an impossible (c) a possible (d) a certain

c $\varnothing$ is the empty set, then $P(\varnothing)$ = ... (Port Said 2015)

(a) 0 (b) 2 (c) 1 (d) 0.5

d If A = S, then P (A) = ... (Kafr El-Sheikh 2016)

(a) zero (b) 1 (c) 2 (d) 3

e Which of the following can be a probability of an event ? (El-Beheira 2017)

(a) zero % (b) 1.2 (c) $\frac{17}{16}$ (d) 101 %

f A coin was tossed once, then the probability of getting a head is .... (Red Sea 2014)

(a) zero (b) 2 (c) $\frac{1}{2}$ (d) 0.25

g In the experiment of rolling a fair die once, if A is the event of getting a number less than 4, then P (A) = ... (Qena 2013)

(a) $\frac{5}{6}$ (b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{6}$

h If a fair die is rolled once, then the probability of getting a number > 6 = ... (El-Kalyoubia 2017)

(a) $\varnothing$ (b) zero (c) $\frac{1}{6}$ (d) $\frac{1}{3}$

i If the probability of success of a student is 70 % , then the probability of his failure = ...

(a) 0.7 (b) 0.07 (c) 0.3 (d) 0.03

j A regular coin is tossed 1000 times , then the most expected number to get a head equals ...... (Alexandria 2013)

(a) 496 (b) 503 (c) 600 (d) 999

18 In the experiment of forming a 2-digit number from the set of digits $\{5 , 6\}$

What is the probability :

a The event A : the units digit is an odd number.

b The event B : the sum of the two digits is 11

c The event C : the two digits are equal.

19 In one of the factories that produce saving energy electric bulbs , if the average of the daily production is 600 bulbs and the ratio of the damaged bulbs is about 5 % of the production.

Complete :

a The number of the damaged bulbs during the daily production = ...

b The number of the working bulbs during the daily production = ...

c The probability that the bulb is working = ...

d The probability that the bulb is damaged = ...

e If the production of the factory during few days is 2500 bulbs, so the number of the damaged bulbs ......

20 The set $\{2 , 3 , 5\}$ is used to write a 2-digit number.

Find the probability of the following events :

a The tens digit is odd.

b The units digit is odd.

c The sum of the two digits is 7

d The product of the two digits is 15

LESSON 3

21 In the experiment of forming a number consisting of two digits without repeating the number using the set of numbers {1 , 2 , 3} Find :

a The probability of getting an odd prime number.

b The probability of getting an even number.

22 A class of 40 students has got a maths exam whose maximum mark is 50 , if 30 students got less than 40 marks , and 10 students got (40 up to 50). Calculate the probability of :

a The event A : where A is a student who has got less than 40 marks.

b The event B : where B is a student who has got a mark satisfying the inequality : $B \geq 40$

23 In one of the fitness centres , 10 ladies suffering from over weight were waiting to meet the specialized doctor , if the weights of 4 of them are between 100 , 110 kg. , the weights of the others are between 110 , 120 kg. Find the following probabilities :

a Entrance of a lady of weight less than 110 kg.

b Entrance of a lady of weight more than 110 kg.

c Entrance of a lady of weight 90 kg.

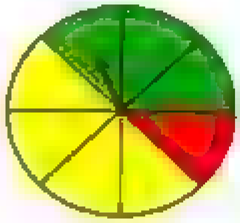
24 In a class of 42 students , we found that 20 students play football , 8 students play basketball and the rest of students play other sports. One of those students is chosen randomly , find :

a The probability that the student is one of those players who play football.

b The number of students who play other sports if the total number of students at school is 600

25 Two players play in a football team. During training , one of them kicked 21 penalty kicks and he scored 18 goals and the other kicked 32 penalty kicks and he scored 25 goals. Which of them should you choose to kick a penalty kick in the game ? Why ?

26 The opposite figure represents a spinner game



a Find the probability that the pointer stops at :

(1) the red colour.

(2) the green colour.

(3) the yellow colour.

b Find the probability that the pointer does not stop at the red colour.

27 The following table shows a sample formed from 200 TV viewers of TV programs, they were asked about their prefered program :

Program	Sports	News	Series	Films	Songs
Number of viewers	70	20	45	35	30

If a viewer is chosen at random, what is the probability that he is a viewer of ?

a News. b Songs. c Sports. d Series. e Films.

28 In one of the classes of the sixth primary, the teacher of Maths classified the levels of the students in his subject into (weak - intermediate - advanced) their number is 40 students and recorded his data in the opposite table :

The level	Number of the students
Weak	5
Intermediate	25
Advanced	10
The sum	40

One of the students in this class is chosen at random.

Calculate the probability of :

a The event A : where A is a weak student.

b The event B : where B is an advanced student.

c The event C : where C is not an intermediate student.

Test on Unit Four

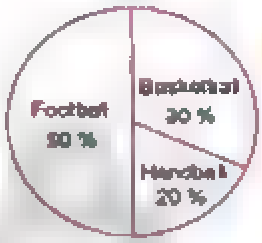
Answer the following questions

1 Choose the correct answer

a The sum of measures of the accumulative angles about the centre of a circle = ... (90° or 180° or 270° or 360°)

b A regular die is tossed once , then the probability of appearance an even number = ... (1 or $\frac{1}{2}$ or $\frac{1}{3}$ or $\frac{1}{6}$)

c The opposite figure represents the percentages of distribution of the sport activities for the pupils in a class of a school , their number is 40 pupils , then the number of pupils who participated in basketball = ... pupils. (20 or 12 or 8 or 5)



d The measure of the central angle of the circular sector whose area represents $\frac{1}{8}$ from the area of the circle = ... $^\circ$ (30 or 45 or 60 or 90)

e The probability of the impossible event = ... ($\varnothing$ or 1 or 0 or 2)

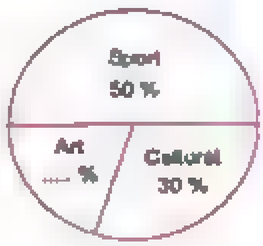
2 Complete each of the following

a In the experiment of tossing a regular coin once , then the set of the sample space (S) = ...

b **In the opposite figure :**

(1) The percentage of the members who prefer art is .... %

(2) The measure of the central angle of cultural = ... $^\circ$



c A class of 50 pupils , if the probability of success for those pupils in the end year exam is 0.8 , then the expected number for the pupils who will succeed = ...

d If S is the sample space of a random experiment , then P (S) = ...

e A basket contains 15 balls numbered from 1 to 15 , a ball is drawn randomly , then the probability that the drawn ball carries a number divisible by 5 is ....

3 The following table shows the percentages for chickens production in four farms in a month :

Farm	1st	2nd	3rd	4th
Percentage of production	25 %	30 %	35 %	......

(1) Complete the table.

(2) Represent these data by a pie chart.

4 A box contains 4 white balls , 3 blue balls and 8 red balls , all of them are symmetric , a ball is selected without looking inside the box.

Find the probability that the selected ball is :

(1) Blue. (2) Not red.

(3) Green. (4) Blue or red.

5 The following table shows the number of studying hours that Tamer done in a week :

Subject	Arabic	Maths	Science	English	Social studies	Total
Number of hours	6	10	7	9	4	36

Represent these data by circular sectors.

Activity of Unit Four

1. Ask your classmates about their times of getting up from the following times :
(Before 6 : 00 a.m. - at 6 : 00 a.m. - at 6 : 30 a.m. - at 7 : 00 a.m.)
Then tabulate the data you get in a simple frequency table and represent these data by a pie chart using Excel program, then print the sheet.

2. Toss a coin 30 times, record what you have got in the following table :

The event	The tally	The frequency
Head		
Tail		
The sum	30	

a Calculate the probability of the event A where A is the event of appearance of a head.

b Calculate the probability of the event B where B is the event of appearance of a tail.

c What is your expectation about the chance of appearing of each of the head and the tail if the number of tossing times increases to be : 100 times - 500 times - 1000 times.

A research project

on unit four

Project aims

- Collecting and organizing data.
- Doing surveys on a sample of society.
- Calculating probabilities.
- Predicting results through studying samples.
- Linking mathematics with life.

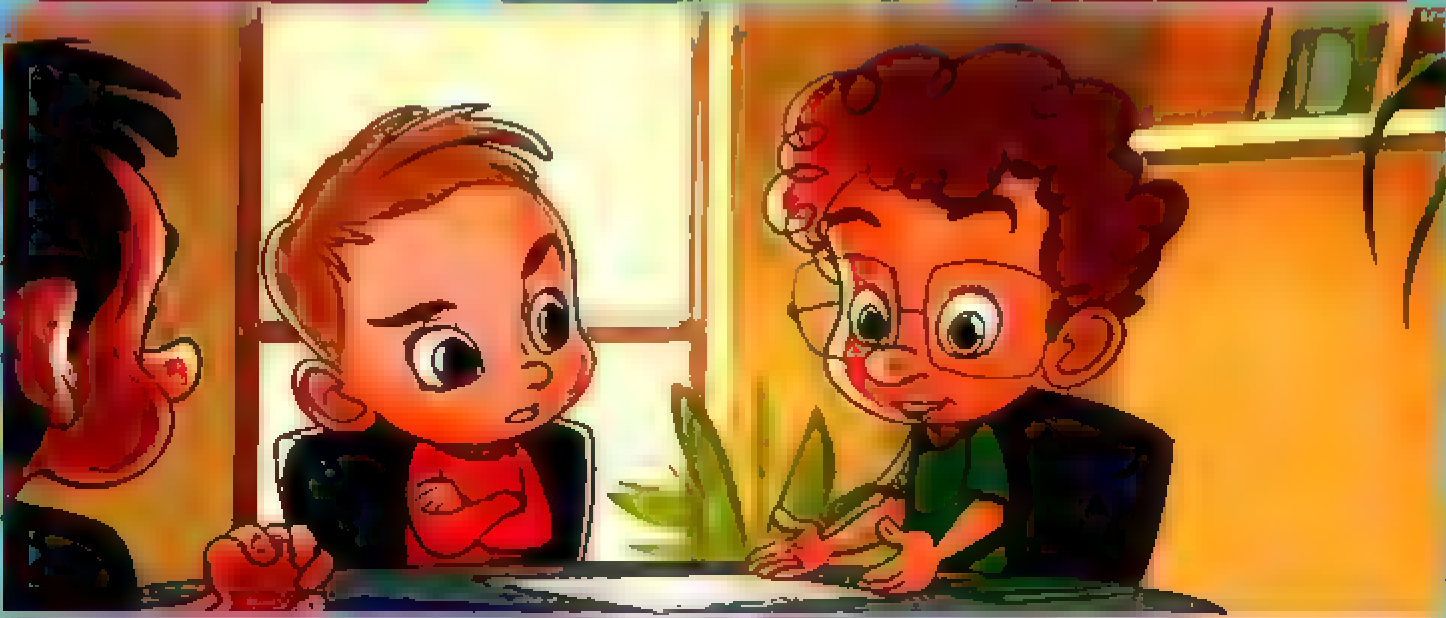
Do a research project on the following topic

"Probabilities play an important role in our daily life. They let us predict whether events will occur or not".

Discuss the following points using available resources

- Do a survey about your classmates' favourite sport.
- Record their answers in a tally table.
- Calculate the probability of each sport being preferred
- According to the number of the pupils in your school, predict the number of your school pupils that will prefer every sport through your previous calculation of probabilities.
- Write a short note on the importance of doing sports in our life.

TIMSS Questions



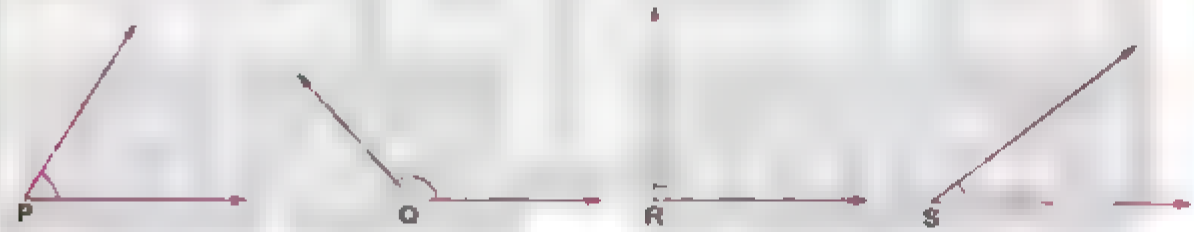
TIMSS* Questions

First : Choose the correct answer :

1 $\frac{4}{5}$ = ... % (Aswan 2017)

(a) 90 80 50 40

2 In which of the following are the angles ordered by measure , from smallest to greatest ?



Q , P , R , S (b) Q , R , P , S

S , P , R , Q (d) S , R , P , Q

3 Twice the number y subtracted from it 4 the symbolic expression for this situation is .... (Alexandria 2016)

(a) y – 4 (b) 2 y – 4 (c) y + 4 (c) 2 y + 4

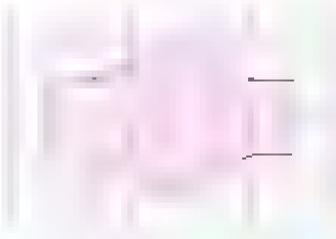
4 If the pattern 3 , 6 , 9 , 12 was continued , which of these numbers would be one of the numbers in the pattern ?

(a) 26 27 28 29

* TIMSS : Trends of the International Mathematics and Science Studies.

TIMSS QUESTIONS ?

5 How many lines of symmetry does the opposite figure have ?



(a) 4 (b) 3
(c) 2 (d) 1

6 The smallest prime number is .... (El-Fayoum 2011)

(a) zero (b) 1 (c) 2 (d) 3

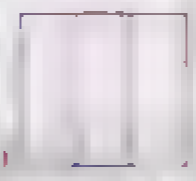
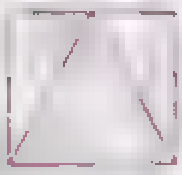
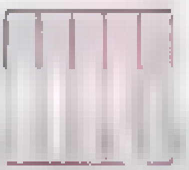
7 How much do the apples weigh in grams ?

(a) 200 (b) 202
(c) 210 (d) 220

8 In which number does 8 have the value of 800 ?

(a) 1468 (b) 2587 (c) 3809 (d) 8634

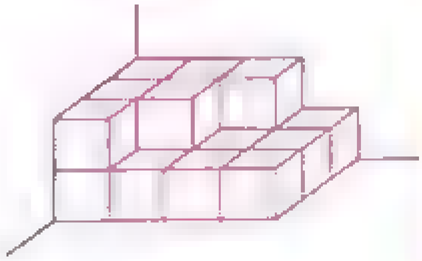
9 Which of the following figures the shaded area represents $\frac{2}{3}$ of the square ? (Cairo 2016)

(a)  (b)  (c)  (d) 

10 Which fraction is not equal to the others ?

(a) $\frac{1}{2}$ (b) $\frac{4}{8}$ (c) $\frac{2}{4}$ (d) $\frac{2}{8}$

11 Mariam stacks these boxes in the corner of the room , all the boxes are the same size. How many boxes did she use ?



(a) 25 (b) 19
(c) 18 (d) 13

12 Salma paid L.E. x for buying three pens , then the price of each pen is L.E. .... (El Dakahlia 2011)

(a) $\frac{3}{x}$ (b) $\frac{x}{3}$ (c) $3x$ (d) $3 + x$

13 The highest common factor for the two numbers 18 and 24 is ....

(a) 6 (b) 3 (c) 2 (d) 4

14 Which number is 100 more than 5432 ?

(a) 6 432 (b) 5 532 (c) 5 442 (d) 5 433

15 3 .... {1 , 33 , 35} (Ismailia 2017)

(a) $\in$ (b) $\notin$ (c) $\subset$ (d) $\not\subset$

16 The perimeter of a rectangle is 16 cm. , its width is 3 cm. , then its area = ... cm^2. (Red Sea 2016)

(a) 15 (b) 39 (c) 48 (d) 24

17 The angle between the two hands of the clock is right when the time is .... o'clock.

(a) 12 (b) 6 (c) 3 (d) 2

18 54.76 ≃ .... (to the nearest tenth)

(a) 50 (b) 55 (c) 54.7 (d) 54.8

19 The number of symmetry lines of the isosceles triangle = ... (Beni Suef 2016)

(a) 3 (b) 1 (c) 2 (d) zero

20 All .... numbers are divisible by 2

(a) even (b) odd (c) prime (d) decimal

Second : Complete each of the following :

1 The smallest odd number is ....

2 The side lengths of a triangle are 3 cm. , 4 cm. and 5 cm. , then its perimeter = ... cm. (El-Sharkia ...)

3 The place value of the digit 5 in the number 256 374 is ....

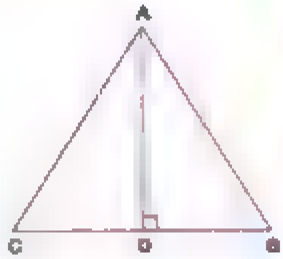
4 5 764 + ... = 8 253

5 Pentagon is a polygon of .... sides.

TIMSS QUESTIONS

6 In the opposite figure :

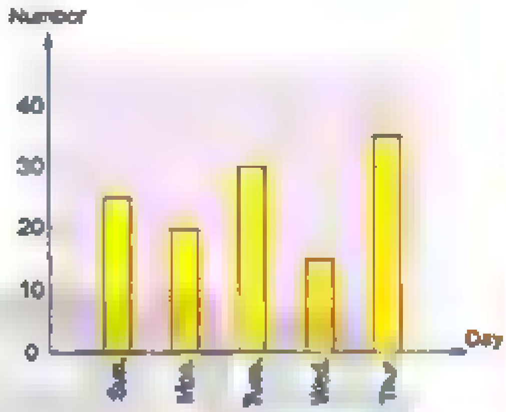
$\overline{AD} \perp \overline{BC}$, then the length of $\overline{AD}$ is called ········ of $\triangle ABC$ (Damietta 2017)



7 If $\{3 , x\} = \{5 , 3\}$, then $x + 1 =$ ········

8 The lowest common multiple for the two numbers 10 and 15 is ········

9 The opposite graph shows the number of cartons of milk sold each day of week at a school , the number of cartons of milk the school sold on Wednesday is ········



10 The number of lines of symmetry of the rhombus = ········ ([illegible] 2017)

11 3 105 ÷ ········ = 23

12 A prime number between 1 to 10 is ········ ([illegible] 2016)

13 One day and two hours = ········ hours.

14 3.26 km. = ········ m. ([illegible] 2017)

15 Six sevenths = ········

16 8 + 8 + 8 + 8 = 8 × ········

17 3.75 + 2.5 = ········ ≃ ········ (to the nearest $\frac{1}{10}$) (El Menia 2016)

18 The opposite calendar for December , Jana's birthday is on Thursday , December 2 , she is going on a trip exactly 3 weeks later , then she will go on the trip on the date ········

DECEMBER

			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

19 89.25 = ······ (to the nearest tenth) (Aswan 2016)

20 If $\{2, 3, 5\} \cap \{5, 7, 3\} = \{x, 3\}$, then $x = \cdots\cdots$

Third : Answer the following questions :

1 In Sara's class, there are twice as many girls as boys, there are 8 boys in the class. What is the total number of boys and girls in the class ?

2 Bassem is arranging squares in the following way :

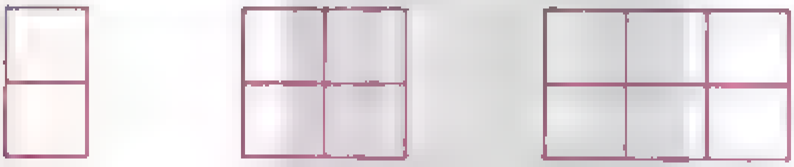


Figure 1 Figure 2 Figure 3

How many squares ☐ would Bassem need to make figure 15 ?

3 Amgad ate $\frac{1}{2}$ of a cake and Amal ate $\frac{1}{4}$ of the cake. How much of the cake did they eat altogether ?

4 In a football league, teams get :

3 points for a win, 1 point for a tie, 0 points for a loss

Stars team has 11 points, what is the smallest number of games Stars team could have played ?

5 George practiced tennis six days a week.

For 3 of the days he practiced for 45 minutes each day.

For 3 of the days he practiced for 20 minutes each day.

In hours and minutes, what is the total amount of time George practiced on these six days ?

Glossary

A

able to	قادر على
absolute	مطلق
according to	بالنسبة إلى
additive	جمعي
advanced	متقدم
afternoon	بعد الظهر
age	عمر/سن
agree	يوافق
always	دائمًا
apparent	ظاهر
appearance	ظهور
appropriate	مناسب
arc	قوس
area	مساحة
associative	دمج
Atlantic	المحيط الأطلنطي
average	معدّل
axes	محاور
axis	محور

B

backward	خلفيًا
balance	توازن
balloon	بالون
base	قاعدة
begin	يبدأ
behind	خلف
bell	جرس
belong to	ينتمي إلى
board	لوح / لافتة / كرتون
both	كلًا
bound	مرتبط
box	صندوق
bulb	بصلة

C

calculator	الآلة الحاسبة
cardboard	كرتون / ورق مقوى
carry	يحمل
carton	علبة كرتون
case	علبة / حالة
ceiling	سقف
cell	خلية
central	مركزي
ceramic	خزفي
certain	مؤكد
chart	رسم بياني
check	يتأكد
circle	دائرة
circular	دائري
closure	إغلاق
circumference	محيط الدائرة
click	ينقر
close	يغلق
coin	عملة
coloured	ملوّن
commutative	إبدال
compare	يقارن
comparing	مقارنة
compasses	فرجار
competition	منافسة/مسابقة
complement	مكمّل
component	جزء
concept	مبدأ
consecutive	متتالي
constant	ثابت
container	وعاء
coordinate	إحداثي
copy	ينسخ / نسخة

correspond to	يماثل / يناظر
cost	تكلفة
cover	يغطي / غطاء
cube	مكعب
cuboid	متوازي مستطيلات

D

damage	ضرر / تلف
data	معلومات
decimal	كسر عشري
decrease	ينقص
deduce	يستنتج
deep	عميق
degree	درجة
denote	يدل على
deposit	أودع / وديعة
depression	هبوط / ضغط
depth	عمق
describe	يوصف
description	وصف
design	رسم / صورة
determine	يحدد
diagram	شكل هندسي توضيحي
diameter	قُطر الدائرة
dice	حجر النرد
die	حجر النرد
difference	الفرق
dimension	بُعد
direction	اتجاه
directly	مباشرةً
discover	يكتشف
displacement	إزاحة
distance	مسافة
distribution	توزيع
distributive	توزيع
diver	غوّاص
dividend	المقسوم
diving	غطس
divisor	المقسوم عليه
dot	نقطة
double	ضعف
down	أسفل
drag	يجرّ / يسحب
drawing	سحب / مختار
drop	يقع / يسقط
during	خلال

E

edge	حافة
electric	كهربائي
electronic	إلكتروني
element	عنصر
elevation	ارتفاع
energy	طاقة
entrance	دخول / مدخل
equality	تساوي
equation	معادلة
event	حدث
except	ماعدا
exchange	يبدّل
existence	وجود / بقاء
expenditure	تكلفة / إنفاق
experiment	تجربة
explain	يفسر
exponent	أُس
express	يعبر عن
expression	تعبير

F

face	وجه
factory	مصنع
farm	مزرعة
farther	الأبعد
favourite	مفضّل
few	قليل
fitness	لياقة
fold	يطوي
forecast	يتنبأ
form	يكون
formula	شكل / قاعدة
forward	أمامي
fridge	ثلاجة

G

gain	يربح / مكسب
general	عام
geometric	هندسي
give	يعطي
given	معطى
graph	رسم بياني
graphically	بيانياً
grocery	بقالة

H

hand	يد / طرف
head	رأس / وجه العملة
height	ارتفاع
helicopter	هليكوبتر
high	مرتفع
hill	تل
hit	ضربة
hobby	هواية
horizontal	أفقي
hundredth	جزء من مئة

I

ideal	نموذجي
identity	محايد
image	صورة
important	مهم
impossible	مستحيل
incline	يميل
include	يحصر
increase	يزيد
index	أس
inequality	متباينة
inner	داخلي
inside	داخل
integer	عدد صحيح
interesting	ممتع
intermediate	وسط
internally	داخلي
intersect	يتقاطع
inverse	عكسي
iron	حديد

K

kick	يركل / ركلة

L

largest	الأكبر
lateral	جانبي
lattice	شبكة تربيعية
layer	طبقة
least	الأقل
left	شمال / باقي
length	طول
level	مستوى
lid	غطاء
lie	يقع
listing	سرد
loss	خسارة

M

magnitude مقدار
major أكبر
marble بلية اللعب
maximum الأقصى
measure يقيس / قياس
mention يحدد
merchant تاجر
midnight منتصف الليل
minor أصغر
missing مفقود
most معظم
mountain جبل
move يتحرك
movement تحرك
multiplicative ضربى

N

natural طبيعى
nearest الأقرب
necessary ضرورى
negative سالب
neutral محايد
news أخبار
next التالى
non-negative غير سالب
non-positive غير موجب
numerical عددى

O

observe يلاحظ
occur يحدث
once مرة
opinion رأى
opposite مقابل
order يرتب / ترتيب
ordering ترتيب
orthogonal متعامد
outcome نتيجة
outer خارجى

P

painting طلاء / دهان
pair زوج
pan كفة الميزان
parallel يوازى / موازى
parallelogram متوازى الأضلاع
participate يشارك
particular خصوصى
pattern تسلسل
peach خوخ
penalty عقاب
percentage نسبة مئوية
perform يقوم به / يُجرى
perimeter محيط
pie charts قطاعات دائرية
plane مستوى
pointer مؤشر
polar قطبى
pool حوض
position موضع
positive موجب
possibility إمكانية
possible ممكن
power طاقة / أُس
practice تمرين
preceding السابق
predict يتوقع
prelude تمهيد

prevent	يمنع
previous	السابق
print	يطبع
probability	الاحتمال
product	منتج / حاصل الضرب
production	إنتاج
profit	مكسب
program	برنامج
property	خاصية

Q

quantity	كمية
quotient	خارج القسمة

R

radii	أنصاف أقطار الدائرة
radius	نصف قطر الدائرة
raise	يرفع
random	عشوائي
reason	سبب
record	يسجل
rectangle	مستطيل
reflection	انعكاس
relation	علاقة
repeat	يكرر
repeated	متكرر
represent	يمثل
required	مطلوب
respect to	بالنسبة إلى
respectively	على الترتيب
rest	باقي
rhombus	معين
right	يمين / صحيح
rise	يرتفع
rotation	دوران
round	دورة / حول
row	صف
rule	قاعدة
rust	صدأ

S

sample	عينة
satisfy	يحقق
scale	ميزان
sector	قطاع
select	يختار
semicircle	نصف دائرة
sentence	عبارة / جملة
sequence	تسلسل
series	مسلسلات
set	مجموعة
shaded	مظلل
sheet	صفحة
side	جانب
sign	علامة
simplest	أبسط
simplify	يختصر
situation	موقف
size	حجم / قياس
space	فضاء / فراغ
specialized	متخصص
spinner	لعبة الدوارة
square	مربع
starting	بداية
statement	عبارة / جملة
statistical	إحصائي
statistics	إحصاء
stick	عود / يلصق
store	محل / مخزن

string	خيط
submarine	غواصة
subset	مجموعة جزئية
substitute	يعوض / يستبدل
substitution	تعويض
succession	تعاقب
suffer	يعاني من
suitable	مناسب
sum	مجموع / مقدار
summarize	ملخص
supply	يجهز / يمد بـ
sure	مؤكد
surface	سطح
survey	يفحص
symbol	رمز
symmetric	متماثل
symmetry	تماثل

T

tail	ظهر العملة
tank	خزان
target	هدف
tart	تورتة
technological	تكنولوجي
tick	نقطة / علامة صغيرة
tie	ربطة
tile	بلاطة
tossing	قذف
total	مجموع
trade	تجارة
training	تمرين
transform	يتحول
transformation	تحويل
translate	ينتقل
translation	انتقال
triangle	مثلث
truck	عربة نقل
twice	ضعف
twin	توأم

U

uncoloured	غير ملون
unfold	غير مطوي
unique	وحيد
unknown	غير معلوم
upper	علوي
usually	عادة

V

valley	وادي
value	قيمة
variable	متغير
verify	يتحقق
vertex	رأس
vertical	رأسي
vertices	رؤوس

W

wall	حائط
weather	الطقس
weight	وزن
wheel	عجلة
whether	إذا كان
wide	عريض
width	عرض
withdraw	يسحب
withdrawal	سحب
without	بدون

WORKSHEETS

First Worksheets on unit 1 and unit 2

Second Worksheets on unit 3 and unit 4

First

Worksheets

on unit 1 and unit 2

SHEET 1 On lesson 1 unit 1

Total mark 25

1 Complete each of the following : 5

[a] $\mathbb{Z}^+ \cap \mathbb{Z}^- =$ ....

[b] $|-13| =$ ....

[c] The opposite of 7 is ....

[d] $\mathbb{Z} - \mathbb{N} =$ ....

[e] $\mathbb{N} \cap \mathbb{Z} =$ ....

2 Put the suitable sign "$\in$, $\notin$, $\subset$ or $\not\subset$" : 5

[a] -7 ☐ $\mathbb{N}$

[b] $\frac{9}{3}$ ☐ $\mathbb{Z}$

[c] $\mathbb{Z}^+$ ☐ $\mathbb{N}$

[d] $|-35|$ ☐ $\mathbb{N}$

[e] $\mathbb{N}$ ☐ $\mathbb{Z}^-$

3 Write an integer to represent each situation : 5

[a] A temperature of 2 degrees below zero. (..........)

[b] An increase of L.E. 7 (..........)

[c] 9 m. above the sea level. (..........)

[d] A loss of P.T. 80 (..........)

[e] A bank deposit of L.E. 95 (..........)

4 Find the result of each of the following : 5

[a] $3 + |-3| =$ ....

[b] $|25| + |-5| =$ ....

[c] $|-11| + 22 =$ ....

[d] $|-6| \times |3| =$ ....

[e] $|-9| \times 0 =$ ....

5 Represent the following numbers on the number line : 5

[a] -3

[b] $|-5|$

[c] $-|2|$

[d] $\{7, 8, 9\}$

[e] $\{-3, -4, -5, -6, ...\}$

SHEET 2

From lesson 1 unit 1
to lesson 2 unit 1

Total mark 25

1 Put the suitable relation ">, = or <" : 5

[a] – 8 ☐ 4 [b] 0 ☐ – 2

[c] 5 ☐ | – 5 | [d] – 3 ☐ – 5

[e] | – 9 | ☐ – | – 10 |

2 [a] Arrange the following numbers in an ascending order . 5

– 6 , 15 , 0 , | – 9 | and – 18

[b] Arrange the following numbers in a descending order :

– 9 , 17 , | – 9 | , – 15 and 16

3 Complete each of the following : 5

[a] The number .... is neither positive nor negative.

[b] – 4 , – 3 , – 2 , ... , ... , ... (in the same pattern)

[c] $\mathbb{Z} - \mathbb{Z}^- =$ ....

[d] The smallest positive integer is ....

[e] | – 12 | + | – 21 | = ...

4 Write : 4

[a] The previous integer and the next integer of – 27

[b] The integers between the two integers – 5 and 3

5 Write using the listing method each of the following sets : 6

[a] The set of integers greater than – 4

[b] The set of integers smaller than – 1

[c] The set of non-negative integers.

[d] The set of integers smaller than 5 and greater than – 6

SHEET 3

From | lesson 1 unit 1
to | lesson 3 unit 1

Total mark 25

1 Find the result of each of the following : 5

[a] (– 7) + 2 = ...

[b] (– 4) + (– 5) = ...

[c] 14 – 27 = ...

[d] 16 – (– 3) = ...

[e] 12 + (– 12) = ...

2 Write the property of addition in Z in each of the following : 4

[a] (– 5 + 6) + 9 = – 5 + (6 + 9) (..........)

[b] (– 8) + 7 = 7 + (– 8) (..........)

[c] – 11 + 0 = – 11 (..........)

[d] 14 + (– 14) = 0 (..........)

3 Complete each of the following : 5

[a] 4 + (– 7) – 2 = ...

[b] – 8 + 5 + 8 = ...

[c] | – 7 | + ... = 0

[d] 4 + ... = – 1

[e] The additive inverse of (– 5) is

4 Use the properties of addition in Z to find : 6

[a] – 15 + 29 + 15

[b] 55 + (– 255) + 45 + 255

5 Arrange each of the following in an ascending order : 5

[a] | – 3 | , 5 , – 3 , 0 and 4

[b] 1 , 11 , – 1 and – 11

SHEET 4

From lesson 1 unit 1
to lesson 4 unit 1

Total mark 25

1 **Complete :** 5

[a] The product of two negative integers is ………

[b] $17 \times$ ……… $= (-5) \times 17$

[c] $-|-2| =$ ………

[d] $\mathbb{Z}^{+} \cup \{0\} =$ ………

[e] $5 \times (-2) =$ ………

2 **Find the result of each of the following :** 4

[a] $9 \times (-3)$

[b] $(-36) \div (-4)$

[c] $[8 + (-5)] \times 6$

[d] $6 \times [-2 + (-7)]$

3 [a] **Use the properties of multiplication of integers to find each of the following :** 6

(1) $50 \times 14 \times 2$

(2) $8 \times (-9) \times 125 \times 3$

[b] **Use the distributive property to find the result of each of the following :**

(1) $3 \times (-2) + 3 \times 5$

(2) $112 \times 98 + 112 \times (-97)$

4 [a] **Arrange the following numbers in an ascending order :** 4

$-2 , 4 , 0 , -5 , |-3|$ and 6

[b] If $a = 4$, $b = -3$ and $c = 5$, then find the value of :

(1) $2a + b - c$

(2) $\frac{2c - a}{b}$

5 [a] **Write using the listing method each of the following sets :** 6

(1) The set of integers greater than -3

(2) The set of integers included between -4 and 2

[b] **Use the properties of addition in $\mathbb{Z}$ to find :**

(1) $5 + 4 + (-5)$

(2) $45 + 36 + 55 + 64$

SHEET 5

From lesson 1 unit 1
to lesson 5 unit 1

Total mark 25

1 Choose the correct answer : 5

[a] $(-7)^2$ .... $\mathbb{N}$ ($\in$ or $\notin$ or $\subset$ or $\not\subset$)

[b] The additive inverse of $(-3)^2$ is .... (9 or 3 or -3 or -9)

[c] $(-9)^2 =$ .... (-81 or -18 or 81 or 18)

[d] If $|-4| = x$, then $x =$ .... (4 or -4 or 16 or -16)

[e] If $-7 + n = -7$, then $n =$ .... (1 or 7 or -7 or 0)

2 Find the value of each of the following : 6

[a] $5^4 \times 5^3 =$ ....

[b] $9^9 \div 9^7 =$ ....

[c] $8^3 \times 8 \times 8^2 =$ ....

[d] $\frac{(-7)^6}{(-7)^6} =$ ....

3 Simplify each of the following : 6

[a] $\frac{3^5 \times 3^4}{3^7}$

[b] $\frac{6^3 \times 6^5}{6^7 \times 6}$

4 Put the suitable relation "> , = or <" : 6

[a] -12 .... $(-6)^2$

[b] $(-1)^2$ .... $(-1)^3$

[c] $\frac{9^3}{9^3}$ .... $(-10)^{zero}$

[d] $|-6| + (-5)^2$ .... 2^5

5 Arrange in a descending order : 2

$(-2)^5$, $(-4)^0$, $(-3)^4$, $(-1)^{15}$ and 3^2

SHEET 6

From lesson 1 unit 1
to lesson 6 unit 1

Total mark 25

1 Complete in the same pattern : 5

[a] 2 , 6 , 10 , 14 , ... , ...

[b] 1 , 4 , 9 , ... , ...

[c] $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, ... , ...

[d] 10 000 , 1 000 , 100 , ... , ...

[e] −2 , 0 , 2 , 4 , ... , ...

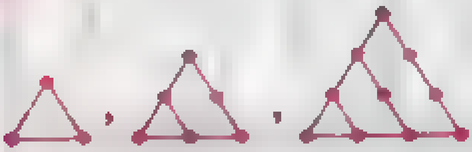
2 Describe the pattern , then complete in the same pattern : 6

[a] 5 , 13 , 21 , 29 , ... , ... , ...

[b] 25 , 21 , 17 , 13 , ... , ... , ...

[c] 1 , 2 , 4 , 8 , 16 , ... , ... , ...

3 Look at the pattern of dots , then answer : 5



, ... , ...

[a] Draw the 4th and the 5th shapes.

[b] How many dots will be there in the 4th and the 5th shapes ?

4 Arrange each of the following numbers in an ascending order : 4

[a] $(-4)^2$, −5 , |−5| , 0 , 6 and −14

[b] 11 , 5^0 , −|7| , −11 , 0 and 3^3

5 Choose the correct answer : 5

[a] $\{-3 , \frac{7}{11}\}$ .... $\mathbb{Z}$ ($\in$ or $\notin$ or $\subset$ or $\not\subset$)

[b] If m × 7 = 0 , then m = ... (1 or 0 or 2 or −7)

[c] 8 + ... = −2 (6 or −10 or 10 or −6)

[d] $\mathbb{N} \cup \mathbb{Z}^- =$ .... ($\mathbb{Z}$ or $\mathbb{Z}^+$ or $\{0\}$ or $\mathbb{N}$)

[e] 6 − (−9) = ... (−3 or 3 or 15 or −15)

Sheet 7

From lesson 1 unit 1
to lesson 1 unit 2

Total mark 25

1 Find the solution set of each of the following equations : 6

[a] $x + 5 = 12$, if the substitution set is : $\{3, 5, 6, 7\}$

[b] $3x - 4 = 8$, if the substitution set is : $\{3, 5, 6\}$

[c] $2x + 1 = x - 3$, if the substitution set is : $\{2, 4, -1, -4\}$

[d] $3(x - 2) = -6$, if the substitution set is : $\{-1, 0, 1\}$

2 Find the solution set of each of the following inequalities : 6

[a] $3x + 5 > 2$, if the substitution set is : $\{-2, -1, 0, 1\}$

[b] $3x - 1 > -2$, if the substitution set is : $\{-2, -1, 0, 1, 2\}$

[c] $5x - 1 > 4$, if the substitution set is : $\{2, 3, 4, 5, 6\}$

[d] $x + 3 < 5$, if the substitution set is : $\{0, 1, 2, 3, 4\}$

3 Considering the set of substitution is $A = \{0, 1, 2, 3\}$ 3

Find the solution set of each of the following :

[a] $2x - 7 = -1$ [b] $x + 4 > 5$

4 Complete : 5

[a] The additive inverse of $-4 =$ ....

[b] $|-9| + 3 =$ ....

[c] 1 , 2 , 4 , 8 , 16 , ... , ... , ... (in the same pattern)

[d] $\mathbb{Z}^+ \cap \mathbb{Z}^- =$ ....

[e] The multiplicative neutral element in $\mathbb{Z}$ is ....

5 [a] Simplify : $\frac{5^4 \times 3^6}{3^4 \times 5^2}$ 5

[b] Determine the degree of each of the following equations :

(1) $4b - 7 = 8$ (2) $x^3 - 3x^2 = 4$

(3) $x - 2y = 9$ (4) $x^4 + 3x^5 = 19$

Sheet 8

From lesson 1 unit 1
to lesson 2 unit 2

Total mark 25

1 Find the solution set of each of the following equations in $\mathbb{N}$: 6

[a] $2x - 1 = 9$

[b] $3x + 2 = 17$

[c] $3x - 4 = 11$

[d] $4x - 3 = -7$

2 Find the solution set of each of the following equations in $\mathbb{Z}$: 6

[a] $x + 8 = -3$

[b] $3x + 2 = -19$

[c] $2x + 4 = -4$

[d] $2x + 1 = 13$

3 Complete : 5

[a] The degree of the equation : $3x^2 + 4x - 1 = 0$ is ......

[b] If $|x| = 7$, then $x =$ .... or $x =$ ......

[c] $\mathbb{Z}^+ \cap \mathbb{Z}^- =$ ......

[d] The number ...... is neither positive nor negative.

[e] 3 , 9 , 27 , 81 , ... , ... (in the same pattern)

4 [a] Use the properties of addition in $\mathbb{Z}$ to find : 4

(1) $25 + 13 + (-25)$

(2) $5 + (-3) + 7 + (-9)$

[b] Arrange ascendingly :

-5 , $|-5|$, $(-2)^2$, 0 and $-(3)^2$

5 [a] Use the multiplication properties of integers to find : 4

(1) $50 \times (-31) \times 2$

(2) $(-25) \times 9 \times (-4)$

[b] Simplify : $\dfrac{2^{11}}{2^5 \times 2^4}$

SHEET 9

From lesson 1 unit 1
to lesson 3 unit 2

Total mark 25

1 Find the solution set of each of the following inequalities : 8

[a] $2x + 1 < 7$, where $x \in \mathbb{N}$

[b] $2x - 3 \geq 5$, where $x \in \mathbb{Z}$

[c] $3x + 1 \leq 13$, where $x \in \mathbb{N}$

[d] $1 - 2x > 5$, where $x \in \mathbb{Z}$

2 Find the solution set of the inequality $x + 2 \leq 5$ where : 4

[a] $x \in \mathbb{N}$

[b] $x \in \mathbb{Z}$

, then represent the solution set on the number line.

3 Use the distributive property to find the result of each of the following : 4

[a] $23 \times (-121) + 23 \times 21$

[b] $(-35) \times (-72) + (-35) \times 82$

4 Complete : 5

[a] The degree of the equation : $2x + 1 = 5$ is ....

[b] The additive inverse of $(-8)^0$ is ....

[c] $3^5 + 3^5 =$ ....

[d] The greatest negative integer is ....

[e] 27 , 9 , 3 , 1 , ... , ... (In the same pattern)

5 Use the multiplication and addition properties of integers to find : 4

[a] $4 \times (-16) \times 25$

[b] $-15 + 15 + 29$

Second Worksheets

on unit 3 and unit 4

SHEET 1

On lesson 1 unit 3

Total mark 25

1 Determine the position of each of the following points $A(1, -1)$, $B(4, -1)$ and $C(4, 5)$, then find : 5

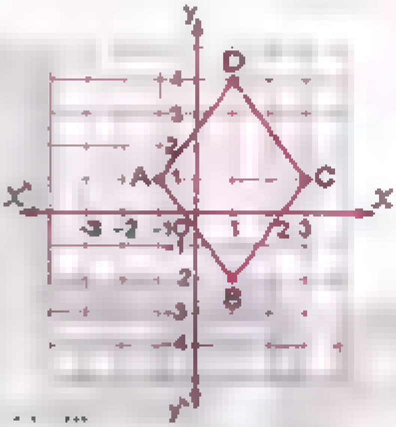
[a] The length of each of $\overline{AB}$ and $\overline{BC}$

[b] The type of the triangle ABC with respect to its side lengths and its angles.

[c] The area of the triangle ABC

2 In the opposite figure : 5

ABCD is a rhombus , complete :



[a] A (........ , ...) ,

B (........ , ...) ,

C (........ , ...) ,

and D (........ , ...)

[b] The length of $\overline{AC}$ = ...

[c] The length of $\overline{BD}$ = ...

[d] The surface area of the rhombus ABCD = ...

3 Determine the positions of $X(-2, 2)$, $Y(-2, -3)$, $Z(3, -3)$ and $L(3, 2)$, then find : 5

[a] The name of the shape XYZL

[b] The perimeter and the area of the shape XYZL

[c] The number of axes of symmetry for the shape XYZL

4 Determine the positions of $L(-2, -1)$, $M(1, -1)$, $N(1, 3)$ and $P(-2, 3)$, then find : 5

[a] The length of each of $\overline{LP}$ and $\overline{PN}$

[b] The perimeter and the area of the shape LMNP

5 On a square lattice , draw Δ QRS where $Q(-1, -3)$, $R(3, -3)$ and $S(1, 6)$, then find : 5

[a] The length of $\overline{QR}$

[b] The type of the triangle QRS according to its side lengths.

[c] The number of axes of symmetry for the triangle QRS

SHEET 2

From lesson 1 unit 3
to lesson 2 unit 3

Total mark 25

1 On a square lattice , draw Δ ABC where A (5 , 3) , B (1 , 1) and C (6 , – 3) , then find its image by translation $(x , y) \longrightarrow (x - 4 , y + 1)$ 5

2 On the coordinate plane , determine the points A (1 , 2) , B (– 2 , 2) and C (– 2 , – 4) , then find : 5

[a] The length of $\overline{AB}$

[b] The length of $\overline{BC}$

[c] The image of Δ ABC by translation (3 , – 1)

3 If A (1 , – 1) and B (– 1 , 3) , write the mapping rule of the translation that makes B the image of A 5

4 On a lattice , plot the vertices of the triangle ABC where A (1 , 1) , B (– 3 , – 1) and C (0 , – 5) , then draw its image by translation $(x + 4 , y + 1)$ 5

5 On a square lattice , draw Δ MNT where M (1 , – 3) , N (– 3 , 1) and T (– 2 , – 5) , then draw its image by translation of magnitude 3 units in the positive direction of y-axis. 5

SHEET 3

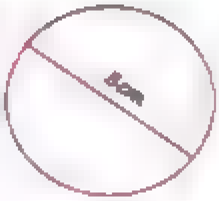
From lesson 1 unit 3
to lesson 3 unit 3

Total mark 25

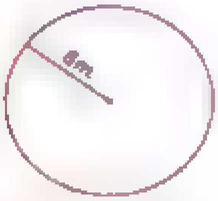
1 Find the area of each of the following circles (Consider $\pi = 3.14$) : 4

[a]

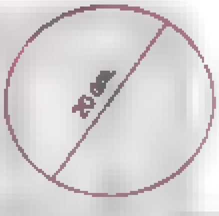
[b]



[c]



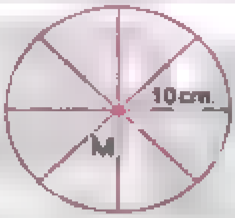
[d]



2 In the opposite figure : 5

A circle M of radius 10 cm. is divided into 8 equal circular sectors.

Calculate the area of one sector. (Consider $\pi = 3.14$)



3 [a] If the length of the diameter of a circle is 14 cm. Calculate : 6

(1) The circumference of the circle.

(2) The surface area of the circle. $\left(\text{Consider } \pi = \frac{22}{7}\right)$

[b] Find the area of the opposite figure

$\left(\text{Consider } \pi = \frac{22}{7}\right)$

4 In the opposite figure : 5

Find the area of the shaded part (Consider $\pi = 3.14$)

5 [a] Determine in the coordinates plane the image of the line segment $\overline{AB}$ where A (2 , 3) , B (– 2 , 0) by translation (X + 3 , y – 2) 5

[b] A circle , its circumference is 88 cm.

Calculate its radius length and its surface area. $\left(\text{Consider } \pi = \frac{22}{7}\right)$

SHEET 4

From : lesson 1 unit 3
to : lesson 4 unit 3

Total mark 25

1 [a] A cube-shaped box is of edge length 5 cm. Find : (4)

(1) Its lateral area.

(2) Its total area

[b] A cuboid is with length 7 cm. , width 5 cm. and height 8 cm. Find :

(1) Its lateral area.

(2) Its total area

2 [a] If the sum of the edges of a cube is 108 cm (6)

Find its lateral and total area.

[b] A cuboid is with square base of side length 3 cm and height 6 cm.

Find its lateral area and total area.

3 [a] The perimeter of the base of a couboid is 20 cm. and its height is 6 cm. (6)

Calculate the lateral area of the cuboid.

[b] If the lateral area of a cube is 100 cm^2 Find its total area.

4 [a] A cuboid with a square base whose perimeter is 20 cm. (5)

and its height is 8 cm. Find :

(1) The lateral area.

(2) The length of its base side.

(3) The total area.

[b] Find the area for the circle with diameter length 14 cm. (Consider $\pi = \frac{22}{7}$)

5 [a] A cuboid whose total area = 132 cm^2 and its lateral area = 112 cm^2. (4)

Find the area of its base

[b] A cuboid whose lateral area 140 $cm.^2$ and the dimensions of its base are 6 cm. and 4 cm. Find its height.

SHEET 5

From lesson 1 unit 3
to lesson 1 unit 4

Total mark 25

1 The following table shows the rate of the score of 300 pupils in one school : 5

Rate	Excellent	Good	Pass	Weak
Percentage	20 %	45 %	25 %	10 %

[a] Represent these data by a pie chart.

[b] Find the number of excellent pupils.

2 The following table shows the percentage of the time Ayman spent studying some subjects during a week : 5

Subject	Arabic	Math	English	Science
Percentage	30 %	25 %	20 %	

[a] Complete the table.

[b] Represent these data by a pie chart.

3 The monthly income of a family is L.E 1800 ، the family spends 25 % of its income on rent ، 40 % on food ، 20 % on others and saves the rest. 5

[a] Represent these data using the circular sectors.

[b] Find the capital which this family saves monthly.

4 The following table shows the number of studying hours that Mohamed has done in a week : 5

Subject	Arabic	Maths	Science	English	Social studies	Total
Number of hours	9	10	6	7	4	36

Represent these data by a pie chart.

5 [a] A cuboid ، its length is 3 cm. ، its width is 2 cm. and its height is 4 cm. 5

Find its total area.

[b] A diameter length of a circle is 20 cm.

Calculate its surface area. (Consider $\pi = 3.14$)

SHEET 6

From lesson 1 unit 3
to lesson 2 unit 4

Total mark 25

1 A bag contains 8 equal cards , have the same colour , numbered from 1 to 8 Write the sample space for this experiment. 3

2 From the set of digits $\{1, 5, 7\}$, a number is formed from two digits , determine the sample space of this experiment showing the number of its elements. 3

3 Determine the sample space of tossing three distinct coins once and observing the sequence of appearance of heads and tails. 3

4 [a] In the coordinates plane , draw the rectangle ABCD where A (4 , 2) , B (4 , 4) , C (1 , 4) and D (1 , 2) , then : 8

(1) Draw its image by the translation $(x + 2 , y + 2)$

(2) Calculate the perimeter of the image of the rectangle ABCD

[b] Find the area of a circle with diameter length 28 cm. $\left(\text{Consider } \pi = \frac{22}{7}\right)$

5 [a] Find the total area of a cuboid with square base of side length 6 cm. and height 8 cm. 8

[b] The following table shows the percentage of production of electric sets :

Set	1st	2nd	3rd	4th
Percentage	40 %	15 %	30 %	15 %

Represent these data by a pie chart.

SHEET 7

From lesson 1 unit 3
to lesson 3 unit 4

Total mark 25

1 A bag contains 15 cards numbered from 1 to 15 , if one of the cards is chosen randomly , write the sample space for this experiment and the number of its elements , then find the probability that the chosen card : 4

[a] Carried an even number.

[b] Carried a number divisible by 4

[c] Carried a number satisfying the inequality : $3 \leq X - 1 < 9$

2 A bag contains 25 balls (4 balls are yellow , 9 balls are red and the remainder is black) , if a ball is drawn randomly , find the probability that the drawn ball is : 4

[a] black. [b] yellow or black. [c] not black. [d] brown.

3 A bag contains 20 similar marbles. Tarek drew a marble randomly and he found it red. If the probability of drawing a red marble = $\frac{3}{5}$ Find the number of red marbles in the bag. 4

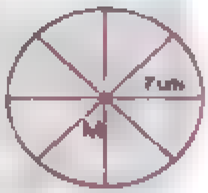
4 [a] The length of a cuboid is 3 cm. , its width is 2 cm and its height is 4 cm. Find the total surface area of the cuboid. 6

[b] In the opposite figure :

A circle of radius length 7 cm. is divided into 8 equal circular sectors. Find :

(1) The area of one circular sectors. $\left(\text{Consider } \pi = \frac{22}{7}\right)$

(2) The measure of the central angle of sector.



5 [a] If the perimeter of one face of a cube is 20 cm. Find its lateral and total area. 7

[b] The following table shows the ratio for producing chickens in four farms in a month :

Farm	1^{st}	2^{nd}	3^{rd}	4^{th}
The ratio of production	10 %	35 %	25 %	30 %

Represent these data by a pie chart.

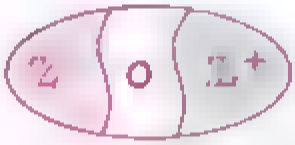
SUMMARY OF THE SECOND TERM

Summary of Unit One

The set of integers "$\mathbb{Z}$" = {... , − 3 , − 2 , − 1 , 0 , 1 , 2 , 3 , ...}
It is formed from the union of three sets $\mathbb{Z}^-$, {0} and $\mathbb{Z}^+$

i.e. $\mathbb{Z} = \{\underbrace{\ldots, -3, -2, -1}_{\mathbb{Z}^-}, \underbrace{0}_{\{0\}}, \underbrace{1, 2, 3, \ldots}_{\mathbb{Z}^+}\}$

$\mathbb{Z} = \mathbb{Z}^- \cup \{0\} \cup \mathbb{Z}^+$



- $0 \notin \mathbb{Z}^+$ and $0 \notin \mathbb{Z}^-$
- The set of non-negative integers = {0 , 1 , 2 , ...} = $\{0\} \cup \mathbb{Z}^+ = \mathbb{N}$
- The set of non-positive integers = {0 , − 1 , − 2 , − 3 , ...} = $\{0\} \cup \mathbb{Z}^-$
- The set of odd integers = {... , − 3 , − 1 , 1 , 3 , ...}
- The set of even integers = { ... , − 4 , − 2 , 0 , 2 , 4 , ...}

Representation of the integers on the number line

Negative integers "$\mathbb{Z}^-$" are the left of O

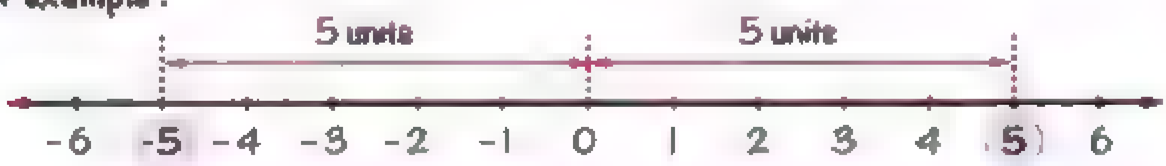
The origin

Positive integers "$\mathbb{Z}^+$" are the right of O

Opposites (inverses) and absolute value

On the number line , any two numbers that are at the same distance from O and on two opposite positions of it are called **opposites** or **inverses**.

For example :



i.e. The opposite of 5 is − 5 and the opposite of − 5 is 5

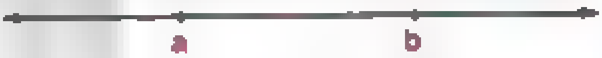
The absolute value

1. The absolute value of a number is its distance from 0 on the number line.
2. The absolute value of any number X is denoted by $|X|$
3. The absolute value of any number (except 0) is always positive

Examples : • $|4| = 4$ • $|-4| = 4$ • $|0| = 0$

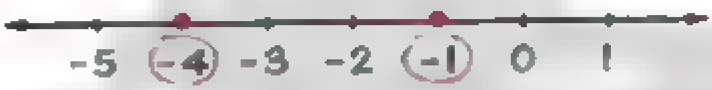
Ordering and comparing integers

- For any two integers a and b, if the point representing a is to the left of the point representing b, then $a < b$



For example :

$-4 < -1$



because the point representing -4 lies on the left of the point representing -1

- Any positive integer is greater than any negative integer
- Zero is smaller than any positive integer and is greater than any negative integer.
- The least positive integer is "1" and we cannot determine the greatest positive integer.
- The greatest negative integer is "-1" and we cannot determine the least negative integer.

Operations on integers

First Addition

1. To add integers have the same sign, keep the same sign and add the absolute value of each number.

   For example : • $5 + 4 = 9$ • $(-5) + (-4) = -9$

2. To add integers with different signs, keep the sign of the number with the largest absolute value and subtract the smallest absolute value from the largest.

   For example : • $(-8) + 6 = -2$ (because $|-8| > |6|$)

   • $9 + (-3) = 6$ (because $|9| > |-3|$)

Summary

Fourth Division

- The division is not always possible in $\mathbb{Z}$
- The quotient of two integers with the same sign is positive.
  i.e. $(+) \div (+) = (+)$ and $(-) \div (-) = (+)$
- The quotient of two integers with different signs is negative.
  i.e. $(+) \div (-) = (-)$ and $(-) \div (+) = (-)$
- The quotient of zero divided by any non-zero integer is zero.
- Division by zero has no meaning
- The division operation in $\mathbb{Z}$ is not commutative.
- The divison operation in $\mathbb{Z}$ is not associative.

Repeated multiplication

- If a is an integer and $n \in \mathbb{Z}^+$, then $a \times a \times a \times \cdots$ to n times $= a^n$ where a is called the base and n is called the power , index or exponent.

For example :

$$3 \times 3 \times 3 \times 3 = \underset{\text{Base}}{3}^{\overset{\text{Power}}{4}}$$

- Any number to the first power is that number itself.
  For example : • $9^1 = 9$ • $(-3)^1 = -3$
- Any number except 0 to the zero power is 1
  For example : • $5^0 = 1$ • $(-7)^0 = 1$
- If the base is one and $n \in \mathbb{Z}$, then $1^n = 1$
  For example : • $1^5 = 1$ • $1^{12} = 1$
- If $a \in \mathbb{Z}$ and $n \in \mathbb{Z}^+$, then $(-a)^n = \begin{cases} (a)^n & \text{if n is even} \\ -(a)^n & \text{if n is odd} \end{cases}$

  i.e. • **A negative integer** raised to the **power** of an **even** integer gives a **positive** integer
  • **A negative integer** raised to the **power** of an **odd** integer gives a **negative** integer.

Rules of power

Rule 1

If $a \in \mathbb{Z} - \{0\}$, $n \in \mathbb{Z}^+$, $m \in \mathbb{Z}^+$, then : $\boxed{a^m \times a^n = a^{m+n}}$

For example :

- $3^2 \times 3^3 = 3^{2+3} = 3^5 = 243$
- $a^3 \times a^5 = a^{3+5} = a^8$

Rule 2

If a is an integer and $a \neq 0$, $n \in \mathbb{Z}^+$, $m \in \mathbb{Z}^+$, $m \geq n$, then : $\boxed{\frac{a^m}{a^n} = a^{m-n}}$

For example :

- $2^5 \div 2^2 = 2^{5-2} = 2^3 = 8$
- $\frac{(-4)^8}{(-4)^6} = (-4)^{8-6} = (-4)^2 = 16$

Numerical patterns

- Numerical pattern is a sequence of numbers according to a particular rule.
- Describing of the pattern is discovering the rule of the pattern and expressing it in words.

For example :

- $1 \xrightarrow{+3} 4 \xrightarrow{+3} 7 \xrightarrow{+3} 10 \xrightarrow{+3} 13 \xrightarrow{+3} 16$ (1 , 4 , 7 , 10 , 13 , 16)

Description of the pattern

Each number is more than its preceding by 3

- $3 \xrightarrow{\times 2} 6 \xrightarrow{\times 2} 12 \xrightarrow{\times 2} 24 \xrightarrow{\times 2} 48 \xrightarrow{\times 2} 96$ (3 , 6 , 12 , 24 , 48 , 96)

Description of the pattern

Each number is twice of its preceding.

Summary of Unit Two

The equation

It is a mathematical statement that has two experssions separated by an equal sign. One (or both) of the expressions contains one unknown (or more).

For example :

$x + 5 = -7$ is an equation , the letter "x" is called the unknown or the variable.

The degree of an equation

It is determined by the highest power of the unknown (symbol) in the equation.

For example :

- $3x - 1 = 8$ is an equation of the first degree in one unknown x
- $a^2 - 2b - 10 = 0$ is an equation of the second degree in two unknowns a and b

Solving first degree equation in one unknown

The solution of the equation is the number which satisfies the equation.
i.e. which makes the two sides of the equation equal

Example

Find the solution set of the equation :

$x + 4 = 6$ if the substituion set is $\{-3 , 1 , 2\}$

Solution

Substitute in the left hand side of the equation for x by the elements of the substitution set as follows :

When $x = -3$

$\therefore$ The left hand side $= -3 + 4 = 1 \neq 6$

When $x = 1$

$\therefore$ The left hand side $= 1 + 4 = 5 \neq 6$

When $x = 2$

$\therefore$ The left hand side $= 2 + 4 = 6$

$\therefore$ The solution set of the equation is $\{2\}$

Example

Find the solution set of the equation :

$3x - 1 = -7$, where $x \in \mathbb{Z}$

Solution

$\because 3x - 1 = -7$ (Adding 1 to each of the two sides)

$\therefore 3x - 1 + 1 = -7 + 1$

$\therefore 3x = -6$ (Dividing each of the two sides by 3)

$\therefore \frac{3x}{3} = \frac{-6}{3}$ $\therefore x = -2 \in \mathbb{Z}$ $\therefore$ The S.S. $= \{-2\}$

The Inequality

It is a mathematical statement that has two expressions separated by an inequality sign (< or >). One (or both) of the expressions contains one unknown (or more).

For example :

- $2x + 1 > 7$
- $3 - x \leq 8$

The degree of an inequality

It is determined by the highest power of the unknown (symbol) in the inequality.

For example :

$2x + 1 < -8$ is an inequality of the first degree in one unkown x

Solving first degree inequality in one unknown

Example

Find the solution set of the inequality :

$x - 3 > 1$ if the substitution set is $\{6, 5, 4, 3\}$

Solution

When $x = 6$

$\therefore$ The left hand side $= 6 - 3 = 3$ is greater than 1

When $x = 5$

$\therefore$ The left hand side $= 5 - 3 = 2$ is greater than 1

Summary

When $x = 4$

$\therefore$ The left hand side $= 4 - 3 = 1$ is not greater than 1

When $x = 3$

$\therefore$ The left hand side $= 3 - 3 = 0$ is not greater than 1

$\therefore$ The solution set $= \{6 , 5\}$

Example

Find the solution set of each of the following inequalities :

1. $2x - 3 \geq 5$, where $x \in \mathbb{Z}$
2. $1 - 2x > -7$, where $x \in \mathbb{N}$
3. $-5 < 3x + 1 \leq 10$, where $x \in \mathbb{Z}$

Solution

1. $\because 2x - 3 \geq 5$ (Adding 3 to each of the two sides)

$\therefore 2x - 3 + 3 \geq 5 + 3$

$\therefore 2x \geq 8$ (Dividing each of the two sides by 2)

$\therefore \frac{2x}{2} \geq \frac{8}{2}$ $\therefore x \geq 4$

$\therefore$ The S.S. $= \{4 , 5 , 6 , 7 , ...\}$

2. $\because 1 - 2x > -7$ (Subtracting 1 from each of the two sides)

$\therefore 1 - 2x - 1 > -7 - 1$

$\therefore -2x > -8$ (Dividing each of the two sides by -2)

$\therefore \frac{-2x}{-2} < \frac{-8}{-2}$

$\therefore x < 4$ $\therefore$ The S.S. $= \{3 , 2 , 1 , 0\}$

3. $\because -5 < 3x + 1 \leq 10$ (Subtracting 1 from each of the three sides)

$\therefore -5 - 1 < 3x + 1 - 1 \leq 10 - 1$

$\therefore -6 < 3x \leq 9$ (Dividing each of the three sides by 3)

$\therefore \frac{-6}{3} < \frac{3x}{3} \leq \frac{9}{3}$

$\therefore -2 < x \leq 3$ $\therefore$ The S.S. $= \{-1 , 0 , 1 , 2 , 3\}$

Summary of Unit Three

The distance between two points on the number line

The distance between two points on the number line = | Number of the ending point − number of the starting point |

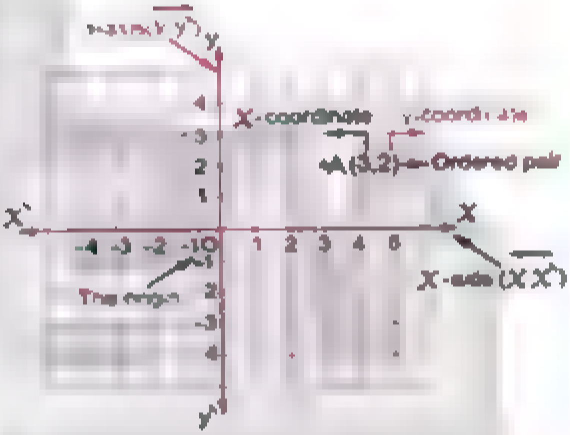
For example :

In the opposite figure :

MN = | 2 − (− 1) | = | 2 + 1 | = 3 units.

Graphing points in the coordinate plane

The position of any point in the coordinate plane is determined by a unique ordered pair as in the opposite figure.



The distance between two points in the coordinate plane

For example :

In the opposite figure :

- $\overline{AB} \parallel \overleftrightarrow{xx'}$

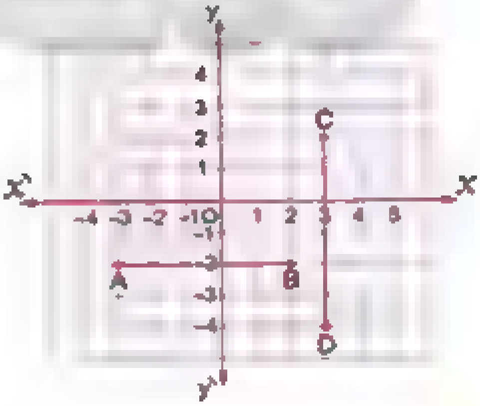
  $\therefore AB = | B - A | = | 2 - (-3) |$

  $= | 2 + 3 | = 5$ units.

- $\overline{CD} \parallel \overleftrightarrow{yy'}$

  $\therefore CD = | D - C | = | -4 - 2 |$

  $= | -6 | = 6$ units.

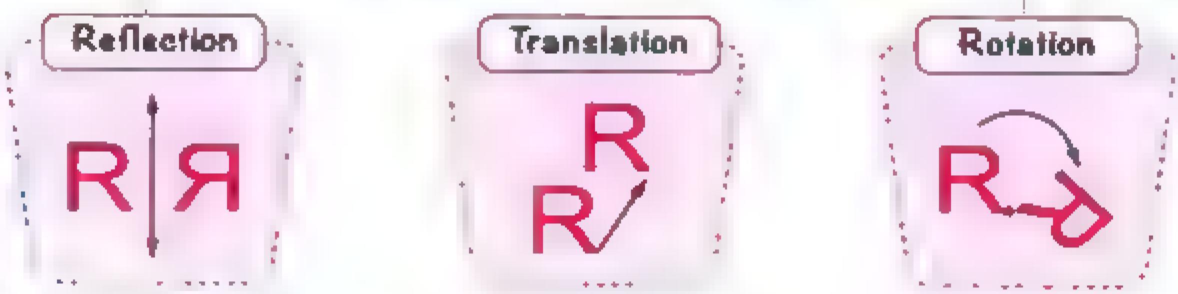


Geometric transformations

There are three types of the geometric transformations which are shown in the following diagram :

Summary

The geometric transformations



Translation

The translation is a geometric transformation which slides a shape from a place to another place (image) such as every point of the original shape moves the same distance in the same direction to form the image.

First Translation in the plane

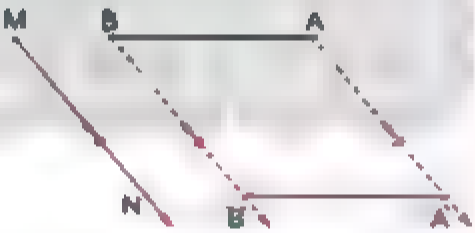
A Finding the image of a point by a given translation

A' is the image of A by translation of magnitude MN in the direction of $\overrightarrow{MN}$

B Finding the image of a line segment by a given translation

$\overline{A'B'}$ is the image of $\overline{AB}$ by translation of magnitude MN in the direction $\overrightarrow{MN}$

Check that : $AB = A'B'$ and $\overline{AB} \parallel \overline{A'B'}$

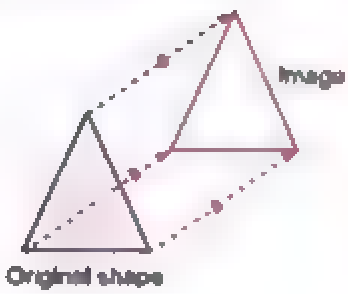


C Finding the image of a geometric shape by a given translation

The opposite figure shows the image of a triangle by a certain translation

Every point of the shape must move :

- The same distance.
- In the same direction



Second Translation in the coordinates plane

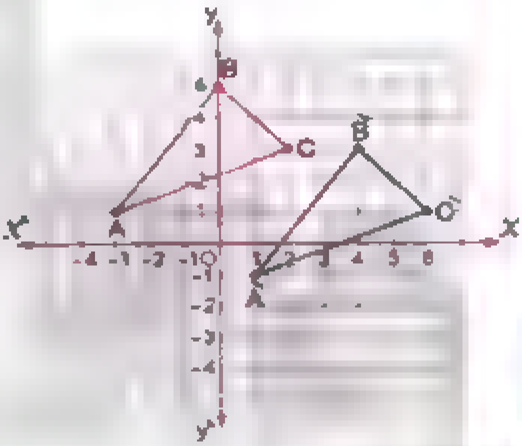
The image of the point A (x , y) $\xrightarrow{\text{By translation } (a , b)}$ the point $\acute{A}$ $(x + a , y + b)$

Example

On a square lattice , draw △ ABC where :

A (– 3 , 1) , B (0 , 5) and C (2 , 3) , then find its image by the translation $(x , y) \longrightarrow (x + 4 , y - 2)$ "The translation (4 , – 2)"

Solution

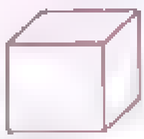


A (– 3 , 1) ⟶ $\acute{A}$ (– 3 + 4 , 1 – 2)
i.e. $\acute{A}$ (1 , – 1)
B (0 , 5) ⟶ $\acute{B}$ (0 + 4 , 5 – 2)
i.e. $\acute{B}$ (4 , 3)
C (2 , 3) ⟶ $\acute{C}$ (2 + 4 , 3 – 2)
i.e. $\acute{C}$ (6 , 1)
△ $\acute{A}\acute{B}\acute{C}$ is the image of △ ABC
by the translation $(x , y) \longrightarrow (x + 4 , y - 2)$

Area of the circle

The area of the circle = πr^2 where $\pi = \frac{22}{7} \simeq 3.14$

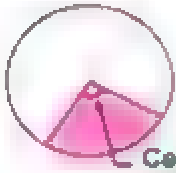
Lateral area and total area for each of the cube and the cuboid

Solid	Lateral area (L.A.)	Total area (T.A.)
Cube 	Area of one face × 4 = Edge length × itself × 4	Area of one face × 6 = Edge length × itself × 6
Cuboid 	Perimeter of base × height	Lateral area + 2 × (the area of the base)

- If a cube without a lid , then :
  The total area = The area of one face × 5 = Edge length × itself × 5
- If a cuboid without a lid , then :
  The total area = The lateral area + the area of one base.

Summary of Unit Four

Representing the statistical data by using the circular sectors



Central angle

- Each circular sector has an angle whose vertex is the centre of the circle which is called a central angle
- The sum of the measures of the angles accumulating around at a point as the centre of the circle is equal to 360°

Example

The following table shows the percentage of the production of a factory of house electrical sets :

The kind of set	Washing machine	Heater	Oven	Mixer
The percentage	20 %	15 %	40 %	25 %

Represent these data by circular sectors "pie chart".

Solution

The measure of the central angle

of washing machine = $\frac{20}{100} \times 360° = 72°$

The measure of the central angle of heater = $\frac{15}{100} \times 360° = 54°$

The measure of the central angle of oven = $\frac{40}{100} \times 360° = 144°$

The measure of the central angle of mixer = $\frac{25}{100} \times 360° = 90°$

Sample space "outcomes space"

It is the set of all possible outcomes for a random experiment.
It is usually denoted by the symbol (S) and the number of all elements of the sample space is denoted by n (S).

Example

Write the sample space of each of the following random experiments and give the number of its elements :

1. Rolling a die once and observing the number appearing on the upper face.
2. Choosing a prime number less than 20
3. Tossing two distinct coins once.

Solution

① S = {1 , 2 , 3 , 4 , 5 , 6} , n (S) = 6

② S = {2 , 3 , 5 , 7 , 11 , 13 , 17 , 19} , n (S) = 8

③ S = {HH , HT , TH , TT} , n (S) = 4

Probability of occurrence of an event

$$P(A) = \frac{\text{The number of elements of A}}{\text{The number of elements of S}} = \frac{n(A)}{n(S)}$$

- The impossible event = ∅ while the probability of the impossible event = 0 i.e. P (∅) = 0
- The certain event = S while the probability of the certain event = 1 i.e. P (S) = 1
- The probability of the possible event = proper fraction
- For any event A , we found that : $0 \le P(A) \le 1$
- The sum of probabilities of all outcomes of a random experiment = 1
- If the probability of occurrence of an event A is P (A) , then the probability that it doesn't occur = 1 – P (A)

Example

25 cards are numbered from 1 to 25 , a card is drawn at random , find the probability that :

① The number is even.

② The number is divisible by 5

③ The number is less than 26

④ The number is 30

Solution

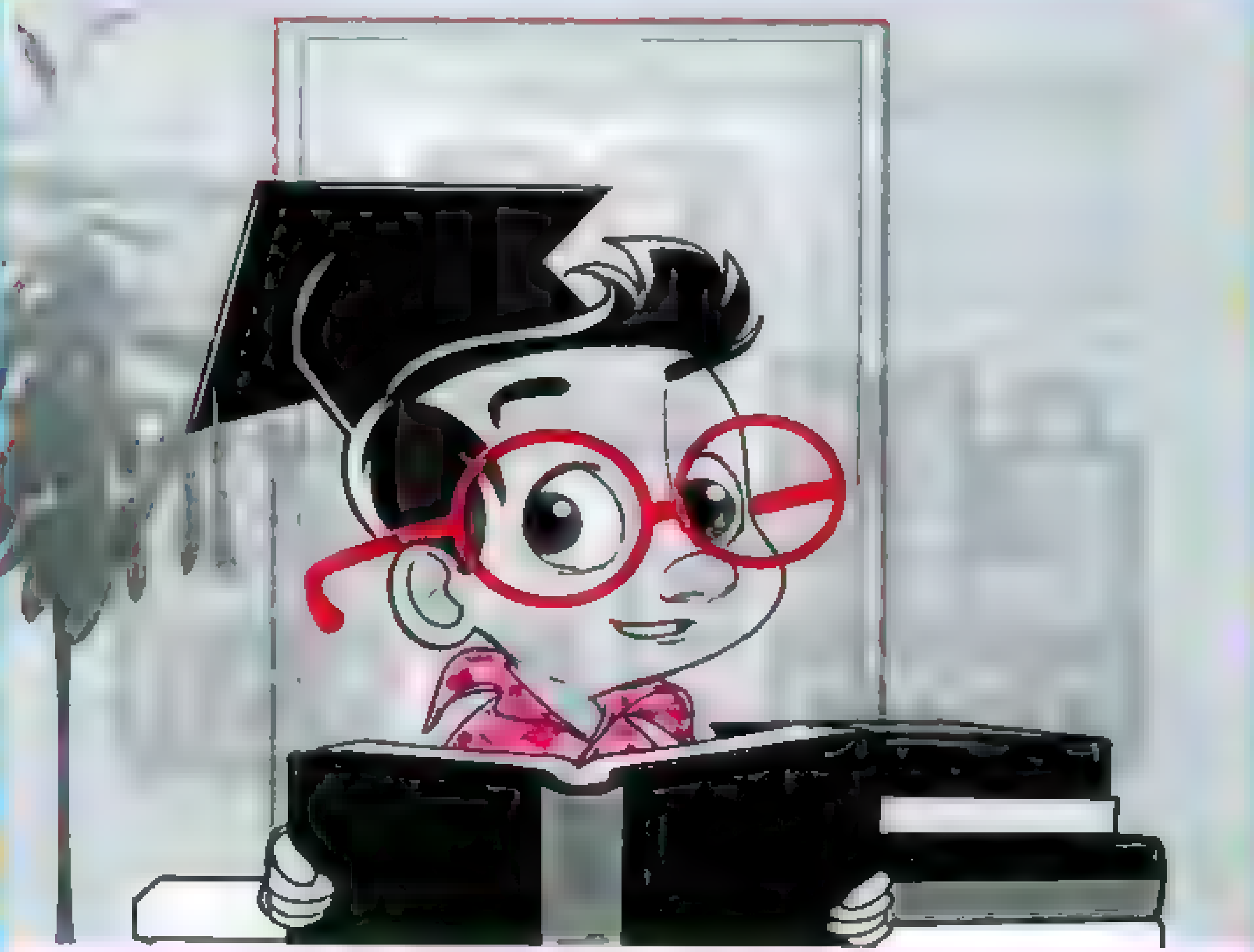
① The probability that the number is even = $\frac{12}{25}$

② The probability that the number is divisible by 5 = $\frac{5}{25} = \frac{1}{5}$

③ The probability that the number is less than 26 = $\frac{25}{25} = 1$ (Sure event)

④ The probability that the number is 30 = $\frac{0}{25} = 0$ (Impossible event)

FINAL EXAMINATIONS



- Model Examinations of the School Book
  (2 models + model for the special needs students)
- 5 Model Examinations.
- 20 Schools' Examinations from Different Governorates.

Model Examinations of the School Book

Model 1

Answer the following questions :

1 **Choose the correct answer from those given :**

(1) $(-1)^8 + (-1)^9 =$ .... (zero or -1 or 1 or 2)

(2) The image of the point $(-3, 4)$ by translation $(x, y-4)$ is ....

($(-3, 0)$ or $(-7, 4)$ or $(-3, 8)$ or $(-1, 4)$)

(3) $\{0\}$ .... $\mathbb{N}$ ($\in$ or $\notin$ or $\subset$ or $\not\subset$)

(4) When tossing a die once, then probability of getting a number on the upper face more than 6 = ... ($\varnothing$ or zero or $\frac{1}{6}$ or $\frac{1}{3}$)

2 **Complete the following :**

(1) $\left|\frac{5-11}{3}\right|$ .... $\mathbb{Z}$

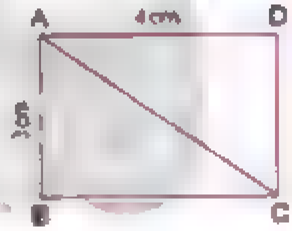
(2) If $x + 6 = 2$, $x \in \mathbb{Z}$, then $x =$ ....

(3) **In the opposite figure :**

ABCD is a rectangle

, then the area of Δ ABC

= .... cm^2



(4) A box contains 5 white balls, 3 blue balls and 8 red balls all of them are symmetric. One ball is drawn from the box at random. Then the probability that the drawn ball is red = ...

3 [a] Find the result of : $4 \times 3^2 + 3^2 - 7 \times 3$

[b] Find the solution set of the inequality : $x - 2 \geq 3$, $x \in \mathbb{Z}$

4 [a] A cuboid-shaped box with a square base its length is 10 cm. and its height is 7 cm. Calculate the lateral area.

[b] The circumference of a circle is 88 cm. Calculate its area.

5 [a] Find the solution set of the equation : $3x + 9 = 3$, $x \in \mathbb{Z}$

[b] The following table shows the percentage of the production of a factory of house electrical sets :

The kind of set	Washig machine	Heater	Oven	Mixer
The percentage	30 %	15 %	40 %	15 %

Represent these data by circular sectors.

Model 2

Answer the following questions :

1 Choose the correct answer from those given :

(1) If $2x = -6$, then $x \in$ .... ($\mathbb{N}$ or $\varnothing$ or $\mathbb{Z}^+$ or $\mathbb{Z}^-$)

(2) The circumference of the circle = ... $\times \pi$ (r or 2 r or r^2 or r + 2)

(3) When tossing a die once , then the probability of getting the number 5 = ... (zero or $\frac{1}{6}$ or $\frac{5}{6}$ or 1)

(4) The number which satisfies the inequality : $x > -2$ is .... (-1 or -2 or -3 or -4)

2 Complete the following :

(1) $\frac{2^3 \times 2^5}{2^2} =$ ....

(2) The set of counting numbers (C) .... $\mathbb{N}$

(3) A cube of total area 150 cm^2 , then the length of its edge is .... cm.

(4) In a 6^{th} primary class , the marks of the students are given in the following table :

Excellent	Very good	Good	Weak
8	18	16	6

If one of students is randownly chosen , then the probability that this pupil got good degree is ....

3 [a] Find the result of : $6 \times -5 - (2 \times 3) + 3$

[b] Find the solution set of the inequality : $x - 2 \geq 3$ where $x \in \mathbb{Z}$, then represent it on the number line.

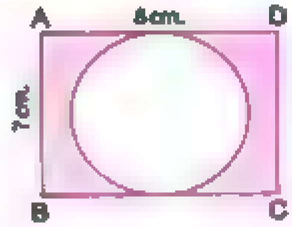
4 [a] Find the solution set of the equation : $2x + 9 = 5$, where $x \in \mathbb{Z}$

[b] In the opposite figure :

ABCD is a rectangle where its length = 8 cm.

and its width = 7 cm.

Calculate the area of shaded part.



5 [a] In a Cartesian coordinates plane , locate the points A (2 , 3) , B (4 , 3) and C (4 , 7) , then find :

(1) The length of $\overline{BC}$

(2) The image of Δ ABC by translation (0 , – 4)

[b] The following table shows the number of students partcipating in the school activties :

The activity	Cultural	Sports	Social	Arts
The percentage	5 %	45 %	15 %	35 %

Represent these data by circular sectors.

Model examination for the special needs students

Answer the following questions :

1 Complete the following :

(1) $|3| =$ ....

(2) The probability of the impossible event = ...

(3) If $x + 2 = 3$, $x \in \mathbb{N}$, then $x =$ ....

(4) The perimeter of the base of a cuboid is 10 cm. , its height is 4 cm.
, then its lateral area = ... cm²

2 Choose the correct answer from those given :

(1) $2^5 \times 2^2 =$ .... (2^7 or 4^7 or 1)

(2) The surface area of a circle = $\pi \times$ .... (r or r^2 or 2 r)

(3) $\mathbb{Z}^+ \cup \{0\} =$ .... ($\mathbb{Z}^-$ or $\mathbb{N}$ or $\mathbb{Z}$)

(4) When tossing a fair die once , then the probability of getting an odd number = ... ($\frac{1}{6}$ or $\frac{1}{3}$ or $\frac{1}{2}$)

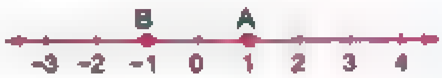
3 Put true (✔) or false (✘) :

(1) $|-5| + 5 = 10$ ( )

(2) If $3x = 9$, then $x = -3$ ( )

(3) The probability of the sure event = zero ( )

(4) In the opposite figure :



The distance between the points A and B = 2 units. ( )

4 Join from column (A) to column (B) :

A	B
(1) The sum of the measures of the angles of the sectors about the centre of the circle = …………	$\in$
(2) 2 ………… $\mathbb{Z}^+$	360°
(3) The solution set of the inequality : $x + 2 < 5$, where $x \in \mathbb{N}$ is …………	(4 , 4)
(4) The image of the point (3 , 2) by translation (1 , 2) is …………	{0 , 1 , 2}

5 [a] **Complete the following :**

The length of the edge of a cube is 4 cm. Calculate its total area and lateral area :

The total area = 6 × ………… = ………… cm²

The lateral area = 4 × ………… = ………… cm²

[b] Find the result of : $\frac{2^3 \times (-2)^4}{2^6}$

$\frac{2^3 \times 2^4}{2^6} = \frac{2^{\cdots + \cdots}}{2^6} = 2^{\cdots} = \cdots\cdots$

Model Examinations

Model (1)

Answer the following questions :

1 Choose the correct answer :

(1) A fair die is thrown once , then the probability of appearing the number 3 equals .... (0 or $\frac{1}{6}$ or $\frac{1}{3}$ or $\frac{1}{2}$)

(2) The solution set of the equation : $2x = -6$ in $\mathbb{N}$ is .... ($\{-3\}$ or $\{3\}$ or $\{2\}$ or $\emptyset$)

(3) $\{|-13|\}$ .... $\mathbb{Z}$ ($\in$ or $\notin$ or $\subset$ or $\not\subset$)

(4) If $x + 5 \geq 2$, then $x \geq$ .... (3 or -3 or 7 or -7)

(5) The integer that lies between -4 and -1 is .... (-2 or -5 or 3 or -4)

(6) $(-5)^2 \times (2)^2 =$ .... (10^0 or 10 or $(10)^2$ or $(10)^3$)

(7) If A is an event in a sample space S , P (A) = 1 , then A is .... event. (impossible or simple or sure or independent)

2 Complete each of the following :

(1) $\mathbb{Z}^+ - \mathbb{Z}^- = \mathbb{N} -$ ....

(2) $14 + 213 + (-14) =$ ....

(3) The sum of edge lengths of a cube is 84 cm. , then its lateral area equals .... cm^2

(4) The image of the point $(2 , -1)$ by translation 3 units in the positive direction of y-axis is ....

(5) If $x + 6 = 2$, where $x \in \mathbb{Z}$, then $x =$ ....

(6) $(4 \times 3 + 3) - (7 \times 3) =$ ....

(7) If $x = |-3|$, $y = -2$, then $2xy =$ ....

(8) $\frac{1}{3}$, $\frac{2}{3}$, 1 , $\frac{4}{3}$, ... , ... (in the same pattern)

3 Choose the correct answer :

(1) The multiplicative identity element in $\mathbb{Z}$ is .... (-1 or 1 or 0 or 2)

(2) $\mathbb{Z}^+ \cap \mathbb{Z}^- =$ .... ($\{0\}$ or $\emptyset$ or $\mathbb{Z}$ or zero)

(3) The surface area of the circle = ... (πr or πr^2 or $2\pi r$ or $2\pi r^2$)

(4) $3 - -3 =$ (0 or 1 or 3 or 6)

(5) The additive inverse of $(-5)^2$ is (25 or 5 or -5 or -25)

(6) $27 \div (-3)^2 =$ (-9 or 24 or 3 or 81)

(7) The measure of the angle for the sector of third of a circle is

(90° or 120° or 180° or 270°)

4 Answer the following :

(1) The circumference of a circle is 88 cm. Calculate its area. $\left(\text{Consider } \pi = \frac{22}{7}\right)$

....

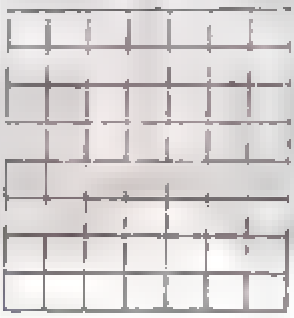
....

(2) Find the solution set of the inequality : $2x + 1 \le 7$ where $x \in \mathbb{Z}$

....

....

(3) In the cartesian coordinates plane , locate each of the following points A (1 , 1) , B (3 , 1) and C (3 , 3) , then find the image of Δ ABC by translation $(x - 2 , y + 2)$



(4) The following table shows the percentage of egg production in three farms , a merchant collected these eggs to distribute them on the grocery stores :

The farm	First	Second	Third
The percentage of the production	25%	35%	40%

Represent these data by using the circular sectors.

....

....

....

Model 2

Answer the following questions :

1 Choose the correct answer :

(1) If $x + 2 = -3$, then $x = \dots\dots$ (-1 or 1 or 5 or -5)

(2) $\mathbb{Z} = \mathbb{N} \cup \dots\dots$ ($\mathbb{Z}^+$ or $\mathbb{Z}^-$ or $\{0\}$ or $\varnothing$)

(3) If the lateral area of a cube is 36 cm^2 , then its total area = $\dots\dots$ cm^2

(144 or 81 or 54 or 96)

(4) $-8 \dots\dots \mathbb{Z}$ ($\in$ or $\notin$ or $\subset$ or $\not\subset$)

(5) $|-5| + 7 = \dots\dots$ (2 or zero or 7 or 12)

(6) $(-1)^3 + 2 = \dots\dots$ (3 or -1 or -3 or 1)

(7) If S is the sample space of a random experiment , then P (S) = $\dots\dots$

($\varnothing$ or zero or -1 or 1)

2 Complete each of the following :

(1) At throwing a fair die once , then the probability of appearing an even prime number = $\dots\dots$

(2) 1 , 4 , 7 , 10 , $\dots\dots$, $\dots\dots$ (in the same pattern)

(3) A cuboid its lateral area 120 cm^2 and the perimeter of its base 20 cm. , then its height = $\dots\dots$ cm.

(4) If X $(-4 , 1)$ and Y $(-4 , -3)$, then the length of $\overline{XY} = \dots\dots$ units.

(5) The measure of the angle of the sector whose area represents $\frac{1}{8}$ the surface area of the circle = $\dots\dots°$

(6) $\frac{8^3 \times 8^4}{8^7} = \dots\dots$

(7) The image of the point (2 , 4) by the translation $(x - 1 , y + 1)$ is $\dots\dots$

(8) The equation $2x^3 + 2x = 1$ is of the $\dots\dots$ degree.

3 Choose the correct answer :

(1) An integer between -1 , 2 is $\dots\dots$ (-2 or 3 or zero or -3)

(2) The set of counting numbers $\dots\dots \mathbb{N}$ ($\in$ or $\notin$ or $\subset$ or $\not\subset$)

(3) The multiplicative neutral element in $\mathbb{Z}$ is $\dots\dots$ (0 or 1 or -1 or 2)

(4) $|-11| \dots\dots 11$ ($>$ or $<$ or $=$ or $\leq$)

(5) The number that satisfies the inequality $x < -2$ is ....

(-3 or -2 or -1 or 0)

(6) $5^2 \times 2^2 =$ ....

(5^4 or 2^4 or 10^2 or 10^4)

(7) $\{(-1)^{zero}, (zero)^2\}$ .... $\mathbb{Z}$

($\in$ or $\notin$ or $\subset$ or $\not\subset$)

4 Answer the following :

(1) Find the solution set of the equation : $2x - 3 = -9$ where $x \in \mathbb{Z}$

....

....

(2) A cuboid box with a square base of side length 6 cm. and its height is 10 cm. Calculate its lateral surface area and its total surface area.

....

....

....

(3) Use the distributive proberty to find the result of : $32 \times 117 - 32 \times 17$

....

....

(4) The following table shows the degrees of a classroom in maths test in one month :

Assessment	Excellent	Very good	Good	Weak
Number of pupils	9	14	10	7

Represent these data by a pie chart.

....

....

....

....

Model 3

Answer the following questions :

1 Choose the correct answer :

(1) The image of point $(3 , -2)$ by translation $(4 , 2)$ is ....

$((7 , 0)$ or $(-7 , 0)$ or $(-1 , 4)$ or $(1 , 7))$

(2) The measure of the angle for the circular sector of a quarter of the circle = ...

$(30°$ or $45°$ or $60°$ or $90°)$

(3) Which of the following can be probability of an event ?

$(1.2$ or $\frac{17}{18}$ or 5^0 or $101\%)$

(4) The number which satisfies the inequality $x - 2 > 3$ is ....

$(3$ or 4 or 5 or $6)$

(5) A class of 50 pupils. If the probability of success for those pupils at the end year exam is 0.9 , then the expected number for the pupils who will success equals ....

$(50$ or 45 or 25 or $9)$

(6) $(5)^{zero} =$ ....

(zero or 5 or 1 or 50)

(7) $\frac{3}{5}$ .... $\mathbb{Z}$

$(\in$ or $\notin$ or $\subset$ or $\not\subset)$

2 Complete each of the following :

(1) If $X(-3 , 2)$, $Y(-3 , -4)$, then the length of $\overline{XY}$ = ... units.

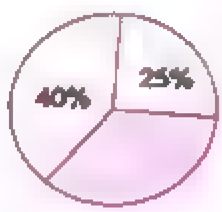
(2) The sum of edge lengths of a cube is 96 cm. , then its lateral area = ... cm^2.

(3) $(4 \times 3 + 3) - (7 \times 3) =$ ....

(4) The surface area of the circle of diameter 20 cm. = ... π cm^2.

(5) In the opposite figure :

The percentage of the shaded circular sector equals .... %



(6) $(-1)^2 - 1 =$ ....

(7) 25 , 21 , 17 , 13 , ... , ... (in the same pattern)

(8) If $2y = 8$, then $y + 3 =$ ....

3 Choose the correct answer :

(1) $|-3|+|3| =$ .... (zero or 1 or -6 or 6)

(2) If $x+1=2$, then $x=$ .... where $x \in \mathbb{N}$ (3 or 1 or -1 or -3)

(3) $3^5 \div 3^2 =$ .... (3^7 or 3^{10} or 3^3 or 3^2)

(4) $\mathbb{N}^+ \cap \mathbb{Z}^- =$ .... ($\mathbb{Z}$ or $\mathbb{Z}^*$ or $\mathbb{N}$ or $\varnothing$)

(5) The number of integers between -1 and 3 is ....

(-2 or -1 or 3 or -3)

(6) {zero} .... $\mathbb{N}$ ($\in$ or $\notin$ or $\subset$ or $\not\subset$)

(7) The equation : $2x-1=15$ is of the .... degree.

(first or second or third or fourth)

4 Answer the following :

(1) A box without a lid, in the form of a cuboid its length is 16 cm.

, its width is 7 cm. and its height is 19 cm.

Calculate each of its lateral area and its total area.

....

....

....

(2) In the experiment of forming a 2-digit number from the digits {3 , 5}

Write the sample space , then find the probability of each of the following :

[a] The envent A is the units digit equals the tens digit.

[b] The event B is the tens digit is an odd digit.

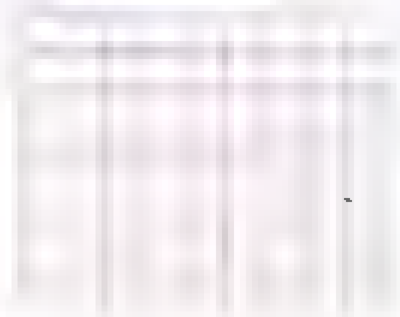
[c] The event C is the units digit is an even digit.

....

....

....

(3) In the corrdinates plane , find the image of the line segment $\overline{AB}$ where A (2 , 3) , B (– 2 , 0) by translation (x + 3 , y – 2)



(4) The following table shows the percentage of the production of a factory of house electrical sets :

The kind of set	Washing machine	Heater	Oven	Mixer
The percentage	20 %	15 %	40 %	25 %

Represent these data by circular sectors.

....

....

....

....

Model 4

Answer the following questions :

1 Choose the correct answer :

(1) 9^2 .... $(-3)^4$ (> or < or = or ≥)

(2) If zero ∈ {5 , x – 2} , then x = ... (zero or – 5 or 2 or – 2)

(3) $(-1)^3 - (1)^2$ = ... (– 2 or 1 or 0 or 2)

(4) The circumference of the circle = ...

(πr or πr^2 or $2\pi r$ or $2\pi r^2$)

(5) The multiplicative neutral element in $\mathbb{Z}$ is ....

(0 or 1 or 2 or – 2)

(6) The probability of getting a tail when throwing a coin once is ....

(0 or $\frac{1}{6}$ or 1 or $\frac{1}{2}$)

(7) A circle is of diameter length 10 cm. , then its area = ... cm^2.

(50 or 100 or 78.5 or 25)

2 **Complete each of the following :**

(1) .... is the set of all possible outcomes for a random experiment.

(2) $(2)^3 \times (-1)^2 + 8 =$ ....

(3) $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{12}$, $\frac{1}{24}$, ... , ... (in the same pattern)

(4) The measure of the central angle of the circular sector whose area represents $\frac{3}{5}$ from the surface area of the circle = ... °

(5) If $x + 2 = |-4|$, then the solution set = ...

(6) If $2y = 6$, then $y - 5 =$ ....

(7) $-4\left[3 + (-1)\right] =$ ....

(8) The solution set of the inequality $x + 1 \le 5$, where $x \in \mathbb{N}$ is ....

3 **Choose the correct answer :**

(1) The number that satisfies the inequality $x > -4$ is ....
(-5 or -6 or -4 or -3)

(2) The image of the point $(4 , -2)$ by translation $(x + 2 , y - 1)$ is ....
($(2 , -1)$ or $(6 , -3)$ or $(2 , -2)$ or $(2 , -3)$)

(3) $(-100)^{zero} =$ .... (-100 or 100 or zero or 1)

(4) $|-4| - |4| =$ .... (zero or 1 or 8 or -8)

(5) If $x + 1 = 2$, then $x =$ .... where $x \in \mathbb{N}$ (3 or 1 or -1 or -3)

(6) A cuboid with a square base , its lateral area is 224 cm^2 , its height is 14 cm. , then the side length of its base is .... cm. (14 or 4 or 2 or 3)

(7) $\left\{\frac{2}{3-4}\right\}$ .... $\mathbb{Z}$ ($\in$ or $\notin$ or $\subset$ or $\not\subset$)

4 **Answer the following :**

(1) Find the area of the opposite figure :
$\left(\text{Consider } \pi = \frac{22}{7}\right)$

....

(2) Find the solution set of the inequality : $2 - x > 3$, where $x \in \mathbb{Z}$

....

....

(3) Use the distributive property to find the result of : $43 \times 44 + 43 \times 56$

....

(4) The following table shows the percentage of four favorite sports in one of a youth center :

The favorite sports	Football	Volleyball	Basketball	Swimming
The percentage of players	40%	20%	15%	......%

Complete the table , then represent these data by circular sectors.

....

....

....

....

Model 5

Answer the following questions :

1 **Choose the correct answer :**

(1) If $3X = -9$, $X \in \mathbb{Z}$, then $X + 1 =$ .... (-3 or -2 or -1 or 4)

(2) The lateral area of the cube = area of one face × ....

(6 or 5 or 4 or 3)

(3) If X $(-2 , 1)$ and Y $(3 , 1)$, then the length of $\overline{XY}$ = ... units.

(0 or 1 or 3 or 5)

(4) If $\varnothing$ is the empty set , then P $(\varnothing)$ = ... (zero or 5 or 1 or 2)

(5) $(-3) \times |-5|$ = ... (15 or -15 or 8 or -8)

(6) $9^7 + 9^5$ = ... (9^{-12} or 9^2 or 9^{zero} or 9^{35})

(7) The next number in the pattern : 2 , 3 , 5 , 8 , 13 is ....

(18 or 19 or 20 or 21)

2 **Complete each of the following :**

(1) The measure of the angle of the sector whose area represents $\frac{3}{4}$ the surface area of the circle = ...°

(2) If the probability of success of a pupil is $\frac{2}{3}$, then the probability of his failure is ....

(3) The solution set of the inequality $x + 1 < 5$, $x \in \mathbb{N}$ is ....

(4) $(-1)^2 - 1 =$ ....

(5) The height of a cuboid whose total surface area is 400 cm^2 and its base is in the shape of a square of side length = 10 cm. equals .... cm.

(6) $85 = 5 + (8 \times 1) + (8 \times$ ....$)$

(7) If $x = |-12|$, $y = -3$, then $x + y =$ ....

(8) The greatest negative integer is ....

3 Choose the correct answer :

(1) The image of the point $(-3 , 4)$ by translation $(x , y - 4)$ is ....
$((-3 , 0)$ or $(-7 , 4)$ or $(-3 , 8)$ or $(-1 , 4))$

(2) A circle of diameter length 8 cm. , then its area = ... π cm^2.
(4 or 8 or 16 or 64)

(3) The number that satisfies the inequality : $x - 2 > 3$ is ....
(3 or 4 or 5 or 6)

(4) If $a < b$, then $-3a$ .... $-3b$ $(<$ or $>$ or $=$ or $\leq)$

(5) $\mathbb{Z} \cap \mathbb{N} =$ .... $(\mathbb{Z}^+$ or $\mathbb{Z}$ or $\{0\}$ or $\mathbb{N})$

(6) $-|-6| + 6$ .... $\mathbb{Z}^+$ $(\in$ or $\notin$ or $\subset$ or $\not\subset)$

(7) The equation : $x^3 + 1 = 10$ is of the .... degree.
(first or second or third or fourth)

4 Answer the following :

(1) Find the solution set of : $2x - 8 = -26$, where $x \in \mathbb{N}$

....

....

(2) Find the value of : $\frac{(-2)^4 \times (2)^6}{(2)^5 \times (-2)}$

....

....

(3) A box contains 4 white balls , 6 red balls and 5 blue balls , all the balls are identical , a ball is chosen randomly , find the probability that the chosen ball is :

[a] White. [b] Not red.

(4) The following table shows the percentage of the number of students in one classroom according to their favorite activities :

Activity	Sports	Reading	Music	Computer
Percentage	10%	15%	35%	40%

Represent these data by a pie chart.

Some Schools' Examinations from Different Governorates

1 Cairo Governorate

Answer the following questions :

1 Choose the correct answer :

(1) $(-19)^0 + (19)^0 =$ .... $(-1$ or zero or 1 or $2)$

(2) $\mathbb{Z} - \mathbb{N} =$ .... $(\mathbb{Z}^+$ or $\{0\}$ or $\mathbb{Z}^-$ or $0)$

(3) The height of the cuboid whose lateral area is 160 cm^2 and the dimensions of its base are 3 cm. and 7 cm. equals .... cm.

$(6$ or 8 or 10 or $16)$

(4) The image of the point A $(-4, 3)$ by translation $(-1, -4)$ is ....

$((-5, -7)$ or $(-5, -1)$ or $(-7, 3)$ or $(-3, -1))$

(5) If $a \in \{2, -5, -3\} \cap \{5, -2, -3\}$, then $a =$ ....

$(2$ or -3 or -5 or $5)$

(6) The probability of impossible event = ... $(0$ or 1 or 0.5 or $1.2)$

2 Choose the correct answer :

(1) $(|-9| + 3) \div 2$ .... $\mathbb{Z}$ $(\in$ or $\notin$ or $\subset$ or $\not\subset)$

(2) A cube the perimeter of its base is 36 cm., then its lateral area = ... cm^2

$(9$ or 324 or 36 or $486)$

(3) The number which satisfies the inequality : $x > -2$ is ....

$(1$ or -4 or -3 or $-2)$

(4) The measure of the angle of the sector which represents $\frac{1}{4}$ the circle equals .... $(30°$ or $45°$ or $90°$ or $60°)$

(5) $(-1)^{104} + (-1)^{103} =$ .... $(0$ or 2 or -1 or $1)$

(6) $3^2 + 3^2 + 3^2 =$ .... $(2^6$ or 4^6 or 3^3 or $2^9)$

3 Complete the following :

(1) $\mathbb{Z} = \mathbb{N} \cup$ ....

(2) If $x + 3 = |-7|$, then $x =$ ....

(3) The edge length of the cube whose total area is 600 cm^2 is ....

(4) The set of solution of the inequality : $-2 < x \leq$ zero in $\mathbb{Z}$ is ....

(5) The lateral area of the cuboid whose length is 6 cm. and width is 4 cm. and its height is 5 cm. equals ....

(6) A fair die is thrown once , then the probability of appearing the number 5 equals ....

(7) A circle of diameter length 14 cm. , then its area = ... cm^2 $\left(\pi = \frac{22}{7}\right)$

(8) If a = 3 , b = − 2 , then 3 a b = ...

4 [a] Find the result of : $\frac{5^{11} \times 5^4}{5^7 \times 5^6}$

[b] Find in ℕ the set of solution of the inequality : $3x - 2 < 7$

[c] A circle of radius length 10 cm. is divided into 8 equal circular sectors.

Find the area of one circular sector. (consider $\pi = 3.14$)

5 [a] In a Cartesian coordinates plane , locate the points A (0 , 4) , B (2 , 1) , C (− 2 , 1) , then find the image of Δ ABC by translation (0 , − 2)

[b] The following table shows the percentage of the production of a factory of house electrical sets :

The kind of set	Washing machine	Heater	Oven	Mixture
The percentage	30 %	15 %	40 %	15 %

Represent these data by circular sectors.

2 Cairo Governorate

Answer the following questions :

1 Choose the correct answer :

(1) The set of non-negative integers is .... (ℂ or ℤ or {0} or ℕ)

(2) The equation : $2^5 + x^5 = 100$ is of the .... degree (11^{th} or 5^{th} or 6^{th} or 1^{st})

(3) If ∅ is the empty set , then P (∅) = ... (1 or 2 or 0 or 0.5)

(4) The area of the circle whose radius length is 2 π cm. is .... cm^2. (4π or $2\pi^2$ or 12.56 or $4\pi^3$)

(5) The integer which satisfies the inequality : $y < -3$ is .... (−2 or −8 or 0 or 1)

(6) If $3X = -9$, then $-5X$ = ... (15 or 9 or −15 or −|−15|)

2 Choose the correct answer :

(7) The image of the point (4 , −2) by translation two units in the positive direction of the y-axis is .... ((4 , 2) or (2 , −2) or (6 , −2) or (4 , 0))

(8) The L.S.A. of the cuboid whose dimensions are 3 cm. , 4 cm. and 0.6 dm. is .... (72 cm^2 or 8.4 dm^2 or 84 dm^2 or 84 cm^2)

(9) -9^3 .... $(-3)^2$ (< or = or > or ≥)

(10) $ℤ^+ \cap ℤ^-$ = ... (ℤ or ℕ or 0 or { })

(11) Half the T.S.A. of a cube whose sum of its edge lengths is 36 cm. is .... cm^2. (108 or 27 or 54 or 18)

(12) A box contains 14 balls , 5 red , 3 green and the rest are yellow , then the probability of selecting a non-red ball is .... ($\frac{3}{7}$ or $\frac{5}{14}$ or $\frac{9}{14}$ or $\frac{4}{7}$)

3 Complete :

(1) The ratio between the T.S.A. and L.S.A. of the cube is ....

(2) If A (2 , 9) , B (−4 , 9) , then the length of $\overline{AB}$ = ... length units.

(3) The probability of appearing an odd prime number when rolling a die once is ....

(4) The circumference of the circle whose area is 452.16 cm^2 is .... (π = 3.14)

(5) $((-7)^3 \times 7^4) \div (-7)^5 = \ldots\ldots\ldots$

(6) The S.S. of the inequality $3 + 4\,x > -9$ in $\mathbb{Z}$ is $\ldots\ldots\ldots$

(7) The volume of a cube whose L.S.A. is 144 cm^2 is $\ldots\ldots\ldots$ cm^3

(8) The measure of the central angle which represents $\frac{1}{9}$ of the circle is $\ldots\ldots\ldots$

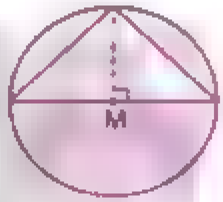
4 **Answer the following :**

(1) Find the S.S of the equation : $2\,x - 3 = -9$ in $\mathbb{Z}$ and in $\mathbb{N}$

(2) Use the distributive property to find the result : $25 \times 9 + 25 - 25 \times 9$

(3) Find the area of the shaded part

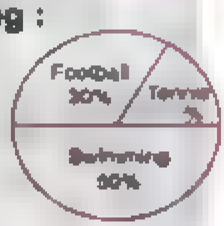
, if the radius length = 7 cm. $\left(\pi = \frac{22}{7}\right)$



(4) **Notice the opposite pie chart , then complete the following :**

[a] The percentage of the tennis players is $\ldots\ldots\ldots$

[b] The measure of the angle of the sector which represents the football players is $\ldots\ldots\ldots$



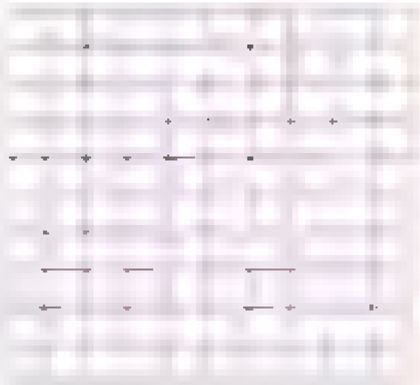
(5) In the coordinate plane

, draw the figure ABCD , where :

A (3 , 1) , B (1 , 3) , C (3 , 5) and D (5 , 3)

, then draw its image by translation $(x - 4 , y - 4)$

What is the area of the image of the figure ?



3 Giza Governorate

Answer the following questions :

1 Choose the correct answer :

(1) $(-1)^{12} + (-1)^{13} = \dots\dots$ (0 or 1 or 2 or −1)

(2) $5 \times 5^2 =$ (25^2 or 25^3 or 5^2 or 5^3)

(3) If $x - 5 = 7$, $x \in \mathbb{N}$, then $x =$ (2 or 12 or −12 or 35)

(4) The image of the point (4 , 5) by translation (0 , −4) is
((4 , 9) or (5 , 1) or (4 , 1) or (4 , −1))

(5) When tossing a dice once , then the probability of getting a number less than 1 = ... ($\varnothing$ or 0 or $\frac{1}{6}$ or 1)

(6) The set of odd numbers ∩ the set of even numbers =
(0 or $\mathbb{N}$ or $\mathbb{Z}$ or $\varnothing$)

(7) A circle , its circumference is 44 cm. , then the length of its radius = ... cm. $\left(\pi = \frac{22}{7}\right)$ (22 or 11 or 7 or 14)

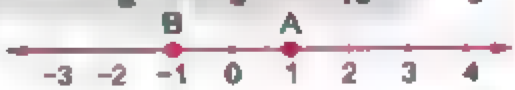
(8) $\left|\frac{6-12}{3}\right|$ .... $\mathbb{N}$ ($\notin$ or $\in$ or $\not\subset$ or $\subset$)

(9) If $2x = 6$, then $4x =$ (3 or 6 or 12 or 18)

(10) If $x + 2 < 2$, then $x \in$ .... ($\mathbb{N}$ or $\varnothing$ or $\mathbb{Z}^+$ or $\mathbb{Z}^-$)

(11) A box contains 10 cards numbered from 1 to 10 , one card is selected at random , then the probability of getting a number divisible by 5 = ...
($\frac{1}{2}$ or $\frac{1}{5}$ or $\frac{3}{10}$ or $\frac{2}{5}$)

(12) In the opposite figure :



The distance between the two points A and B = ... units. (2 or −2 or 1 or 3)

2 Complete :

(1) $4 \times 3^2 + 3^2 - 7 \times 3 =$ ....

(2) If $x + 3 = |-6|$, then $x =$ ....

(3) The sum of the measures of the angles of the sectors about the centre of the circle = ...

(4) The equation : $x^2 + 3 = 8$, then the equation is of .... degree.

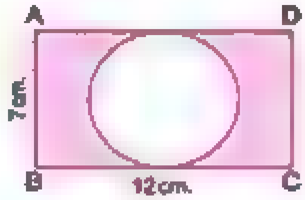
(5) A box contains 15 balls all of them are symmetric , 5 white balls , 4 blue balls and the rest are red balls , one ball is drawn from the box at random , then the probability that the drawn ball is red = ...

(6) The image of the point $(-1, 2)$ by translation of 3 units in the positive direction of the X-axis is ....

(7) The lateral area of a cuboid with a square base its length is 10 cm. and its height is 9 cm. = ...

(8) **In the opposite figure :**

ABCD is a rectangle, its length is 12 cm., its width is 7 cm. A circle is drawn to touch the sides $\overline{AD}$ and $\overline{BC}$, then the area of the shaded part = ... $\left(\pi = \frac{22}{7}\right)$



3 **Answer the following :**

(1) Find the result of : $\frac{(-4)^{11} \times 4^3}{4^{12}}$

....

(2) Find the solution set of the inequality : $2X + 9 < 1$ in $\mathbb{Z}$ and represent it on the number line.

....

(3) A container water tank in the form of a cube, its inner edge length is 1.5 m. It is wanted to paint it to prevent the rust. The cost price of one square metre is L.E. 15, calculate the cost of painting.

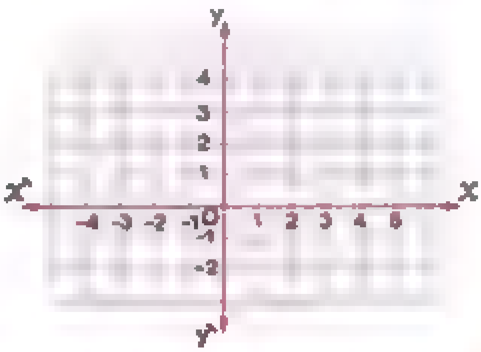
....

(4) **On the coordinate plane :**

Locate the points A $(3, -2)$, B $(1, 1)$ and C $(3, 1)$, then :

[a] Find the length of $\overline{BC}$

[b] Draw the image of Δ ABC by translation $(X + 2, y + 3)$



....

(5) The following table shows the percentage of the favourite sport for your class students :

The favourite sport	Football	Basketball	Volleyball	Swimming
The percentage	45 %	10 %	25 %	20 %

Represent these data by using the circular sectors.

4 Alexandria Governorate

Answer the following questions :

1 Choose the correct answer from those between brackets :

(1) $\mathbb{Z} = \mathbb{N} \cup$ .... ($\{0\}$ or $\varnothing$ or $\mathbb{Z}^+$ or $\mathbb{Z}^-$)

(2) $\{0\}$ .... $\mathbb{Z}$ ($\in$ or $\notin$ or $\subset$ or $\not\subset$)

(3) If $x \in \{2, 5, -3\} \cap \{-5, -2, -3\}$, then $x =$ .... (-5 or -3 or -2 or 2)

(4) $(9)^2$ .... $(-3)^4$ ($>$ or $<$ or $=$ or otherwise)

(5) (-7) .... $(-|-5|)$ ($>$ or $<$ or $=$ or otherwise)

(6) The solution set of the equation : $x - 2 = 3$ in $\mathbb{Z}$ is .... (5 or 1 or $\{5\}$ or $\{3\}$)

(7) The number which satisfies the inequality : $x + 4 > 2$ is .... (-1 or -2 or -3 or -4)

(8) A cube of edge length 6 cm. , then its lateral area = ... cm^2. (216 or 180 or 144 or 108)

(9) The image of the point (........ , ...) by translation $(x - 3, y + 4)$ is $(-5, -3)$ ($(-8, 15)$ or $(-2, 7)$ or $(-8, 7)$ or $(-2, -7)$)

(10) The lateral area of the cube = Area of one face × .... (2 or 4 or 6 or height)

(11) The sum of measures of the angles of the sectors about the centre of the circle = ... (100° or 150° or 180° or 360°)

(12) If $\varnothing$ is empty set , then P ($\varnothing$) = ... (0 or 2 or 1 or 0.5)

2 **Complete each of the following :**

(1) $|-5| + |7| =$ ....

(2) $5 \times (-3 + 7) = 5 \times (-3) + 5 \times$

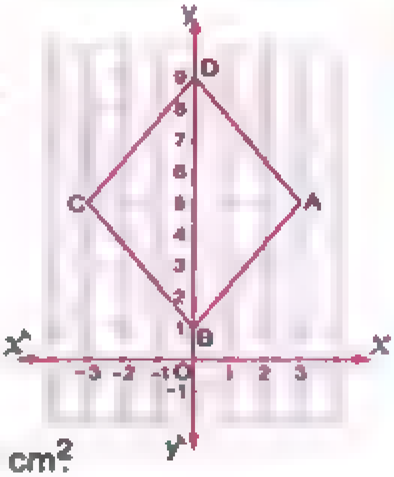
(3) The S.S. of the inequality : $x + 4 < 7$ in $\mathbb{N}$ is ....

(4) **In the opposite coordinate plane :**

A (.............. , ...)

(5) **In the opposite coordinate plane :**

The length of $\overline{AC}$ = ... units.



(6) If the lateral area of a cube is 100 cm^2 , then its total area = ... cm^2.

(7) The perimeter of the base of a cuboid is 10 cm. , its height is 4 cm. , then its lateral area = ... cm^2.

(8) When tossing a die once , then probability of getting a number 5 = ...

3 **Answer the following :**

(1) Arrange the following numbers in an ascending order :

-9 , 17 , $|-9|$, -15 and 16

....

(2) Find the result in the simplest form by using the basic laws of repeated multiplication : $\dfrac{(-5)^3 \times (-5)^2}{(-5)^4}$

....

....

(3) A circle , its diameter length is 7 cm. , calculate its surface area where $\pi = \frac{22}{7}$

....

....

(4) **In the coordinate plane :**

ABCD is a rectangle where

A (4 , 1), B (4 , 3), C (1 , 3) and D (1 , 1)

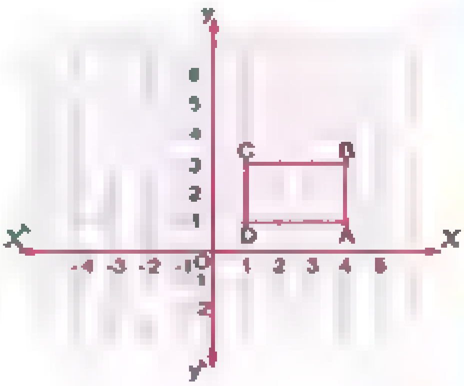
, find its image by translation ($x - 5$, $y + 3$)

....

....

....

....



(5) The following table shows the number of students participating in the school activities :

The activity	Cultural	Sports	Social	Arts
The percentage	5 %	45 %	15 %	35 %

Represent these data by circular sectors.

5 El-Kalyoubia Governorate

Al-Obour Educational Zone
Al-Rowda Language School

Answer the following questions :

1 Choose the correct answer :

(1) $\{-3, -\frac{1}{3}\}$ .... $\mathbb{Z}$ ($\subset$ or $\in$ or $\not\subset$ or $\notin$)

(2) $(-1)^7 \times 2^3 =$ .... (2^5 or 8 or -8 or -2^5)

(3) If $2x = 10$, then $x + 2 =$ .... (7 or 3 or 5 or 6)

(4) The equation : $x^2 + 3 = 4$ is of .... degree.
(1^{st} or 3^{rd} or 2^{nd} or 4^{th})

(5) The image of the point $(3, -2)$ by translation $(-3, 2)$ is ....
((0, 0) or (3, 0) or (2, 0) or (6, 4))

(6) The sum of the measures of the accumulative angles at the centre of a circle is .... (90° or 360° or 180° or 70°)

(7) When throwing a fair die once, the probability of appearing number less than 4 = ... ($\frac{5}{6}$ or $\frac{1}{2}$ or $\frac{2}{3}$ or $\frac{1}{6}$)

(8) The lateral area of a cube whose side length is 3 cm. = ... cm.²
(27 or 48 or 36 or 54)

(9) The number which satisfies the inequality : $x - 2 > 3$ is ....
(3 or 5 or 4 or 6)

(10) $2^6 \times 2^4 =$ .... (2^2 or 2^{12} or 2^{10} or 2^{24})

2 Complete the following :

(1) $12 \times$ .... $= -72$

(2) $3^7 + 3^7 =$ ....

(3) A circle, its diameter length is 14 cm., then its area = ... cm.² $(\pi = \frac{22}{7})$

(4) $\mathbb{N} \cup \mathbb{Z}^- =$ ....

(5) The solution set of the equation $3X + 2 = 8$ in $\mathbb{N}$ is ....

(6) The solution set of the inequality : $X + 5 \leq 7$ where $X \in \mathbb{Z}$ is ....

(7) A cuboid whose length is 9 cm. , width is 7 cm. and its height is 10 cm. , then its lateral area = ... and its total area = ...

(8) The greatest negative integer is ....

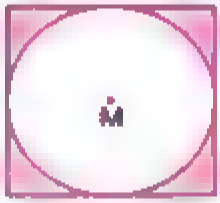
3 **Answer the following :**

(1) A box contains 5 white balls , 9 red balls and 4 black balls. If a ball is selected randomly , then calculate the probability that the selected balls is :

[a] White = ... [b] Black or red = ...

[c] Yellow = ... [d] Not black = ...

(2) A circle M is drawn inside a square of side length 14 cm and touches its sides. Calculate the area of the shaded part. ($\pi \simeq 3.14$)



....

(3) Arrange in an ascending order : $(-2)^3$, $(-3)^2$, $(-1)^{15}$ and $(-5)^2$

....

(4) In a Cartesian coordinate plane locate the points A (4 , 3) , B (4 , 1) , C (1 , 1) and D (1 , 3) , then find :

[a] Its image by translation $(X - 2 , y - 3)$

....

[b] Area of the figure and its perimeter.

The area = ... , the perimeter = ...

[c] Name of the figure. (..........)

6 El-Sharkia Governorate

Answer the following questions :

1 **Choose the correct answer :**

(1) $(-1)^8 + (-1)^9 =$ .... (zero or 1 or -1 or 2)

(2) If the radius length of a circle is 10 cm. , then its surface area = ... cm^2 (Given that : $\pi = 3.14$) (3.14 or 31.4 or 314 or 3140)

(3) $\varnothing$ .... {a , b} ($\in$ or $\notin$ or $\subset$ or $\not\subset$)

(4) All the following numbers satisfy the inequality : $x > -3$ except ....
(zero or -1 or -2 or -3)

(5) The image of the point $(-3 , 4)$ by translation $(0 , -4)$ is (........ , ...)
($(-3 , 0)$ or $(-7 , 4)$ or $(-3 , 8)$ or $(-1 , 4)$)

(6) $\mathbb{Z} - \mathbb{Z}^- =$ .... ($\varnothing$ or $\mathbb{N}$ or $\mathbb{Z}^+$ or $\{0\}$)

(7) The measure of the angle for the circular sector of half of a circle is ....
(90° or 120° or 180° or 360°)

(8) The equation , $x + 2 = 10$ is of the .... degree.
(first or second or third or fourth)

(9) If a die is rolled once , then the probability of getting a number 5 is ....
(1 or $\frac{5}{6}$ or $\frac{1}{6}$ or $\frac{1}{5}$)

(10) If the edge length of a cube is 6 cm. , then its total area = ... cm^2
(24 or 36 or 144 or 216)

(11) $(-5) \times |-4| =$ .... (20 or -20 or 9 or -9)

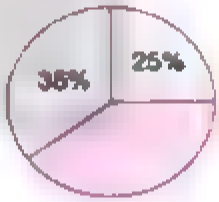
(12) $(3)^7 \div (3)^4 =$ .... ($(3)^3$ or $(3)^5$ or $(3)^{11}$ or $(3)^2$)

2 Complete each of the following :

(13) $\mathbb{Z} = \mathbb{Z}^- \cup$ .... $\cup$ ....

(14) The lateral surface area of a cuboid = ... × height.

(15) In the opposite figure :
The percentage of the shaded circular sector = ... %



(16) The probability of the impossible event equals ....

(17) If $x + 6 = 2$, where $x \in \mathbb{Z}$, then $x =$ ....

(18) The sum of measures of angles accumulative around the centre of the circle = ...°

(19) $\frac{2^3 + 2^5}{2^2} =$ ....

(20) The circumference of the circle = ... × ....

3 Answer the following :

(21) Find the solution set of the equation : $2x + 9 = 5$ where $x \in \mathbb{Z}$

....

....

....

(22) Use the properties of addition in $\mathbb{Z}$ to find the result of :
$-17 + 19 + 17$ (state the property used in each step).

....

....

....

(23) A cuboid with a square shaped base of side length 7 cm. and its height is 10 cm. , calculate its lateral surface area.

....

....

(24) Find the solution set of the inequality : $x + 4 < 7$, where $x \in \mathbb{N}$

....

....

....

(25) The following table shows the favorite sport in youth centre :

Sports	Football	Basketball	Handball	Volleyball
Percentage	40 %	20 %	30 %	10 %

Represent these data by circular sector.

....

....

....

....

7 El-Monofia Governorate

Answer the following questions :

1 Choose the correct answer from those between brackets :

(1) $\mathbb{Z} - \mathbb{Z}^- =$ .... ($\mathbb{Z}^+$ or $\mathbb{N}$ or $\{0\}$ or $\varnothing$)

(2) The number which satisfies the inequality : $x > -2$ is ....
(-1 or -2 or -3 or -4)

(3) The surface area of a circle $= \pi \times$ .... (r or r^2 or $2r$ or $2r^2$)

(4) When tossing a die once , then the probability of getting a number 5 = ...
(zero or $\frac{1}{6}$ or $\frac{5}{6}$ or 1)

(5) $(-1)^8 + (-1)^9 =$ .... (zero or -1 or 1 or 2)

(6) If $2x = -6$, then $x \in$ .... ($\mathbb{N}$ or $\varnothing$ or $\mathbb{Z}^+$ or $\mathbb{Z}^-$)

(7) If A (– 2 , 1) and B (3 , 1) , then the length $\overline{AB}$ = ... length units.
(0 or 1 or 3 or 5)

(8) If ∅ is the empty set , then P (∅) = ... (zero or 0.5 or 1 or 2)

(9) (–5) × |4| = ... (20 or – 20 or 9 or – 9)

(10) If a < b , then : – 3 a .... – 3 b (< or > or = or ∈)

(11) The image of the point (– 3 , 4) by translation (X , y – 4) is ....
((– 3 , 0) or (– 7 , 4) or (– 3 , – 8) or (– 1 , 4))

(12) The lateral surface area of the cube = area of one face × ....
(6 or 5 or 4 or 3)

2 Complete :

(1) The probability of apperance a head when tossing a coin once = ...

(2) A circle of diameter length 8 cm. , then its area = ... π cm^2.

(3) The lateral area of the cuboid = perimeter of the base × ....

(4) The equation : $4X^3 - X = 29$ is of .... degree.

(5) A circular sector represents $\frac{1}{3}$ of a circle , then the measure of its central angle = ...

(6) If the area of one face of a cube equal 9 cm^2 , then its total area = ... cm^2

(7) The solution set of the inequality $-2 < X \leq$ zero in $\mathbb{Z}$ is ....

(8) The perimeter of one face of a cube is 12 cm. , then its total area = ... cm^2

3 Answer the following :

(1) A cuboid-shaped box with a square base its length is 10 cm. and its height is 7 cm. Calculate the lateral area.

....

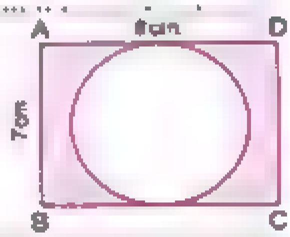
....

(2) Find the solution set of the equation : $2X + 9 = 3$, $X \in \mathbb{Z}$

....

....

(3) **In the opposite figure :**



ABCD is a rectangle where its length = 8 cm. and its width = 7 cm.

Calculate the area of shaded part.

....

....

....

(4) Use the properties of addition in $\mathbb{Z}$ to find :

$$116 + 190 + (-116)$$

(5) The following table shows the number of students participating in the school activities :

The activity	Cultural	Sports	Social	Arts
The percentage	5 %	45 %	15 %	35 %

Represent these data by circular sectors.

8 El-Gharbia Governorate

Answer the following questions :

1 Choose the correct answer :

(1) A fair die is thrown once , then the probability of appearing the number 6 equals .... (0 or $\frac{1}{6}$ or $\frac{1}{3}$ or $\frac{1}{2}$)

(2) The solution set of the equation : $3x = -6$ in $\mathbb{N}$ is .... ($\{-3\}$ or $\{3\}$ or $\{2\}$ or $\varnothing$)

(3) If $x + 5 \geq 2$, then $x \geq$ .... (3 or -3 or 7 or -4)

(4) The integer that lies between -4 and -1 is .... (-2 or -5 or 3 or -4)

(5) $(-5)^2 \times (2)^2 =$ .... (10^0 or 10 or 10^2 or 10^3)

(6) If A is an event in a sample space S , P (A) = 1 , then A is .... event. (impossible or possible or sure)

(7) The multiplicative identity element in $\mathbb{Z}$ is .... (-1 or 1 or 0 or 2)

(8) $\mathbb{Z}^+ \cap \mathbb{Z}^- =$ .... ($\{0\}$ or $\varnothing$ or $\mathbb{Z}$ or zero)

(9) The surface area of the circle = ... (π or πr^2 or $2\pi r$ or $2\pi r^2$)

(10) The additive inverse of $(-5)^2$ is .... (25 or 5 or -5 or -25)

(11) $27 + (-3)^2 =$ .... (-9 or 24 or 3 or 81)

(12) The measure of the angle for the sector of third of a circle is ....

(90° or 120° or 180° or 270°)

2 **Complete each the following :**

(1) $\mathbb{Z}^+ - \mathbb{Z}^- = \mathbb{N} -$ ....

(2) $14 + 213 + (-14) =$ ....

(3) The sum of edge lengths of a cube is 84 cm. , then its lateral area equals .... cm^2

(4) The result of : $2^3 \times (-1)^2 + 8 =$ ....

(5) If $x + 6 = 2$, where $x \in \mathbb{Z}$, then $x =$ ....

(6) $(4 \times 3 + 3) - (7 \times 3) =$ ....

(7) If $x = |-3|$, $y = -2$, then $2\,x\,y =$ ....

(8) If $-5\,x = 35$, where $x \in \mathbb{Z}$, then $x =$ ....

3 **Answer the following :**

(1) The circumference of a circle is 88 cm. Calculate its area. $\left(\text{Consider } \pi = \frac{22}{7}\right)$

....

(2) Find the solution set of the inequality : $2\,x + 1 \leq 7$ where $x \in \mathbb{Z}^+$

....

(3) In the Cartesian coordinates plane , locate each of the following points A (1 , 1) , B (3 , 1) and C (3 , 3) , then find the image of Δ ABC by translation $(x - 2 , y + 2)$

....

(4) The following table shows percentage of egg production in three farms , a merchant collected these eggs to distribute them on the grocery stores :

The farm	First	Second	Third
The percentage of the production	25 %	35 %	40 %

Represent these data by using the circular sectors.

....

9 El-Dakahlia Governorate

Answer the following questions :

1 Choose the correct answer :

(1) $|-98|$ .... $\mathbb{Z}^-$ ($\notin$ or $\in$ or $\subset$ or $\not\subset$)

(2) The image of the point (........ , ...) by translation $(x-3, y+4)$ is $(-5, -3)$ ($(-8, 1)$ or $(-2, -7)$ or $(-2, 7)$ or $(2, 7)$)

(3) The equation : $x^2 + x = 5$ is of .... degree
(fourth or third or second or first)

(4) The probability of the impossible event = ... (1 or $\frac{1}{2}$ or $\frac{1}{4}$ or 0)

(5) $(-6)^2$ .... -12 ($>$ or $=$ or $<$ or $\leq$)

(6) A circle, its diameter length is 20 cm., then its area = ... cm^2 ($\pi = 3.14$)
(31.4 or 314 or 23.14 or 43.14)

(7) $2 - (-3)^0 =$ .... (5 or 3 or 1 or 2)

(8) The sum of edge lengths of a cube is 24 cm., then T.S.A. = ... cm^2
(16 or 36 or 4 or 24)

(9) If X (3, 8), Y (3, 4), then the length of $\overline{XY}$ = ... length units.
(4 or 6 or 12 or 5)

(10) If (S) is a sample of a random experiment, then P (S) = ...
(0 or 1 or $\frac{1}{4}$ or $\frac{1}{2}$)

(11) If $3y = 9$, then $y + 5 =$ .... (11 or 32 or 8 or 14)

(12) The additive inverse of $(-3)^2$ is .... (9 or 3 or -3 or -9)

2 Complete :

(1) Two things must be known for the translation to happen .... , ...

(2) The probability of the sure event = ...

(3) $(-1)^{100} + (-1)^{103} =$ ....

(4) If a cuboid shaped box with a square base its length is 9 cm. and its height is 10 cm., then the L.S.A. = ... cm^2

(5) $(-6) \times (-2) =$ ....

(6) The measure of the angle for the sector of third of a circle = ... °

(7) A cube , its volume is 1000 cm^3 , then its lateral area = ... cm^2

(8) $2 \times 3^2 + 3^2 - 4 \times 3 =$ ....

3 **Answer the following :**

(1) Find the solution set of : $3x - 7 \leq 5$, where $x \in \mathbb{Z}$

....

....

....

(2) Find the value of : $\frac{(-3)^7 \times (-3)^4}{(-3)^5}$

....

....

(3) In the coordinate plane :

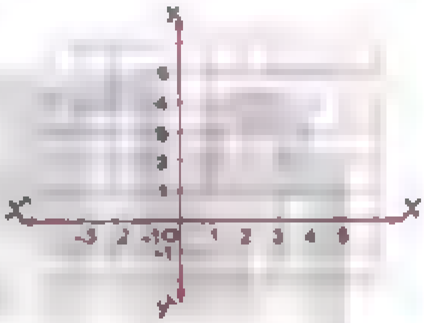
Locate each of the following points

A (2 , 3) , B (4 , 3) and C (4 , 5)

, then find :

[a] The length of $\overline{BC}$ = ... length units.

[b] The image of $\triangle ABC$ by translation (0 , – 2)



....

....

....

(4) Find the lateral area and total area of a cuboid without lid , its length is 16 cm. , its width is 9 cm. and its height is 5 cm.

....

....

....

(5) The following table shows the percentages of production of a factory for three kinds of electric water heaters :

The kind	1^{st}	2^{nd}	3^{rd}
Percentage	25 %	35 %	40 %

Represent data by the circular sectors.

....

....

....

10 Ismailia Governorate

Answer the following questions :

1 Choose the correct answer :

(1) $\mathbb{Z}^+ \cap \mathbb{Z}^- =$ .... ($\varnothing$ or 1 or -1 or 2)

(2) If $2x = 0$, then $x =$ .... (2 or 3 or 5 or 0)

(3) The greatest negative integer is .... (2 or 1 or 0 or -1)

(4) If $x + 6 = 5$, then the solution set in $\mathbb{N}$ is .... ($\{-1\}$ or $\{1\}$ or $\varnothing$ or $\{0\}$)

(5) If $x + 2 = |-5|$, then $x =$ .... (3 or -3 or 7 or 4)

(6) The solution set of the inequality : $x > 0$ in $\mathbb{Z}$ is .... ($\mathbb{Z}$ or $\mathbb{Z}^+$ or $\mathbb{Z}^-$ or $\mathbb{N}$)

(7) The image of the point (3 , 0) by translation of magnitude 3 units in the negative direction of x-axis is .... ((3 , 3) or (0 , 0) or (3 , -3) or (0 , -3))

(8) If $x > y$, then $x + z$ .... $y + z$ ($>$ or $<$ or $=$ or $\leq$)

(9) The probability of the impossible event = ... ($\varnothing$ or 1 or 0 or -1)

(10) The surface area of the circle = $\pi \times$ .... (r or 2 r or r^2 or r^3)

(11) If a fair die is rolled once, then the probability of getting an even number = ... (0 or $\frac{1}{2}$ or $\frac{1}{3}$ or $\frac{1}{4}$)

(12) If the total area of the cube = 54 cm^2, then the area of one face = ... cm^2 (4 or 5 or 8 or 9)

2 Complete :

(1) $\mathbb{Z}^+ - \mathbb{Z}^- = \mathbb{N} -$ ....

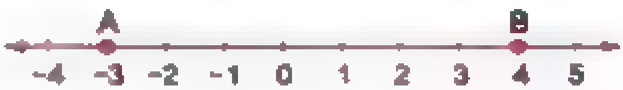
(2) The sum of edge lengths of a cube = 120 cm. , then the lateral area = ... cm^2

(3) $y - 4 < 2$ is an inequality of .... degree.

(4) The area of the circle whose diameter length is 14 cm = ... cm^2

(5) On the number line :

The length of $\overline{AB}$ = ... length units.



(6) If $|x| = 3$, then $x =$ ....

(7) If one of the families spends its salary as the following 40 % for food , 20 % for house rent , 30 % for expenses , then saves the remainder is .... %

(8) A cuboid of length 6 cm. , width 4 cm. and height 5 cm , then its lateral area = ... cm^2

3 **Answer the following :**

(1) Find the value of : $\frac{(-2)^5 \times 3^5}{3^3 \times (-2)^3}$

....

....

(2) Calculate the area of the opposite figure.

$\left(\text{Consider } \pi = \frac{22}{7}\right)$

....

....

....

(3) The perimeter of the base of a cube is 28 cm.

Calculate its lateral area and total area.

....

....

....

(4) Find the solution set of the following equation , where $x \in \mathbb{Z}$: $x + 5 = 4$

....

....

(5) A box contains 25 balls , 6 balls are yellow , 7 balls are red and the remainder is black , if a ball is drawn randomly.

Find the probability that the drawn ball is :

[a] Black = ... [b] Not red = ...

11 Suez Governorate

Answer the following questions :

1 **Choose the correct answer :**

(1) When tossing a die once , then the probability of getting a number on the upper face more than 6 = ... (zero or $\frac{1}{6}$ or $\frac{1}{3}$ or $\varnothing$)

(2) $\{0\}$ .... $\mathbb{N}$ ($\subset$ or $\not\subset$ or $\in$ or $\notin$)

(3) The equation : $x^2 + 3 = 8$ is of .... degree
(first or second or third or fourth)

(4) $|-5|$ .... 5 ($<$ or $=$ or $>$ or otherwise)

(5) $(-1)^8 + (-1)^9 =$ .... (-1 or zero or 1 or 2)

(6) The sum of the measures of the accumulative angles at a point = ... °
(90 or 180 or 270 or 360)

(7) If $2x = -6$, then $x \in$ .... ($\mathbb{N}$ or $\varnothing$ or $\mathbb{Z}^+$ or $\mathbb{Z}^-$)

(8) $\frac{1}{7^5} \times 7^5$ .... 1 ($<$ or $=$ or $>$ or otherwise)

(9) The total area of the cube = Area of one face × ....
(2 or 4 or 6 or 8)

(10) On the number line :
AB = ... units

(8 or 7 or 5 or -2)

(11) $5 \times (-4) =$ .... (-20 or 20 or 9 or -1)

(12) The image of the point $(-3 , 4)$ by translation $(x , y - 4)$ is ....
($(-3 , 0)$ or $(-7 , 4)$ or $(-3 , 8)$ or $(-1 , 4)$)

2 Complete :

(1) $\mathbb{Z} - \mathbb{N} =$ ....

(2) The circumference of the circle = ... $\times \pi$

(3) $\frac{2^2 \times 2^5}{2^2} =$ ....

(4) If $x + 6 = 2$, $x \in \mathbb{Z}$, then $x =$ ....

(5) The lateral area of the cuboid = perimeter of the base × ....

(6) A cube of edge length 10 cm , then its lateral area = ...

(7) .... = (length + width) × 2

(8) A box contains 5 white balls , 3 blue balls and 8 red balls all of them are symmetric. One ball is drawn from the box at random. Then the probability that the drawn ball is red = ...

3 Answer the following :

(1) Use the properties of addition in $\mathbb{Z}$ to find the result of :
$(-7) + 19 + 17$ (state the property used in each step)
....
....
....

(2) Find the solution set of the following inequality in $\mathbb{Z}$: $x - 2 \leq 3$

(3) A circle , its radius length is 7 cm. , calculate its surface area. $\left(\text{where } \pi = \frac{22}{7}\right)$

(4) A cuboid shaped box with a square base. Its length is 10 cm , its height is 7 cm. Calculate the lateral area.

(5) The following table shows the percentages of the production of a factory of house electrical sets :

The kind of set	Washing machine	Heater	Oven	Mixer
The percentage	25 %	15 %	40 %	20 %

Represent these data using circular sectors.

12 Port Said Governorate

Answer the following questions :

1 Choose the correct answer :

(1) The surface area of a circle = $\pi \times$ .... (r or r^2 or 2 r or 3.14)

(2) If $-2x = 6$, then $x \in$ .... ($\mathbb{N}$ or $\varnothing$ or $\mathbb{Z}^+$ or $\mathbb{Z}^-$)

(3) The number which satisfies the inequality : $x - 2 > 3$ is .... (-1 or -2 or 6 or 4)

(4) $(-1)^8 + (-1)^9 =$ .... (zero or -1 or 1 or 2)

(5) $|5 - 11|$ .... $\mathbb{Z}$ ($\notin$ or $\in$ or $\subset$ or $\not\subset$)

(6) $2^5 \times 2^2 =$ .... (2^7 or 2^4 or 2^3 or 1)

(7) When tossing a die once the probability of getting a number on the upper face more than 6 is .... ($\varnothing$ or zero or 1 or 2)

(8) $|-3| =$ .... (3 or -3 or $-|3|$ or $3-3$)

(9) The total area of a cube = area of one face × .... (4 or 5 or 6 or 8)

(10) The probability of the impossible event = ... ($\varnothing$ or zero or 1 or 2)

(11) The image of the point $(2, 3)$ by translation $(x+1, y+2)$ is .... ($(3, 4)$ or $(3, 5)$ or $(4, 3)$ or $(5, 3)$)

(12) If $x + 6 = 2$, $x \in \mathbb{Z}$, then $x =$ .... (4 or $|-4|$ or -4 or $|4|$)

2 Complete :

(1) $3 + |-3| =$ ....

(2) The perimeter of the base of a cuboid is 10 cm. , its height is 4 cm. , then its lateral area = ...

(3) The probability of the sure event = ...

(4) The sum of the measures of the angles of the sectors about the centre of circle = ... °

(5) The circumference of the circle = ... $\times \pi$

(6) A cube of total area 150 cm^2 , then the length of its edge is .... cm.

(7) $\mathbb{Z}^+ \cup \{0\} =$ ....

(8) If $3x = 9$, then $x =$ ....

3 Answer the following :

(1) Find the result of : $(4 \times 3^2 + 3^2 - 7 \times 3)$

....

....

....

(2) In the coordinate plane locate the points

A $(2, 3)$, B $(4, 3)$, C $(4, 7)$, then find :

[a] The length of $\overline{BC}$ = ... units.

[b] The image of Δ ABC by translation $(0, -4)$

....

....

....

(3) Find the solution set of the inequality : $x - 2 \geq 3$ where $x \in \mathbb{Z}$, then represent it on the number line.

....

....

....

(4) A cuboid shaped box with a square base its length side is 10 cm. and its height is 4 cm. , calculate the lateral area.

....

....

(5) The following table shows the percentage of the production of a factory of house electric sets , represent it by circular sectors :

The kind of set	Washing machine	Heater	Oven	Mixer
The percentage	30 %	15 %	40 %	15 %

....

....

....

....

13 Damietta Governorate

Answer the following questions :

1 Choose the correct answer :

(1) $\mathbb{Z} \cap \mathbb{N} =$ .... ($\mathbb{Z}$ or $\mathbb{Z}^+$ or $\{0\}$ or $\mathbb{N}$)

(2) The equation : $x^3 + 4 = 5$ is of the .... degree. (first or second or third or fourth)

(3) A circle , its radius length is 4 cm. , then its area = ... π cm^2 (4 or 8 or 12 or 16)

(4) The image of the point $(-3 , 5)$ by translation $(x + 1 , y - 2)$ is .... ($(-4 , 3)$ or $(-2 , 3)$ or $(-2 , -3)$ or $(2 , 3)$)

(5) If a fair die is tossed once , then the probability of getting an odd number = ... (0 or 1 or $\frac{1}{3}$ or $\frac{1}{2}$)

(6) $|-4| - |4| =$ .... (zero or 1 or 8 or -8)

(7) All the following numbers satisfy the inequality : $x > -3$ except .... (zero or -4 or -1 or 2)

(8) The sum of edge lengths of a cube is 96 cm, then its lateral area = ... cm^2 (8 or 64 or 256 or 384)

(9) A circular sector represents $\frac{1}{3}$ of a circle, then the measure of its central angle = ...° (90 or 120 or 180 or 270)

(10) If $3x = -9$, then $x \in$ .... ($\mathbb{N}$ or $\mathbb{Z}^+$ or $\varnothing$ or $\mathbb{Z}^-$)

(11) $(-1)^8 + (-1)^9 + (-1)^{zero} =$ .... (zero or -1 or 1 or 2)

(12) The solution set of the inequality : $2 \le x < 3$ where $x \in \mathbb{N}$ is ....

({zero} or {2} or {3} or {2, 3})

2 Complete each of the following :

(13) $\frac{(-2)^7 \times (-2)^5}{2^{10}} =$ ....

(14) If $x - 3 = |-7|$, then $x =$ ....

(15) If X (−3, 2), Y (−3, −4), then the length of $\overline{XY}$ = ... units.

(16) The height of a cuboid whose lateral area is 160 cm^2 and dimensions of its base are 7 cm. and 3 cm. = ... cm.

(17) A box contains 5 white balls, 3 blue balls and 8 red balls, all of them are symmetric, one ball is drawn from the box at random, then the probability that the drawn ball is red = ...

(18) The multiplicative identity element in $\mathbb{Z}$ is ....

(19) The image of the point (−1, 2) by translation of magnitude of 3 units in the positive direction of y-axis is ....

(20) The surface area of the circle = ...

3 Answer the following :

(21) Find the solution set of the inequality : $3x - 2 \ge 4$, where $x \in \mathbb{Z}$

....

....

....

(22) Use the properties of addition in $\mathbb{Z}$ to find :

115 + 390 + (− 115) (write the used property)

....

....

....

(23) A cube of edge length 12 cm. Find the total area.

....

....

(24) A circle , its diameter length is 14 cm. Calculate its area where $\left(\pi = \frac{22}{7}\right)$

....

....

(25) The following table shows the rate of the score of 200 students in one school of Cairo governorate :

Rate	Excellent	Good	Pass	Weak
Percentage	15 %	50 %	25 %	10 %

Represent these data by circular sectors.

....

....

....

....

14 Kafr El-Sheikh Governorate

Answer the following questions :

1 Choose the correct answer :

(1) If $X - 2 = 3$, then $X =$ .... (-5 or -1 or 1 or 5)

(2) The lateral area of a cuboid of length 3 cm. , width 2 cm. and height 4 cm. = ... cm^2 (20 or 24 or 40 or 52)

(3) If $a < b$, then $-3a$ .... $-3b$ ($<$ or $>$ or $=$ or $\leq$)

(4) $3 - |-3| =$ .... (0 or 1 or 3 or 6)

(5) The image of the point A (3 , 4) by translation (1 , – 1) is ....
((3 , 3) or (2 , 3) or (4 , 3) or (4 , 5))

(6) $\mathbb{Z}^+ \cap \mathbb{Z}^- =$ .... ($\varnothing$ or $\mathbb{Z}$ or π or $\{0\}$)

(7) $(-1)^{104} + (-1)^{103} =$ .... (zero or -1 or 1 or 2)

(8) A cube of edge length 6 cm. , then its total area = ... cm^2
(36 or 72 or 144 or 216)

(9) If a die is thrown once , then the probability of appearance of the number 5 = ... ($\frac{5}{6}$ or $\frac{1}{6}$ or 0.5 or 1)

(10) The area of the circle = ... × π (r or 2 r or r^2 or r + 2)

(11) The measure of the central angle which represents $\frac{1}{8}$ of the circle = ...
(90° or 36° or 45° or 40°)

(12) If S is a sample space of a random experiment , then P (S) = ...
(0 or 2 or 1 or 0.8)

2 **Complete the following :**

(13) If $x + 5 = 3$, $x \in \mathbb{Z}$, then x = ...

(14) The perimeter of the base of the cuboid is 10 cm. , its height is 4 cm.
, then its lateral area = ... cm^2

(15) The equation : $x^2 - 3 = 6$ is of the .... degree.

(16) $3^2 + 2^3$ = ...

(17) If the perimeter of base of a cube is 20 cm. , then its total area is .... cm^2

(18) A circle of radius length 7 cm. , then its area = ... cm^2

(19) If X (– 3 , 2) , Y (– 3 , 4) , then the length of $\overline{XY}$ = ... length units.

(20) The probability of the impossible event is ....

3 **Answer the following :**

(21) Find the solution set of the inequality : $2x + 1 < 5$, where $x \in \mathbb{N}$

....

(22) Find the result of : $\frac{2^3 \times (-2)^4}{2^5}$

....

(23) If the sum of edge lengths of a cube = 36 cm. Find :

[a] Its lateral area. [b] Its total area

....

(24) A circle of radius length 7 cm. is divided into 8 equal circular sectors.

Find the area of each circular sector. $\left(\pi = \frac{22}{7}\right)$

....

(25) The following table shows the percentage of the number of students who participated in a school activities represent the data by a pie chart :

The activity	Music	Sport	Art
The percentage	25 %	40 %	35 %

15 El-Fayoum Governorate

Answer the following questions :

1 Choose the correct answer from those between brackets :

(1) $\mathbb{N} \cup \mathbb{Z}^- = \dots\dots$ ($\mathbb{Z}^+$ or $\mathbb{Z}^-$ or $\mathbb{Z}$ or $\mathbb{N}$)

(2) All the following numbers satisfy the inequality $x > -3$ except (0 or -2 or -1 or -4)

(3) $(-1)^{11} + (-1)^{10} = \dots\dots$ (zero or -1 or 1 or 2)

(4) If $\frac{x-1}{2} = 3$, $x \in \mathbb{Z}$, then $x = \dots\dots$ (5 or 7 or -7 or 6)

(5) $|-7| + 3 \dots\dots |-7 + 3|$ ($>$ or $=$ or $<$ or $\leq$)

(6) The additive inverse of $(-3)^0$ is $\dots\dots$ (3 or -3 or 1 or -1)

(7) If $x = 4$, $y = -3$, then the negative number of the following is $\dots\dots$ ($x + y$ or $x - y$ or xy or y^x)

(8) The image of the point $(4 , -3)$ by translation $(x - 3 , y + 3)$ is $\dots\dots$ ($(-7 , -6)$ or $(1 , 0)$ or $(0 , 1)$ or $(7 , 6)$)

(9) The probability of appearing a head when tossing a coin once = $\dots\dots$ (zero or 2 or 1 or $\frac{1}{2}$)

(10) If the probability of success of a student in mathematics is 75 % , then the probability of his failure = $\dots\dots$ (25 or 0.35 or 1 or $\frac{1}{4}$)

(11) The ratio between the lateral surface area and the total surface area of a cube = $\dots\dots$ (2 : 3 or 3 : 4 or 6 : 4 or 1 : 2)

(12) The total surface area of a cuboid = 100 cm^2 and area of one base 20 cm^2 , then its lateral surface area = $\dots\dots$ cm^2 (40 or 60 or 80 or 140)

2 Complete each of the following :

(13) The degree of the equation : $x^3 + 3x^2 + x + 4 = 11$ is $\dots\dots$ degree.

(14) The solution set of the inequality : $x \leq 0$ in $\mathbb{N}$ = ...

(15) The solution set of the equation : $x + 6 = 5$ in $\mathbb{N}$ = ...

(16) If the perimeter of one face of a cube is 20 cm. ,
then its total surface area = ... cm^2

(17) In the coordinates plane if the point A (– 2 , 4) and the point B (5 , 4)
, then length of $\overline{AB}$ = ... units.

(18) A cuboid its lateral area is 120 cm^2 and the length is 8 cm. , width is 4 cm.
, then its height = ... cm.

(19) $\dfrac{\text{Circumference of the circle}}{2\pi}$ = ...

(20) .... ≤ the probability of any event ≤

3 Answer the following :

(21) Find the result of : $\dfrac{(-5)^5 \times (-5)^4}{(-5)^7}$

....

(22) Find the solution set of the following equation in $\mathbb{Z}$: $3(x + 2) = 3$

....

(23) Calculate the area of a circle with radius length 10 cm ($\pi = 3.14$)

....

(24) A box in the shape of a cuboid , its length is 10 cm. , its width is 5 cm. and its height is 8 cm. , find its lateral surface area and its total surface area.

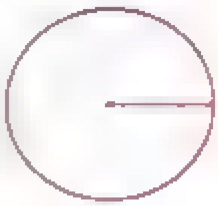
....

(25) The following table shows the percentage of the favorite sports in a school :

Type of the sport	Football	Basketball	Handball
Percentage of students number	40 %	35 %	25 %

Represent these data by circular sectors.

....



16 El-Menia Governorate

Answer the following questions :

1 Choose the correct answer :

(1) If $x - 2 = 3$, then $x = \ldots$ (-5 or -1 or 1 or 5)

(2) A cube of edge length 6 cm. , then its total area = ... cm^2.

(36 or 72 or 144 or 216)

(3) When tossing a die once, then probability of getting a number divisible by 5 equals .... (0 or $\frac{1}{6}$ or $\frac{5}{6}$ or 1)

(4) The equation : $x^2 + 3 = 4$ is of the .... degree.

(first or second or third or fourth)

(5) The smallest natural number is .... (0 or 1 or 2 or 3)

(6) The number which satisfies the inequality : $x > -2$ is ....

(-1 or -4 or -3 or -2)

(7) A circle, its radius length is 4 cm. , then its area = ... π cm^2

(8 or 16 or 64 or $2r$)

(8) The additive identity in $\mathbb{N}$ = ... (zero or 1 or -1 or 2)

(9) The total area of a cube is 324 cm^2 , then the area of face = ...

(54 cm^2 or 81 cm^2 or 54 cm. or 81 cm.)

(10) $(-1)^{104} + (-1)^{103} = \ldots$ (zero or -1 or 1 or 2)

(11) The probability of occurrence of the impossible event = ...

($\varnothing$ or zero or 1 or $\frac{1}{2}$)

(12) If $-3x < 30$, then x .... (-10) ($>$ or $<$ or $=$ or $\leq$)

2 Complete each of the following :

(1) Measure of angle of the circular sector in which its area represents $\frac{1}{8}$ from the area of the circle = ...

(2) If X $(-3 , 2)$, Y $(-3 , 4)$, then length of $\overline{XY}$ = ... length units.

(3) $\mathbb{Z}^{+} - \mathbb{Z}^{-} = \ldots$

(4) The lateral area of a cuboid of length 3 cm. , width 2 cm. and height 4 cm. = ... cm^2.

(5) The sum of the measures of all accumulative angles at the center of a circle equals ....

(6) The image of the point (2 , – 1) by translation (x – 1 , y + 3) is the point (.......... , ...)

(7) If $x + 3 = |-7|$, then x = ...

(8) If $x = |-12|$, $y = -3$, then $x + y$ = ...

3 **Answer the following :**

(1) Find the solution set of the inequality : $3x - 5 \leq 7$ where $x \in \mathbb{Z}^+$, then represent the solution set on the number line.

....

....

....

(2) A cuboid , its length is 6 cm. , its width is 4 cm. and its height is 8 cm. Find :

[a] Its lateral area. [b] Its total area.

....

....

....

(3) Find the result of : $\frac{2^3 \times 2^6}{2^4}$

....

....

(4) A box contains 8 white balls, 7 red balls, all balls are identical, if one ball is drawn randomly, find the probability that this ball is :

[a] Red = ... [b] White = ...

[c] Blue = ... [d] Red or white = ...

(5) The following table shows the percentage of eggs production in three farms during one month :

The farm	First	Second	Third
The percentage of production	25 %	50 %	25 %

Represent these data by circular sectors.

....

....

....

17 Souhag Governorate

Answer the following questions :

1 Choose the correct answer :

(1) .... is the smallest positive integer (– 1 or 0 or 1 or – 10)

(2) $\mathbb{Z}^+ \cap \mathbb{Z}^- =$ .... ($\{0\}$ or $\varnothing$ or $\mathbb{Z}$ or zero)

(3) The probability of getting on the upper face of a die a number which is more than 6 when tossing it once is .... ($\varnothing$ or zero or $\frac{1}{6}$ or $\frac{1}{3}$)

(4) The surface area of the circle whose diameter length is 20 cm.

= .... cm.² (π = 3.14) (314 or 0.314 or 3.14 or 62.8)

(5) $(-1)^8 + (-1)^9 =$ .... (zero or – 1 or 1 or 2)

(6) The probability of the impossible event = ... (0 or 1 or 2 or 3)

(7) A circle, its circumference is 88 cm., then its radius length = ... cm. ($\pi = \frac{22}{7}$)

(28 or 24 or 44 or 14)

(8) The equation : $4x^3 - x = 29$ is of .... degree.

(fourth or third or second or first)

(9) The smallest non-negative integer is .... (1 or 0 or – 1 or 2)

(10) A circle, its radius length is 7 cm., then its area = ... cm.² $\left(\pi = \frac{22}{7}\right)$

(145 or 154 or 22 or 7)

(11) The image of the point (– 4 , 3) by translation (– 1 , – 4) is ....

((– 5 , – 7) or (– 5 , – 1) or (– 7 , 3) or (– 3 , – 1))

(12) $|-9| + 3$ .... $\mathbb{Z}$ ($\in$ or $\notin$ or $\subset$ or $\not\subset$)

2 Complete each of the following :

(1) The lateral surface area of a cuboid of length 3 cm., width 2 cm. and height 4 cm. = ... cm.²

(2) $\dfrac{(-2)^7 \times (-2)^5}{2^{10}} =$ ....

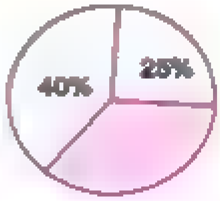
(3) $\mathbb{Z} =$ .... $\cup$ .... $\cup$ ....

(4) If the perimeter of base of a cube is 20 cm., then its lateral area = ... cm²

(5) If A (2 , 4) , B (2 , – 1), then the length of $\overline{AB}$ is .... units.

(6) In the opposite figure :

The percentage of the shaded circular sector = ………… %



(7) The sum of the measures of the accumulative angles at the centre of the circle = …………

(8) The image of the point (2 , 4) by translation ($x - 1$, $y + 1$) is …………

3 **Answer the following :**

(1) Find the solution set of the equation $2x - 3 = -9$, where $x \in \mathbb{Z}$

……………………………………………………

(2) A cuboid box with a square base of side length 6 cm. and its height is 10 cm. Calculate its lateral surface area and its total surface area.

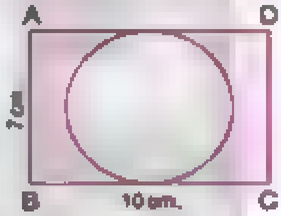
……………………………………………………

(3) Find the solution set of the inequality : $3x - 2 \geq 4$, where $x \in \mathbb{Z}$

……………………………………………………

(4) In the opposite figure :

ABCD is a rectangle where its length = 10 cm. and its width = 7 cm. , calculate the area of the shaded part. ($\pi = \frac{22}{7}$)



……………………………………………………

(5) The following table shows the rate of the score of 200 students in one school of Cairo governorate :

Rate	Excellent	Good	Pass	Weak
Percentage	15 %	50 %	25 %	10 %

Represent these data by a pie chart.

……………………………………………………

18 Qena Governorate

Answer the following questions :

1 Complete :

(1) If the lateral area of a cube is 36 cm^2 , then its total area = ... cm^2

(2) $(-1)^8 + (-1)^9 =$ ....

(3) The distance between the location of a number and the location of zero on the number line is called ....

(4) The additive inverse of zero is ....

(5) The image of the point (3 , 5) by translation $(x + 2 , y - 1)$ is ....

(6) The probability of the impossible event = ...

(7) If A (– 2 , 1) , B (3 , 1) , then AB = ... units

(8) A cube of edge length 6 cm. , then its lateral area = ... cm^2

2 Choose the correct answer :

(1) If S is a sample space of a random experiment , then P (S) = ...
(zero or 2 or 1 or 0.8)

(2) $-|-54| =$ .... (– 54 or 54 or 9 or 1)

(3) The greatest negative integer is .... (0 or 1 or – 1 or – 2)

(4) $-4 >$ .... (4 or – 3 or – 5 or 0)

(5) Type of central angle of a circle is straight angle , then it represents .... from surface area of the circle.
(quarter or half or third or whole one)

(6) 4^2 .... 8 (> or < or = or otherwise)

(7) When tossing a die once , then probability of getting a number 5 = ...
(zero or $\frac{1}{6}$ or $\frac{5}{6}$ or 1)

(8) If the perimeter of base of a cube is 24 cm. , then its total area = ... cm^2
(144 or 36 or 54 or 216)

(9) The equation $x^3 - x = 29$ is of the .... degree.
(first or second or third or fourth)

(10) If $2x = -6$, then $x \in$ .... ($\mathbb{N}$ or $\varnothing$ or $\mathbb{Z}^+$ or $\mathbb{Z}^-$)

(11) $[5 + (-3)] \times (-11) =$ .... (22 or – 22 or 88 or – 88)

(12) $\mathbb{Z}^+$ .... $\mathbb{N}$ ($\in$ or $\notin$ or $\subset$ or $\not\subset$)

3 Answer the following :

(1) A circle , its circumference is 44 cm. Calculate its surface area. ($\pi = \frac{22}{7}$ or 3.14)

(2) A cuboid , its length is 6 cm. , its width is 4 cm. and its height is 8 cm. Find its lateral area and its total area.

(3) Find the result of : $\frac{(-3)^3 \times (-3)^4}{(-3)^5}$

(4) Find the solution set of the inequality : $3X - 2 \geq 4$ where $X \in \mathbb{Z}$, then represent it on the number line

(5) The following table shows the percentage of the production of a factory of house electrical sets :

Marks	Washing machine	Heater	Oven	Mixer
Percentage	30 %	15 %	40 %	15 %

Represent these data by circular sectors.

19 Aswan Governorate

Answer the following questions :

1 Choose the correct answer from those given :

(1) The greatest negative integer is .... (0 or 1 or −1 or 2)

(2) The total area of cube = ... × area of one face (6 or 2 or 4 or 3)

(3) $|-6| + |6| =$ .... (12 or −12 or 1 or 0)

(4) The image of the point (...... , ...) by translation $(x-3, y+4)$ is $(-5, -3)$ ((−8 , 15) or (−2 , −7) or (−8 , 7) or (−2 , 7))

(5) $(-8) \times 1 =$ .... (−7 or −9 or 8 or −8)

(6) The probability of the impossible event = ... (0 or 1 or −1 or $\frac{1}{2}$)

(7) The solution set of the equation : $x + 2 = 7$, where $x \in \mathbb{Z}$ is .... (−5 or 9 or 5 or −9)

(8) $(-36) \div (-4) =$ .... (−9 or 9 or −6 or 4)

(9) $7 - |-3| =$ .... (21 or −10 or 10 or 4)

(10) The previous integer of (− 9) is .... (− 10 or 8 or −8 or 10)

(11) If $\varnothing$ is the empty set then $P(\varnothing) =$ .... (zero or $\frac{1}{2}$ or 1 or 2)

(12) The image of the point (1 , − 3) by translation (...... , ...) is (1 , 0) ((1 , 0) or (0 , 0) or (3 , 0) or (0 , 3))

2 Complete the following :

(1) If $x + 6 = 2$, $x \in \mathbb{Z}$, then $x =$ ....

(2) $(-3)^0 =$ ....

(3) The lateral area of a cube its edge length 5 cm. equals ....

(4) The image of the point (3 , 5) by translation $(x + 2, y - 1)$ is ....

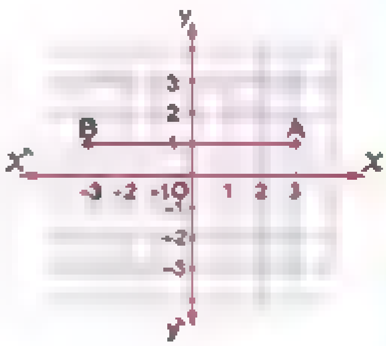
(5) The total area of the cuboid = ... + the sum of the areas of the two bases

(6) When tossing a die once, the probability of getting a number divisible by 3 equals ....

(7) $\mathbb{Z} - \mathbb{N} =$ ....

(8) In the opposite coordinate plane :

AB = ... units.



3 **Answer the following :**

(1) Use the properties of addition operation in $\mathbb{Z}$ to find the result of the following : $37 + 25 + 63 + 75$

....

....

(2) A circle , its circumference 88 cm. Calculate its surface area. ($\pi = \frac{22}{7}$)

....

....

(3) Find the solution set of the inequality : $x - 2 \geq 3$, $x \in \mathbb{Z}$, then represent it on the number line.

....

....

....

(4) A cuboid shaped box with a square base its side length is 9 cm. and the height is 20 cm. Calculate the lateral area and total area.

....

....

....

(5) The following table shows the percentages of the production of house electrical sets :

The kind of set	Washing machine	Heater	Oven	Mixer
The percentage	30 %	15 %	40 %	15 %

Represent these data by circular sectors.

....

....

....

20 South Sinai Governorate

Answer the following questions :

1 **Choose the correct answer :**

(1) 3 .... − 6 (> or < or = or ≤)

(2) If $2x = -6$, then $x \in$ .... ($\mathbb{N}$ or $\mathbb{Z}^+$ or $\mathbb{Z}^-$ or $\{-4\}$)

(3) The image of the point (3 , 5) by translation ($x + 2$, $y - 1$) is ....
((5 , 6) or (5 , 4) or (1 , 4) or (1 , 6))

(4) When tossing a die once, then the probability of getting a number 5 = ... (zero or $\frac{1}{6}$ or $\frac{5}{6}$ or 1)

(5) $|-65|$ .... $\mathbb{Z}^-$ ($\in$ or $\notin$ or $\subset$ or $\not\subset$)

(6) The number which satisfies the inequality : $x > -2$ is ....
(-1 or -2 or -3 or -4)

(7) The circumference of the circle = ... $\times \pi$
(r or $2r$ or r^2 or $r + 2$)

(8) $\mathbb{Z}^+ \cap \mathbb{Z}^- =$ .... ($\mathbb{Z}$ or $\mathbb{N}$ or $\varnothing$ or $\{0\}$)

(9) If x is less than -5, then the symbolic expression is ....
($x > -5$ or $x < -5$ or $x \geq 5$ or $x \leq -5$)

(10) The number of faces of the cube = ... faces.
(6 or 8 or 12 or 4)

(11) The sum of the measures of the accumulative angles at the centre of the circle = ... ($180°$ or $360°$ or $270°$ or $90°$)

(12) If $x - 2 = 1$, then $x =$ .... (1 or -1 or 3 or 2)

2 Complete :

(1) A cube of edge length 6 cm., then its total area = ... cm^2

(2) If the base area of a cube = 49 cm^2, then its lateral area = ...

(3) If $x + 5 > 2$, then $x >$ ....

(4) The probability of the impossible event = ...

(5) The image of the point A (1 , 4) by translation ($x - 2$, $y + 1$) is the point $\acute{A}$ (.......... , ...)

(6) The equation : $3x^2 - 6 = 14$ is of the .... degree.

(7) If the perimeter of the base of a cuboid is 10 cm. and its height is 4 cm., then its lateral area = ... cm^2

(8) If X (-3 , 2), Y (-3 , -4), then the length of $\overline{XY}$ = ... length units.

3 Answer the following :

(1) Find the result of : $\frac{2^6 \times 2^5}{2^3 \times 2}$

....

(2) Find the solution set of the equation : $2x + 9 = 3$, where $x \in \mathbb{Z}$

(3) A circle , its diameter length is 14 cm. , calculate its surface area.

(where $\pi = \frac{22}{7}$)

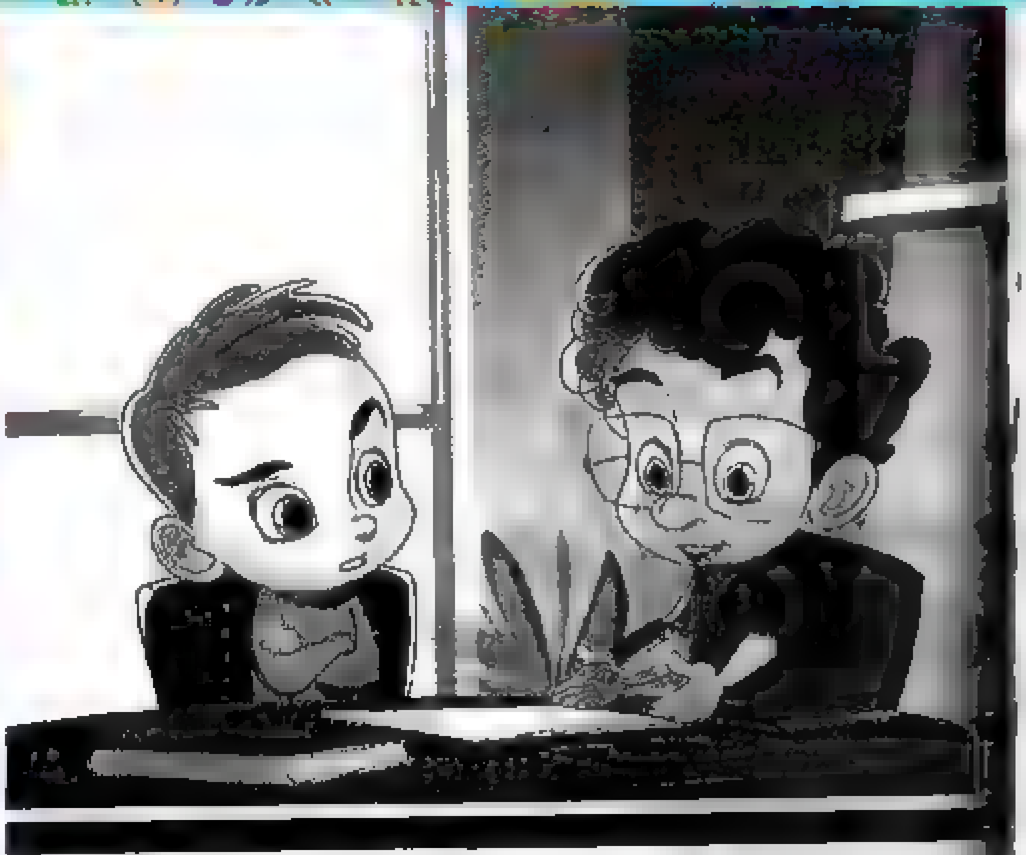
The surface area = ...

(4) In a Cartesian coordinate plane , locate the points A (2 , 3) , B (4 , 3) , C (4 , 7) and join them , then find the length of $\overline{BC}$

(5) The following table shows the percentage of production in three farms :

The farm	The first	The second	The third
The percentage of the production	25 %	35 %	40 %

Represent these data by using the circular sectors.



Guide Answers of the Main Book

Unit One

Exercise 1

1 [a] , [c] , [d] , [e] , [f] and [i] are integers

2 [a] ⊄ [b] ⊂ [c] ∉ [d] ⊄
[e] ⊂ [f] ⊄ [g] ∉ [h] ∉
[i] ⊂ [j] ⊄ [k] ∉ [l] ∈

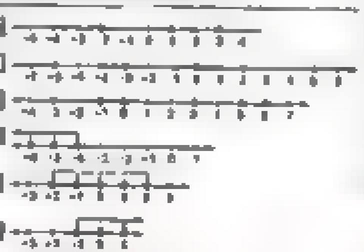
3 [a] 5 [b] 5 , −5 [c] 5 [d] 5
[e] ⊄ [f] Z [g] Z [h] ∅
[i] ⊄ [j] Z^- [k] N [l] N
[m] {0} [n] N [o] N [p] {0}
[q] ∅ [r] {0} [s] ∅ [t] Z
[u] N [v] {0}

4 [a] ⊂ [b] ∉ [c] ∈ [d] ∈
[e] ∈ [f] ∈ [g] 12 [h] 5
[i] 4 [j] 7 [k] ∅ [l] N
[m] Z^- [n] N [o] Z [p] Z
[q] N [r] Z [s] Z^+ [t] {−3}

5 [a] −12 [b] 10 [c] 5 [d] −6000
[e] 4 [f] 3 [g] 80 [h] 750
[i] −20 [j] 17 [k] 100 [l] −8
[m] −200

6 [a] positive , negative [b] positive , negative
[c] negative , positive [d] zero

7 [a]
[b]
[c]
[d]
[e]
[f]



8 [a] 3 [b] −12 [c] 0
[d] 195 [e] −34 [f] 8

9 [a]
[b]

[c]
[d]

10 [a] 7 [b] 2 [c] 0
[d] 10 [e] 21 [f] 100

11 [a] 3 + 2 = 5 [b] 5 − 4 = 1
[c] 2 + 13 = 15 [d] 100 − 50 = 50
[e] 5 + 7 = 12 [f] 12 − 12 = 0
[g] 0 + 5 = 5 [h] 0 + 7 = 7
[i] 2 + 2 = 4 [j] 5 − 5 = 0

12 [a] 3 × 5 = 15 [b] 10 × 2 = 20
[c] 30 ÷ 5 = 6 [d] 4 × 7 = 28
[e] 0 × 3 = 0 [f] 8 × 11 = 88

13 [a] x = 5 or x = −5 [b] x = 12 or x = −12
[c] x = 0 [d] x = 4
[e] x = 8 [f] x = 101

14 [a] ✗ , because zero is neither positive nor negative.
[b] ✗ , because Z^- is a subset of Z , $Z^- \cap Z = Z^-$
[c] ✗ , because $Z^+ \cup N = N$
[d] ✗ , because {−17} is a set and {−17} ⊂ Z
[e] ✓ , because the set of counting numbers = N − {0} = Z^+
[f] ✗ , because zero is neither positive nor negative.

15 [a] −4 [b] −5 [c] −2
[d] 5 or −3 [e] −2
[f] any integer number except 5 , −5 and 3

Exercise 2

1 [a] > [b] > [c] > [d] <
[e] > [f] < [g] < [h] >
[i] > [j] > [k] = [l] >
[m] < [n] < [o] < [p] <
[q] = [r] >

2 [a] 0 [b] 1 , −1
[c] 0 [d] 0
[e] {−2 , −1 , 0 , 1} [f] −3 , −2 , −1 , 0

3 [a] 1 [b] > [c] < [d] −1
[e] − 6 [f] − 6 [g] 24 [h] 3

4 [a] The order is −5 , −1 , 1 and 3
[b] The order is −22 , −11 , 0 , 7 and 11
[c] The order is −9 , −7 , −4 and −1
[d] The order is −7 , −6 , −3 , 2 , 5 and 10
[e] The order is −62 , −42 , −19 , 0 and 8
[f] The order is −15 , −9 , −9 , 16 and 17
[g] The order is −16 , −6 , −8 , 12 and 19
[h] The order is −60 , −22 , −17 , 0 , 2 and 6

5 [a] The order is 7 , 0 , −8 and −15
[b] The order is 8 , 0 , −13 , −16 and −19
[c] The order is 33 , 2 , −28 , −35 and −37
[d] The order is 6 , 3 , 1 , −1 , −5 and −11

6 [c] **7** [e]

8 [a] −10 and −8 [b] 12 and 14
[c] 22 and 24 [d] −1 and 1

9 [a] −3 , −2 , −1 , 0 and 1
[b] 0 , 1 , 2 , 3 and 4
[c] −6 , −5 , −4 , −3 , −2 and −1

10 [a] { −1 , 0 , 1 , 2 , ... }
[b] { −1 , −2 , −3 , −4 , ... }
[c] { −2 , −1 , 0 , 1 }
[d] { −3 , −2 , −1 , 0 , 1 , 2 }
[e] { −5 , −6 , −7 , −8 , ... }
[f] { 0 , 1 , 2 , 3 , ... }
[g] { 0 , −2 , −4 , −6 , ... }
[h] { −2 , −3 , −4 , −5 , ... }
[i] { 0 }

11 [a] −2 , −1 , 0 [b] 0 , −1 , −2
[c] −4 , −3 , −2 [d] −8 , −10 , −12
[e] 6 , 8 , 10 [f] −10 , −5 , 0
[g] −20 , −10 , 0 [h] 3 , 1 , −1

12 [a] P [b] D [c] G
[d] J [e] N [f] J

13 [a] C [b] E [c] G
[d] F [e] D [f] B

14 [a] The order is 16 , 10 , 6 , 5 , 3 and −5
[b] The order is −15 , −5 , −4 , −2 , 3 and 8

15 [a] The both are on the same distance from sea level.
[b] The helicopter.

Exercise 3

1 [a] 
2 + 4 = 6
[b] 
−6 + 4 = −2
[c] 
−3 + (−4) = −7
[d] 
6 + (−7) = −1

2 [a]  5 + 2 = 7
[b]  4 + (−3) = 1
[c] 
−4 + (−2) = −6
[d] 
−7 + 4 = −3
[e] 
8 − 4 = 4
[f] 
5 − (−3) = 5 + 3 = 8
[g] 
−3 − 3 = −3 + (−3) = −6
[h] 
−4 + 4 = 0
[i] 
−10 + 2 = −8

3 [a] 6 [b] −3 [c] 4 [d] 1
[e] −5 [f] 0 [g] −6 [h] −14
[i] −20

4 [a] 2 [b] −5 [c] −10 [d] 30
[e] 1 [f] −17 [g] −7 [h] 3
[i] −6 [j] 0 [k] 22 [l] 14

5 [a] 0 [b] 8 [c] 0 [d] 1
[e] −6 [f] 5 [g] −4 [h] 1

6 [a] Commutative [b] Additive inverse
[c] Additive identity [d] Associative
[e] Additive inverse

7 [a] 4 [b] −5 [c] 7 [d] 0
[e] 0 [f] 6 [g] 5 [h] −7
[i] −5 [j] −8 [k] −5 [l] 4
[m] −6 [n] 0 [o] 0 [p] −9
[q] 1 [r] a [s] −3 [t] 0
[u] additive inverse of b

8 [a] −6 [b] 0 [c] −6 [d] 12
[e] 3 [f] −3 [g] 0 [h] −5

9 [a] −5 + 5 + (−6) "Commutative property"
= (−5 + 5) + (−6) "Associative property"
= 0 + (−6) "Additive inverse"
= −6 "Additive identity"
[b] 10 + [(−5) + (−2)] "Associative property"
= 10 + (−7) = 3
[c] −7 + (−13) + 2 "Commutative property"
= [−7 + (−13)] + 2 "Associative property"
= −20 + 2 = −18
[d] −17 + 17 + 19 "Commutative property"
= (−17 + 17) + 19 "Associative property"
= 0 + 19 "Additive inverse"
= 19 "Additive identity"
[e] [15 + (−3)] + 25 "Associative property"
= 12 + 25 = 37
[f] 5 + 7 + (−3) + (−9) "Commutative property"
= (5 + 7) + [(−3) + (−9)]
"Associative property"
= 12 + (−12) "Additive inverse"
= 0
[g] 25 + (−25) + (−8) + 7
"Commutative property"
= [25 + (−25)] + [(−8) + 7]
"Associative property"
= 0 + (−1) "Additive inverse"
= −1 "Additive identity"
[h] 55 + 45 + (−255) + 225
"Commutative property"
= (55 + 45) + [(−255) + 225]
"Associative property"
= 100 + (−30) = 70
[i] −74 + 74 + 65 + (−65) "Commutative property"
= [−74 + 74] + [65 + (−65)]
"Associative property"
= 0 + 0 "Additive inverse"
= 0 "Additive identity"
[j] −120 + 113 + 17 "Commutative property"
= −120 + (113 + 17) "Associative property"
= −120 + 130 = 10
[k] [2019 + (−1019)] + 180 "Associative property"
= 1000 + 180 = 1180
[l] 63 + 37 + 54 + 46 "Commutative property"
= (63 + 37) + (54 + 46) "Associative property"
= 100 + 100 = 200

10 [a] 10 + 6 = 16 [b] (−8) + (−1) = −9
[c] −3 + (−2) + 6 = −5 + 6 = 1
[d] 4 − 12 = −8
[e] −3 − 5 + 7 = −8 + 7 = −1
[f] −30 + 10 = −20 [g] −3 − 5 = −8
[h] −9 − 3 + 7 = −12 + 7 = −5
[i] 3 − (−3) + 1 = 6 + 1 = 7
[j] −14 + 3 = −11
[k] 2 − (−6) = 8
[l] −9 − (−3) = −6

11 [a] $45 + (-12) = 33$
[b] $-12 + (-12) + 24 = -24 + 24 = 0$
[c] $(-12) + (-21) = -33$
[d] $(-12) + |-8| + 5 = -4 + 5 = 1$

12 [a] $3 + (-4) = -1$ [b] $(-4) + (-2) = -6$
[c] $3 - (-4) = 7$
[d] $(-4) - (-2) = -2$
[e] $3 + (-4) + (-2) = 3 + (-6) = -3$
[f] $3 - (-4) + (-2) = 7 + (-2) = 5$
[g] $-(-2) + 3 - (-4) = 5 - (-4) = 9$
[h] $(3 + (-2)) - (-4) = 1 - (-4) = 5$

13 [a] $1 + 1 = 2 \notin X$
So X is not closed under addition.
[b] $1 + 2 = 3 \notin Y$
So Y is not closed under addition.

14 The increase in the temperature
$= 11 - (-3) = 11 + 3 = 14°C$

15 The temperature on Tuesday
$= -2 - 7 + 5 = -9 + 5 = -4°C$

16 The temperature of that layer
$= -1 + 5 = 4°C$

17 The new depth of the submarine $= 90 - 60$
$= 30$ metres deep below the sea level

18 The balance of Ramy in the bank
$= 6220 - 1211 + 2110 = 5009 + 2110$
$=$ L.E. 7119

Exercise 4

1 [a] positive. [b] positive. [c] negative. [d] 0

2 [a] positive. [b] negative [c] 0 [d] positive.

3

[a] 15	[b] -12	[c] 300
[d] 0	[e] -9	[f] -60
[g] 333	[h] -2400	[i] -12
[j] -20	[k] -30	[l] 0

4

[a] 4	[b] -8	[c] -7
[d] 9	[e] 0	[f] -7
[g] -4	[h] -9	[i] 6
[j] 3	[k] 9	[l] -7

5 [a] Multiplicative identity.
[b] Associative. [c] Commutative.
[d] Distribution.

6

[a] 4	[b] 1	[c] 4
[d] 0	[e] -9	[f] 24
[g] -3	[h] -8	[i] -9
[j] 9	[k] 5	[l] 2

7

[a] 0 , 1	[b] negative , positive	
[c] negative.	[d] 0	[e] 21
[f] 3	[g] -210	[h] -60
[i] -5	[j] 44	[k] 6
[l] -7	[m] -18	[n] -4
[o] $A \times B$	[p] 1	[q] 1
[r] a	[s] additive inverse	

8

[a] 16	[b] 2	[c] -12
[d] -6	[e] -15	[f] zero
[g] 0	[h] 3	[i] 3n
[j] <		

9 [a] $5 \times 2 \times 17$ "Commutative property"
$= (5 \times 2) \times 17$ "Associative property"
$= 10 \times 17 = 170$
[b] $50 \times 2 \times (-45)$ "Commutative property"
$= (50 \times 2) \times (-45)$ "Associative property"
$= 100 \times (-45) = -4500$
[c] $4 \times 3 \times (-5) \times (-2)$
"Commutative property"
$= [4 \times 3] \times [(-5) \times (-2)]$
"Associative property"
$= 12 \times 10 = 120$
[d] $(-2) \times 5 \times (-3) \times (-1)$
"Commutative property"
$= [(-2) \times 5] \times [(-3) \times (-1)]$
"Associative property"
$= -10 \times 3 = -30$

[e] $4 \times 25 \times (-16)$
"Commutative property"
$= [4 \times 25] \times (-16)$
"Associative property"
$= 100 \times (-16) = -1600$
[f] $8 \times (-125) \times 77$ "Commutative property"
$= [8 \times (-125)] \times 77$
"Associative property"
$= -1000 \times 77 = -77000$

10 [a] $3 \times [(-2) + 5] = 3 \times 3 = 9$
[b] $75 \times (37 + 63) = 75 \times 100 = 7500$
[c] $(-6) \times [(-5) + 2] = (-6) \times (-3) = 18$
[d] $69 \times (147 - 47) = 69 \times 100 = 6900$
[e] $112 \times [17 + (-17)] = 112 \times 0 = 0$
[f] $(-35) \times [(-42) + 52] = (-35) \times 10 = -350$
[g] $32 \times (18 - 34 + 17) = 32 \times 1 = 32$
[h] $(-16) \times [45 + (-47) + 1] = (-16) \times (-1) = 16$
[i] $(-3) \times (4 - 5 + 1) = (-3) \times 0 = 0$

11 [a] $26 \times (100 + 1) = 26 \times 100 + 26 \times 1$
$= 2600 + 26 = 2626$
[b] $64 \times (100 - 1) = 64 \times 100 - 64 \times 1$
$= 6400 - 64 = 6336$
[c] $32 \times (100 - 2) = 32 \times 100 - 32 \times 2$
$= 3200 - 64 = 3136$
[d] $72 \times (100 + 10 + 1) = 72 \times 100 + 72 \times 10$
$+ 72 \times 1 = 7200 + 720 + 72 = 7992$

12 [a] $(-5) \times 10 = -50$ [b] $12 \times (-4) = -48$
[c] $3 \times 6 = 18$ [d] $2 \times (-11) = -22$
[e] $6 \times (-6) = -36$ [f] $(-7) \times (-4) = 28$
[g] $0 \div (-4) = 0$ [h] $-2 \div 2 = -1$
[i] $45 \div 9 = 5$ [j] $(-50) \div (-5) = 10$

13 $3x - 2y + z = 3 \times 2 - 2 \times 1 + 5 = 9$

14 $x - 2y + 4 = 6 - 2 \times (-2) + 4 = 6 + 4 + 4 = 14$

15 $(2x + y) \times 3z = (2 \times 3 + (-1)) \times 3 \times (-2)$
$= -6 \times -6 = 36$

Exercise 5

1

[a] 3	[b] 6	[c] 5^4
[d] 7	[e] 10^{12}	[f] 4^3
[g] 5^2	[h] 7^0	[i] 12
[j] 1	[k] 2	[l] 0
[m] m n	[n] 0	[o] 16

2

[a] 8	[b] 9	[c] 125
[d] 10000	[e] 49	[f] -8
[g] -216	[h] -81	[i] 1
[j] 1	[k] 1	[l] -1

3 [a] $2^5 = 32$
[b] $10^3 \times 10^4 = 10^7 = 10000000$
[c] $-(5)^3 \times 5^2 = -(5)^5 = -3125$
[d] $-(2)^6 = -64$ [e] b^{10}
[f] $7^6 = 117\,649$

4 [a] $2^2 = 4$ [b] $3^1 = 3$ [c] $5^0 = 1$
[d] $3^3 = 27$ [e] $(-6)^2 = 36$
[f] $(-4)^3 = -64$
[g] $-(5)^5 \div 5^3 = -(5)^2 = -25$ [h] a^3

5 [a] $8 \times 9 = 72$ [b] $25 \times 4 = 100$
[c] $1 \times (-2) = -2$ [d] $(-64) \times (-1) = 64$
[e] $(-125) \times (-1) = 125$
[f] $(-1) \times 8 = -8$ [g] $-16 \times (-8) = 128$
[h] $8 + 4 = 12$ [i] $-8 + 9 = 1$
[j] $16 + (-27) = -11$ [k] $1 + (-1) = 0$
[l] $1 + 1 = 2$ [m] $16 + 27 - 16 = 27$
[n] $3^6 \times 2^5 = 729 \times 32 = 23328$
[o] $(-2)^2 \times 2^3 \times 2^3 = 2^2 \times 2^3 \times 2^3 = 2^8 = 256$

6 [a] $\frac{5^4}{5^4} = 1$ [b] $\frac{7^9}{7^7} = 7^2 = 49$
[c] $\frac{3^7}{3^5} = 3^2 = 9$ [d] $\frac{2^{11}}{2^4} = 2^7 = 128$
[e] $\frac{(-3)^7}{(-3)^5} = (-3)^2 = 9$
[f] $\frac{(-5)^8}{(-5)^5} = (-5)^3 = -125$
[g] $\frac{(-3)^9}{(-3)^7} = (-3)^2 = 9$

[h] $\frac{-(8)^7 \times 8^0}{-(8)^7} = \frac{-(8)^7}{-(8)^7} = 8^0 = 1$

[i] $\frac{3^2 \times (-(3)^5)}{3^4} = \frac{-(3)^7}{3^4} = -(3)^3 = -27$

[j] $\frac{(-2)^{12}}{2^{10}} = \frac{2^{12}}{2^{10}} = 2^2 = 4$

[k] $\frac{2^6 \times 2^4}{2^7 \times 2} = \frac{2^{10}}{2^8} = 2^2 = 4$

[l] $\frac{9^6 \times (-(9)^7)}{9^7 \times (-(9)^4)} = \frac{-(9)^{13}}{-(9)^{11}} = 9^2 = 81$

[m] $\frac{(-5)^{18}}{(-5)^{17}} = -5$

[n] $(-3)^3 + (-4)^2 = -27 + 16 = -11$

[o] $\frac{4+8}{16} = \frac{12}{16} = \frac{3}{4}$

[p] $2^3 + (-1) = 8 + (-1) = 7$

7 [a] $\frac{a^7}{a^3} = a^4$ [b] $\frac{a^{10}}{a^{10}} = a$

[c] $\frac{x^4}{x^4} = x^0 = 1$ [d] $\frac{x^{14}}{x^6} = x^8$

8 [a] $5^2 \times 3 = 25 \times 3 = 75$

[b] $(-2)^2 \times 3^4 = 4 \times 81 = 324$

[c] $4^2 \times (-3) = 16 \times (-3) = -48$

[d] $x^3 \times y^3$

9 [a] $(-2)^5 = -32$, $(-3)^4 = 81$, $(-4)^0 = 1$, $(-1)^{15} = -1$, $3^2 = 9$

The order is $(-2)^5$, $(-1)^{15}$, $(-4)^0$, 3^2 and $(-3)^4$

[b] $2^3 = 8$, $3^2 = 9$, $(-2)^3 = -8$, $(100)^0 = 1$, $(-1)^5 = -1$

The order is $(-2)^3$, $(-1)^5$, $(100)^0$, 2^3 and 3^2

10 [a] $(-2)^3 = -8$, $(-2)^2 = 4$, $(-2)^0 = 1$, $(-1)^5 = -1$

The order is : $(-2)^3$, $(-2)^0$, $(-1)^5$ and $(-2)^2$

[b] $10^2 = 100$, $(-1)^3 = -1$, $(1000)^{zero} = 1$

The order is 1 000 , 10^2 , $(1000)^{zero}$ and $(-1)^3$

[c] $10^2 = 100$, $(-1)^5 = -1$, $(100)^2 = 10000$, $(-10)^3 = -1000$

The order is 1 000 000 , 100^2 , 10^2 , $(-1)^5$ and $(-10)^3$

11 [a] > [b] > [c] =
[d] > [e] < [f] >
[g] < [h] = [i] <
[j] > [k] = [l] =
[m] < [n] <

12 [a] ∈ [b] ∉ [c] ∈
[d] −1 [e] 1 [f] 5
[g] 2 [h] 1 [i] 2
[j] zero [k] 2 [l] zero
[m] 6 [n] 18 [o] −18
[p] 3^2 [q] 30 [r] zero
[s] 6 [t] 2^4 [u] 2×2^2
[v] 5 [w] 120 + 80 [x] $x^2 + y$
[y] $F^2 + F$

13 [a] $2^2 = 4$ [b] $3 \times 2^7 = 3 \times 128 = 384$

[c] $2^4 + 5 = 16 + 5 = 21$

[d] $2^3 - 1 = 8 - 1 = 7$

[e] $\frac{2^6}{8} = \frac{64}{8} = 8$

[f] $2 \times 2^5 + 1 = 2 \times 32 + 1 = 64 + 1 = 65$

14 [a] $3a^2b = 3 \times 2^2 \times (-3) = 3 \times 4 \times (-3) = -36$

[b] $2a + 3b = 2 \times 2 + 3 \times (-3) = 4 - 9 = -5$

[c] $a^2 + b^2 + ab = 2^2 + (-3)^2 + 2 \times (-3)$
$= 4 + 9 - 6 = 7$

15 $(a-b)^{10} = (3^2 - 2^3)^{10} = (9 - 8)^{10} = (1)^{10} = 1$

16 [a] 17 (17 + 83) = 17 × 100 = 1700

[b] 23 (33 − 23) = 23 × 10 = 230

[c] 27 (27 − 17) = 27 × 10 = 270

[d] 23 (23 + 78 − 1) = 23 × 100 = 2300

Exercise 6

1 [a] 5 , 6 [b] 16 , 19 [c] − 2 , 0
[d] 7 , 10 [e] − 6 , − 3 [f] −10 , −12
[g] 32 , 64 [h] 81 , 243

2 [a] − 7 , 9 [b] 48 , − 96 [c] 0 , 2
[d] − 3 , 6 [e] 0 , − 4 [f] 81 , − 243

3 [a] 162 , − 486 [b] − 25 , − 30 [c] 3 , − 3
[d] − 14 , − 12 [e] 15 , 21 [f] 1 , $\frac{1}{2}$
[g] 13 , 21

4 [a] 46 , 54 , 62 [b] $\frac{1}{32}$, $\frac{1}{64}$, $\frac{1}{128}$
[c] 21 , 34 , 55 [d] 36 , 49 , 64

5 [a] 10 , 5 [b] 31 , 63
[c] 25 , 37 [d] $\frac{3}{2}$, 2

6 [a] 4 , 7 , 10 , 13 , 16 , 19 , 22
[b] 7 , 11 , 15 , 19 , 23 , 27 , 31
[c] 0.5 , 1 , 1.5 , 2 , 2.5 , 3 , 3.5
[d] 128 , 64 , 32 , 16 , 8 , 4 , 2
[e] 18 , 15 , 12 , 9 , 6 , 3

7

The numerical pattern	Description of the pattern
3 , 7 , 11 , 15 , 19 , 23 ,	Each number is more than its preceding by 4
1 , 6 , 11 , 16 , 21 , 26 , "There are other solutions"	Each number is more than its preceding by 5
$\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, 1 , $\frac{5}{4}$, …	Each number is more than its preceding by $\frac{1}{4}$
54 , 50 , 46 , 42 , 38 , … "There are other solutions"	Each number is less than its preceding by 4
3 , 9 , 27 , 81 ,	Each number is three times of its preceding.

8 [a] • Description of the pattern
Each number is more than its preceding by 5
• 22 , 27

[b] • Description of the pattern :
Each number is more than its preceding by 4
• 19 , 23

[c] • Description of the pattern :
Each number is more than its preceding by 3
• 18 , 21

[d] • Description of the pattern
Each number is less than its preceding by 5
• 61 , 56

[e] • Description of the pattern ,
Each number is three times of its preceding.
• 81 , 243

[f] • Description of the pattern
Each number equals half of its preceding
• 12 , 6

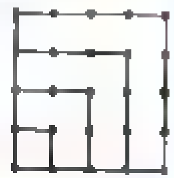
9 Number of line segments 4 , 8 , 12 , 16
The numerical pattern 4 , 8 , 12 , 16 ,
Description of the pattern Each number is more than its preceding by 4

10 Number of triangles 1 , 2 , 3 , 4
The numerical pattern : 1 , 2 , 3 , 4 , …
Description of the pattern Each number is more than its preceding by 1
Do by yourself.

11 Number of line segments 4 , 7 , 10 , 13
The numerical pattern 4 , 7 , 10 , 13 ,
The pattern rule Each number is more than its preceding by 3

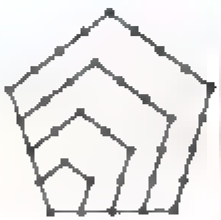
12 The number of dots 1 , 3 , 6 , 10 , 15
The numerical pattern 1 , 3 , 6 , 10 , 15 , ..
The rule of the pattern : Each number is half the product of the order of the number and the order of the next number.

13 [a] 25 dots.
[b]



14 (a)

(b) 70 dots.



15 0 , 1 , 3 , 6

The number of chords in the 6^{th} shape is 15

16 About 8 days.

The numerical pattern 6 , 12 , 18 , 24 ,

Description of the pattern Each number is more than its preceding by 6

17 About 3 months.

The numerical pattern 52 , 104 , 156 , 208 ,

Description of the pattern Each number is more than its preceding by 52

18 The number of months = 7 months.

The numerical pattern 90 , 87 , 84 , 81 ,

Description of the pattern Each number is less than its preceding by 3

Answers of unit test

1 (a) $\mathbb{Z}$ (b) 1 (c) <

(d) 2 (e) 12

2 (a) 30 (b) 17 (c) $\varnothing$

(d) 8 (e) 0

3 (a) The order is : − 16 , − 9 , 9 , $(-4)^2$ and 17

(b) (1) {−3 , −4 , −5 , ...}

(2) {−2 , −1 , 0 , 1 , 2}

4 (a) (1) 23 + (− 15) + 15 (Commutative property)

= 23 + [(− 15) + 15]

(Associative property)

= 23 + 0 (Additive inverse)

= 23 (Additive identity)

(2) 36 + 64 + (− 72) + (− 28)

(Commutative property)

= (36 + 64) + [(− 72) + (− 28)]

(Associative property)

= 100 + (− 100) (Additive inverse)

= 0

(b) $\frac{(-3)^4 \times 3^5}{3^7} = \frac{3^4 \times 3^5}{3^7} = \frac{3^9}{3^7} = 3^2 = 9$

5 (a) (1) −6 , −9 (2) $\frac{1}{48}$, $\frac{1}{96}$

(b) (1) (− 4) × 5 × 2 (Commutative property)

= (− 4) × (5 × 2) (Associative property)

= (− 4) × 10 = − 40

(2) 45 × 117 − 45 × 17 = 45 (117 − 17)

= 45 × 100 = 4500

Unit Two

Exercise 7

1 (a) , (b) , (d) and (g) are equations because each one of them contains a variable and the equality relation.

(c) , (e) , (f) , (h) and (i) are inequalities because each one of them contains a variable and an inequality relation.

2 (a) 1^{st} degree (b) 1^{st} degree

(c) 1^{st} degree (d) 2^{nd} degree

(e) 3^{rd} degree (f) 1^{st} degree

(g) 1^{st} degree (h) 1^{st} degree

(i) 4^{th} degree (j) 3^{rd} degree

(k) 2^{nd} degree (l) 4^{th} degree

3 (a) x < − 3 (b) x ≤ 2

(c) x ≥ 3 (d) − 4 < x < 1

(e) 1 < x ≤ 7 (f) − 2 ≤ x ≤ 5

4 Substituting for x by each element of the substitution set in the equation , we find that

(a) The S.S. = {3} (b) The S.S. = {7}

(c) The S.S. = {2} (d) The S.S. = $\varnothing$

(e) The S.S. = {3} (f) The S.S. = {2}

(g) The S.S. = $\varnothing$ (h) The S.S. = $\varnothing$

(i) The S.S. = {−3} (j) The S.S. = {3}

5 Substituting for x by each element of the substitution set in the inequality , we find that

(a) The S.S. = {0 , 1}

(b) The S.S. = {6 , 7}

(c) The S.S. = {3}

(d) The S.S. = {0 , 1 , 2}

(e) The S.S. = $\varnothing$

(f) The S.S. = {−2 , 0 , 2 , 4}

(g) The S.S. = {−1 , 0 , 1}

(h) The S.S. = {2 , 3 , 4 , 5 , 6}

6 (a) Substituting for x by each element of the substitution set in the equation , we find that The S.S. = {3}

(b) Substituting for x by each element of the substitution set in the inequality , we find that : The S.S. = {7}

7 (a) Substituting for x by each element of the substitution set in the equation , we find that The S.S. = {2}

(b) Substituting for x by each element of the substitution set in the inequality , we find that The S.S. = {−1 , −2 , 0}

8 Substituting for x by each element of the substitution set in the equation , we find that ,

(a) The S.S. = $\varnothing$ (b) The S.S. = {−4}

(c) The S.S. = {7} (d) The S.S. = $\varnothing$

(e) The S.S. = {8}

9 (a) equality (b) first (c) second

(d) {0} (e) {− 3}

10 (a) (d) (b) (c) (c) (b)

(d) (d) (e) (b) (f) (b)

(g) (d) (h) (c) (i) (b)

(j) (d) (k) (d)

Exercise 8

1 (a) ∵ x + 3 = 7 ∴ x = 7 − 3

x = 4 The S.S. = {4}

(b) y + 8 = 19 y = 19 − 8

y = 11 ∴ The S.S. = {11}

(c) m − 7 = 4 m = 4 + 7

m = 11 ∴ The S.S. = {11}

(d) x − 9 = − 5 x = − 5 + 9

∴ x = 4 ∴ The S.S. = {4}

(e) ∵ x + 11 = 2

x = 2 − 11 (impossible in $\mathbb{N}$)

The S.S. = $\varnothing$

(f) ∵ x + 1 = 1 3 ∴ x + 1 = 3

x = 3 − 1 ∴ x = 2

∴ The S.S. = {2}

(g) 8 x = 32 ∴ x = $\frac{32}{8}$

∴ x = 4 ∴ The S.S. = {4}

(h) ∵ 3 y = 27 ∴ y = $\frac{27}{3}$

∴ y = 9 ∴ The S.S. = {9}

(i) ∵ 4 x = |− 8| ∴ x = $\frac{8}{4}$

∴ x = 2 The S.S. = {2}

[j] $\frac{n}{3} = 5$ $\therefore n = 5 \times 3$
$\therefore n = 15$ $\therefore$ The S.S. $= \{15\}$

[k] $\frac{x}{10} = 2$ $\therefore x = 2 \times 10$
$\therefore x = 20$ $\therefore$ The S.S. $= \{20\}$

[l] $\frac{z}{12} = 3$ $\therefore z = 3 \times 12$
$\therefore z = 36$ $\therefore$ The S.S. $= \{36\}$

2 [a] $x + 9 = 3$ $\therefore x = 3 - 9$
$\therefore x = -6$ $\therefore$ The S.S. $= \{-6\}$

[b] $x - 12 = 40$ $\therefore x = 40 + 12$
$\therefore x = 52$ $\therefore$ The S.S. $= \{52\}$

[c] $x + 6 = 0$ $\therefore x = 0 - 6$
$x = -6$ The S.S. $= \{-6\}$

[d] $-4 + x = -8$ $\therefore x = -8 + 4$
$x = -4$ $\therefore$ The S.S. $= \{-4\}$

[e] $n + 17 = |-13|$ $\therefore n = 13 - 17$
$\therefore n = -4$ $\therefore$ The S.S. $= \{-4\}$

[f] $m - (-3) = 1$ $\therefore m + 3 = 1$
$\therefore m = 1 - 3$ $\therefore m = -2$
$\therefore$ The S.S. $= \{-2\}$

[g] $y - (-5) = 3$ $\therefore y + 5 = 3$
$\therefore y = 3 - 5$ $\therefore y = -2$
$\therefore$ The S.S. $= \{-2\}$

[h] $-4x = -24$ $x = \frac{-24}{-4}$
$\therefore x = 6$ $\therefore$ The S.S. $= \{6\}$

[i] $9x = -18$ $x = \frac{-18}{9}$
$\therefore x = -2$ $\therefore$ The S.S. $= \{-2\}$

[j] $2 - x = 9$ $\therefore -x = 9 - 2$
$-x = 7$ $x = -7$
$\therefore$ The S.S. $= \{-7\}$

[k] $7 - m = 12$ $\therefore m = 7 - 12$
$\therefore m = -5$ The S.S. $= \{-5\}$

[l] $\frac{y}{3} = -5$ $y = -5 \times 3$
$\therefore y = -15$ $\therefore$ The S.S. $= \{-15\}$

3 [a] $3x - 2 = 7$ $3x = 7 + 2$
$\therefore 3x = 9$ $x = \frac{9}{3}$
$\therefore x = 3$ $\therefore$ The S.S. $= \{3\}$

[b] $4x + 1 = 17$ $\therefore 4x = 17 - 1$
$\therefore 4x = 16$ $\therefore x = \frac{16}{4}$
$\therefore x = 4$ The S.S. $= \{4\}$

[c] $6x + 7 = 25$ $\therefore 6x = 25 - 7$
$6x = 18$ $\therefore x = \frac{18}{6}$
$\therefore x = 3$ $\therefore$ The S.S. $= \{3\}$

[d] $8x + 12 = 4$ $8x = 4 - 12$
$\therefore 8x = -8$ $x = \frac{-8}{8}$
$\therefore x = -1$ The S.S. $= \{-1\}$

[e] $3x - 13 = 26$ $3x = 26 + 13$
$\therefore 3x = 39$ $\therefore x = \frac{39}{3}$
$\therefore x = 13$ The S.S. $= \{13\}$

[f] $5x + 2 = -6$
$\therefore 5x = -6 - 2$ (impossible in $\mathbb{N}$)
The S.S. $= \varnothing$

[g] $2x - 5 = -21$ $\therefore 2x = -21 + 5$
$2x = -16$ $x = \frac{-16}{2}$
$\therefore x = -8$ $\therefore$ The S.S. $= \{-8\}$

[h] $5x + 4 = 14$ $5x = 14 - 4$
$\therefore 5x = 10$ $\therefore x = \frac{10}{5}$
$\therefore x = 2$ $\therefore$ The S.S. $= \{2\}$

[i] $2x + 9 = -23$ $2x = -23 - 9$
$2x = -32$ $\therefore x = \frac{-32}{2}$
$x = -16$ The S.S. $= \{-16\}$

[j] $2y + 16 = 2^4$ $\therefore 2y + 16 = 16$
$2y = 16 - 16$ $2y = 0$
$\therefore y = \frac{0}{2}$ $\therefore y = 0$
The S.S. $= \{0\}$

[k] $6 + 2m = 16$ $\therefore 2m = 16 - 6$
$\therefore 2m = 10$ $\therefore m = \frac{10}{2}$
$\therefore m = 5$ $\therefore$ The S.S. $= \{5\}$

[l] $3 - 2x = 9$ $-2x = 9 - 3$
$-2x = 6$ $x = \frac{6}{-2}$
$\therefore x = -3$ $\therefore$ The S.S. $= \{-3\}$

[m] $4 + 3x = 3^2$ $4 + 3x = 9$
$\therefore 3x = 9 - 4$ $3x = 5$
$\therefore x = \frac{5}{3}$ (impossible in $\mathbb{Z}$)
$\therefore$ The S.S. $= \varnothing$

[n] $3x - 2 = -19$ $\therefore 3x = -19 + 2$
$3x = -17$
$x = \frac{-17}{3}$ (impossible in $\mathbb{Z}$)
$\therefore$ The S.S. $= \varnothing$

[o] $-6 - 4x = 16$
$\therefore 4x = -6 - 16$ (impossible in $\mathbb{N}$)
$\therefore$ The S.S. $= \varnothing$

[p] $\frac{y}{5} + 2 = -4$ $\frac{y}{5} = -4 - 2$
$\therefore \frac{y}{5} = -6$ $y = -6 \times 5$
$\therefore y = -30$ $\therefore$ The S.S. $= \{-30\}$

[q] $\frac{x}{2} - 4 = 7$ $\frac{x}{2} = 7 + 4$
$\frac{x}{2} = 11$ $\therefore x = 11 \times 2$
$x = 22$ $\therefore$ The S.S. $= \{22\}$

4 [a] $3x - 14 = |-16|$ $\therefore 3x - 14 = 16$
$\therefore 3x = 16 + 14$ $3x = 30$
$\therefore x = \frac{30}{3}$ $\therefore x = 10$
Possible in $\mathbb{N}$ and $\mathbb{Z}$

[b] $2m + 12 = 6$
$2m = 6 - 12$ (impossible in $\mathbb{N}$)
$\therefore 2m = -6$ $m = \frac{-6}{2}$ $\therefore m = -3$
Not possible in $\mathbb{N}$ but possible in $\mathbb{Z}$

[c] $2x + 5 = -27$
$\therefore 2x = -27 - 5$ (impossible in $\mathbb{N}$)
$2x = -32$ $\therefore x = -16$
Not possible in $\mathbb{N}$ but possible in $\mathbb{Z}$

[d] $2L - 15 = 8$ $2L = 8 + 15$
$\therefore 2L = 23$ $\therefore L = \frac{23}{2}$
Not possible in both $\mathbb{N}$ and $\mathbb{Z}$

[e] $\frac{x}{7} = -5$
$\therefore x = 7 \times (-5)$ (impossible in $\mathbb{N}$)
$\therefore x = -35$
Not possible in $\mathbb{N}$ but possible in $\mathbb{Z}$

[f] $4n - 3 = -7$
$\therefore 4n = -7 + 3$ (impossible in $\mathbb{N}$)
$\therefore 4n = -4$ $\therefore n = \frac{-4}{4}$ $n = -1$
Not possible in $\mathbb{N}$ but possible in $\mathbb{Z}$

[g] $3x = 8$ $x = \frac{8}{3}$
Not possible in both $\mathbb{N}$ and $\mathbb{Z}$

5 [a] 2 [b] 3 [c] 2
[d] 5 [e] 14 [f] 7
[g] 10 [h] 2 [i] 2
[j] 1 [k] $\varnothing$ [l] $\{4\}$

6 [a] (d) [b] (d) [c] (c)
[d] (d) [e] (d) [f] (a)
[g] (b) [h] (b) [i] (a)
[j] (d) [k] (c) [l] (c)
[m] (a) [n] (c)

7 $9x = 18$ $\therefore x = \frac{18}{9}$
$\therefore x = 2$ $\therefore x + 1 = 3$

8 $\because x \times (7 - (-2)) = (-8 \times 9) \times (-1)$
$\therefore x \times 9 = 72$ $\therefore x = \frac{72}{9}$
$\therefore x = 8$

9 Let the number be x
, then its triple $= 3x$
So , $x + 3x = 72$ $\therefore 4x = 72$
$\therefore x = \frac{72}{4}$ $\therefore x = 18$
Then , the number is 18

10 Let the three numbers be x , $x + 1$ and $x + 2$
So , $x + x + 1 + x + 2 = 213$
$\therefore 3x + 3 = 213$ $3x = 213 - 3$
$\therefore 3x = 210$ $\therefore x = \frac{210}{3}$ $\therefore x = 70$
Then , the three numbers are 70 , 71 and 72

11 $3x - 1 = 2x + 5$ $\therefore 3x - 2x = 5 + 1$
$x = 6$ $\therefore$ The S.S. $= \{6\}$

12 $2(x + 3) = 4$ $2x + 6 = 4$
$2x = 4 - 6$ $\therefore 2x = -2$
$\therefore x = \frac{-2}{2}$ $\therefore x = -1$
$\therefore$ The S.S. $= \{-1\}$

Exercise 9

1 [a] $x - 3 < 1$ $\therefore x < 1 + 3$
$\therefore x < 4$
The S.S. $= \{0 , 1 , 2 , 3\}$

[b] ∵ $x + 2 > 5$ ∴ $x > 5 - 2$
$x > 3$
∴ The S.S. = {4 , 5 , 6 , 7 , ...}

[c] $x + 4 > 1$ ∴ $x > 1 - 4$
$x > -3$
∴ The S.S = {0 , 1 , 2 , 3 , ...}

[d] ∵ $x + 4 < 7$ ∴ $x < 7 - 4$
$x < 3$
∴ The S.S = {0 , 1 , 2}

[e] $x + 3 \geq 6$ ∴ $x \geq 6 - 3$
∴ $x \geq 3$
∴ The S.S = {3 , 4 , 5 , 6 , ...}

[f] ∵ $x - 4 \leq -1$ ∴ $x \leq -1 + 4$
$x \leq 3$
∴ The S.S = {0 , 1 , 2 , 3}

[g] ∵ $19 < a + 14$ ∴ $19 - 14 < a$
∴ $5 < a$
The S.S. = {6 , 7 , 8 , 9 , ...}

[h] ∵ $m - 5 \geq |-7|$ ∴ $m - 5 \geq 7$
$m \geq 7 + 5$ ∴ $m \geq 12$
∴ The S.S = {12 , 13 , 14 , 15 , ...}

[i] ∵ $-1 \geq x + 3$ ∴ $-1 - 3 \geq x$
∴ $-4 \geq x$ ∴ The S.S. = Ø

[j] ∵ $3x < 12$ $\frac{3x}{3} < \frac{12}{3}$
∴ $x < 4$
The S.S. = {0 , 1 , 2 , 3}

[k] $4k \geq -16$ ∴ $\frac{4k}{4} \geq \frac{-16}{4}$
∴ $k \geq -4$
∴ The S.S = {0 , 1 , 2 , 3 , ...}

[l] $-2y < -14$ ∴ $\frac{-2y}{-2} > \frac{-14}{-2}$
∴ $y > 7$
∴ The S.S. = {8 , 9 , 10 , ...}

3 $x + 3 \leq 6$ $x \leq 6 - 3$ $x \leq 3$
[a] When $x \in \mathbb{Z}$
∴ The S.S. = {3 , 2 , 1 , 0 , −1 , ...}
[b] When $x \in \mathbb{N}$
∴ The S.S. = {0 , 1 , 2 , 3}

4 [a] $2x + 1 < 7$ ∴ $2x < 7 - 1$
∴ $2x < 6$ ∴ $\frac{2x}{2} < \frac{6}{2}$
∴ $x < 3$ The S.S. = {0 , 1 , 2}

[b] $2x - 3 < 5$ ∴ $2x < 5 + 3$
∴ $2x < 8$ ∴ $\frac{2x}{2} < \frac{8}{2}$
∴ $x < 4$
∴ The S.S. = {3 , 2 , 1 , 0 , ...}

[c] $3x - 2 < 1$ ∴ $3x < 1 + 2$
$3x < 3$ $\frac{3x}{3} < \frac{3}{3}$
$x < 1$ The S.S = {0}

[d] $2x + 9 < 1$ ∴ $2x < 1 - 9$
∴ $2x < -8$ ∴ $\frac{2x}{2} < \frac{-8}{2}$
∴ $x < -4$
∴ The S.S = {−5 , −6 , −7 , ...}

[e] ∵ $4x + 2 \geq -10$ ∴ $4x \geq -10 - 2$
$4x \geq -12$ $\frac{4x}{4} \geq \frac{-12}{4}$
$x \geq -3$
∴ The S.S. = {−3 , −2 , −1 , 0 , ...}

[f] $3x + 9 > 0$ ∴ $3x > 0 - 9$
$3x > -9$ ∴ $\frac{3x}{3} > \frac{-9}{3}$
∴ $x > -3$
∴ The S.S. = {0 , 1 , 2 , 3 , ...}

[g] $3x - 5 \leq 4$ $3x \leq 4 + 5$
$3x \leq 9$ $\frac{3x}{3} \leq \frac{9}{3}$
∴ $x \leq 3$
The S.S. = {3 , 2 , 1 , 0 , ...}

[h] $2x - 5 \leq -7$ ∴ $2x \leq -7 + 5$
$2x \leq -2$ ∴ $\frac{2x}{2} \leq \frac{-2}{2}$
$x \leq -1$
∴ The S.S. = {−1 , −2 , −3 , ...}

[i] $4x + 1 < 13$ ∴ $4x < 13 - 1$
∴ $4x < 12$ ∴ $\frac{4x}{4} < \frac{12}{4}$
∴ $x < 3$
The S.S. = {2 , 1 , 0 , −1 , ...}

[j] $3x + 2 \leq 11$ $3x \leq 11 - 2$
∴ $3x \leq 9$ ∴ $\frac{3x}{3} \leq \frac{9}{3}$
∴ $x \leq 3$
∴ The S.S. = {0 , 1 , 2 , 3}

[k] ∵ $1 - 6x < 15$ ∴ $-6x < 15 - 9$
∴ $-6x < 6$ $\frac{-6x}{-6} > \frac{6}{-6}$
∴ $x > -1$
∴ The S.S = {0 , 1 , 2 , 3 , ...}

[l] $1 + 2x \leq -3$ ∴ $2x \leq -3 - 1$
∴ $2x \leq -4$ ∴ $\frac{2x}{2} \leq \frac{-4}{2}$
∴ $x \leq -2$ The S.S. = Ø

[m] ∵ $3x + 2 \geq 12$ ∴ $3x \geq 12 - 2$
∴ $3x \geq 10$ $\frac{3x}{3} \geq \frac{10}{3}$
∴ $x \geq 3\frac{1}{3}$
∴ The S.S. = {4 , 5 , 6 , ...}

[n] ∵ $3x - 5 \leq 7$ ∴ $3x \leq 7 + 5$
∴ $3x \leq 12$ ∴ $\frac{3x}{3} \leq \frac{12}{3}$
∴ $x \leq 4$
The S.S = {4 , 3 , 2 , 1}

[o] $1 - 8x < 33$ ∴ $-8x < 33 - 1$
$-8x < 32$ ∴ $\frac{-8x}{-8} > \frac{32}{-8}$
∴ $x > -4$
∴ The S.S = {−3 , −2 , −1 , ...}

[p] $1 - 3x > 7$ ∴ $-3x > 7 - 1$
∴ $-3x > 6$ $\frac{-3x}{-3} < \frac{6}{-3}$
∴ $x < -2$ The S.S. = Ø

5 $3x + 5 < 2$ ∴ $3x < 2 - 5$
∴ $3x < -3$ $x < \frac{-3}{3}$
∴ $x < -1$
[a] When $x \in \mathbb{N}$ ∴ The S.S. = Ø
[b] When $x \in \mathbb{Z}$
∴ The S.S. = {−2 , −3 , −4 , ...}

6 [a] $3 < x + 2 \leq 6$
∴ $3 - 2 < x + 2 - 2 \leq 6 - 2$
$1 < x \leq 4$
The S.S. = {2 , 3 , 4}

(b) ∵ $-3 \le x - 1 < 3$

∴ $-3 + 1 \le x - 1 + 1 < 3 + 1$

∴ $-2 \le x < 4$

∴ The S.S. = $\{-2, -1, 0, 1, 2, 3\}$

(c) $3 < 2x + 1 \le 9$

∴ $3 - 1 < 2x + 1 - 1 \le 9 - 1$

∴ $2 < 2x \le 8$ ∴ $\frac{2}{2} < \frac{2x}{2} \le \frac{8}{2}$

∴ $1 < x \le 4$

∴ The S.S. = $\{2, 3, 4\}$

(d) $5 \le 1 - 2x \le 11$

∴ $5 - 1 \le 1 - 2x - 1 \le 11 - 1$

∴ $4 \le -2x \le 10$

∴ $\frac{4}{-2} \ge \frac{-2x}{-2} \ge \frac{10}{-2}$

$-2 \ge x \ge -5$

∴ The S.S. = $\{-5, -4, -3, -2\}$

6 (a) > (b) > (c) >
(d) > (e) -3 (f) $3 > a$
(g) $3, 4$ (h) $3, 3$ (i) $\{0, 1\}$
(j) $\{3, 2, 1, 0, -1, ...\}$
(k) $\{2, 3, 4, ...\}$
(l) ∅ (m) $\{0\}$ (n) <

7 (a) (b) (b) (d) (c) (d) (d) (b)
(e) (a) (f) (d) (g) (a) (h) (b)
(i) (c) (j) (a) (k) (c)

8 ∵ $2x - 1 \le x + 3$ ∴ $2x - x \le 3 + 1$

$x \le 4$

The S.S. = $\{4, 3, 2, 1, 0, -1, ...\}$

Answers of unit test

1 (a) $x + 1 = 5$ (b) second (c) -3
(d) 6 (e) >

2 (a) (1) Third degree (2) First degree
(b) (1) $x \le -4$ (2) $-3 < z < 7$

3 (a) At $x = -1$

∵ The left hand side = $3 \times (-1) + 2$
$= -1 \ne 5$

∴ -1 is not a solution to the equation.

At $x = 0$

∴ The left hand side = $3 \times 0 + 2 = 2 \ne 5$

∴ 0 is not a solution to the equation.

At $x = 1$

∴ The left hand side = $3 \times 1 + 2 = 5 \ne 5$

∴ 1 is not a solution to the equation.

At $x = 2$

The left hand side = $3 \times 2 + 2 = 8$

∴ 2 is a solution to the equation.

∴ The S.S. = $\{2\}$

(b) (1) ∵ $4x - 3 = 5$ ∴ $4x = 5 + 3$

∴ $4x = 8$ ∴ $x = \frac{8}{4}$

∴ $x = 2$ ∴ The S.S. = $\{2\}$

(2) $2x + 7 = 1$ ∴ $2x = 1 - 7$

∴ $2x = -6$ $x = \frac{-6}{2}$

∴ $x = -3$

∴ The S.S. = $\{-3\}$

4 (a) At $x = -2$

$3 \times (-2) + 2 = -4$ is smaller than 2

∴ -2 is a solution to the inequality.

At $x = -1$

$3 \times (-1) + 2 = -1$ is smaller than 2

∴ -1 is a solution to the inequality.

At $x = 0$

∴ $3 \times 0 + 2 = 2$

0 is a solution to the inequality.

At $x = 1$

∴ $3 \times 1 + 2 = 5$ is neither smaller than nor equal to 2

5 is not a solution to the inequality.

∴ The S.S. = $\{-2, -1, 0\}$

(b) (1) $5x - 1 > -11$ $5x > -11 + 1$

∴ $5x > -10$ ∴ $x > \frac{-10}{5}$

∴ $x > -2$

The S.S. = $\{-1, 0, 1, ...\}$

(2) $1 \le 2x + 3 < |-7|$

∴ $1 - 3 \le 2x < 7 - 3$

∴ $-4 \le 2x < 4$

∴ $\frac{-4}{2} \le x < \frac{4}{2}$

∴ $-2 \le x < 2$

∴ The S.S. = $\{-2, -1, 0, 1\}$

5 (a) (1) ∵ $3x + 1 = -5$ ∴ $3x = -5 - 1$

∴ $3x = -6$ ∴ $x = \frac{-6}{3}$

$x = -2$

When $x \in \mathbb{N}$ The S.S. = ∅

When $x \in \mathbb{Z}$ The S.S. = $\{-2\}$

(2) $4x - 9 < -1$ ∴ $4x < -1 + 9$

∴ $4x < 8$ ∴ $x < \frac{8}{4}$

∴ $x < 2$

When $x \in \mathbb{N}$ The S.S. = $\{1, 0\}$

When $x \in \mathbb{Z}$

The S.S. = $\{1, 0, -1, -2, ...\}$

(b) Let the number be x

∴ then its double is $2x$

So, $x + 2x = 15$ ∴ $3x = 15$

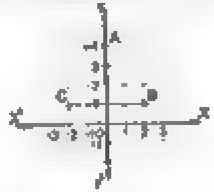
∴ $x = \frac{15}{3}$ $x = 5$

Then, the number is 5

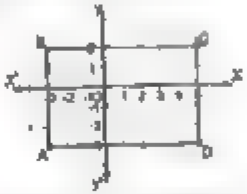
Unit Three

Exercise 10

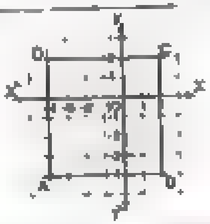
1 [a] 1 unit. [b] 3 units. [c] 6 units.
[d] 10 units. [e] 2 units. [f] 5 units.
[g] 9 units. [h] 7 units. [i] 4 units.

2 

The length of $\overline{BC}$ = 4 units.

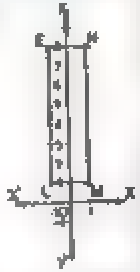
3 [a] 

The name of the shape ABCD is a rectangle.
[b] The length of $\overline{AB}$ = |B − A| = |2 − (−3)| = |5| = 5 units.
, the length of $\overline{BC}$ = |C − B| = |5 − (−3)| = |8| = 8 units.
The perimeter = 2 × (8 + 5) = 26 units.
, the area = 8 × 5 = 40 square units.

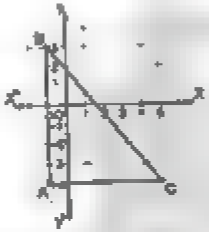
4 [a] 

The name of the shape ABCD is a square.
[b] The length of $\overline{AB}$ = |B − A| = |2 − (−4)| = |6| = 6 units.
The perimeter = 6 × 4 = 24 units.
, the area = 6 × 6 = 36 square units.

[c] Symmetric , because the square has four axes of symmetry.

5 [a] 

[b] The length of $\overline{EL}$ = |L − E| = |1 − 8| = |−7| = 7 units.
, the length of LM = |M − L| = |1 − (−1)| = |2| = 2 units.
∴ The perimeter = 2 × (7 + 2) = 18 units.
, the area = 7 × 2 = 14 square units.
[c] Symmetric , because the rectangle has two axes of symmetry.

6 

[a] The length of $\overline{AB}$ = |B − A| = |3 − (−4)| = |7| = 7 units.
, the length of $\overline{AC}$ = |C − A| = |5 − (−1)| = |6| = 6 units.
[b] The triangle ABC is a right-angled triangle and is a scalene triangle.
[c] The area = $\frac{1}{2}$ × 6 × 7 = 21 square units.

7 [a] E (−5 , −3) , F (2 , −3) , G (2 , 0) and H (−5 , 0)
[b] 7 , 3 , 7 , 3 [c] 21

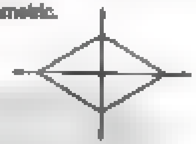
8 [a] A (3 , 5) , B (3 , 1) , C (−3 , 1) and D (−3 , 5)
[b] 6 units , 4 units , 24 square units.

9 [a] A (1 , −2) , B (5 , 2) , C (1 , 6) and D (−3 , 2)
[b] 8 units , 8 units [c] 32 square units.

Exercise 11

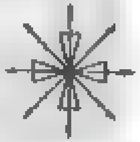
1 [a] Translation [b] Rotation [c] Reflection
[d] Translation [e] Reflection [f] Rotation
[g] Reflection [h] Reflection [i] Translation

2 [a] Symmetric.

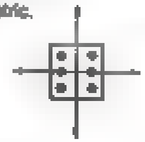


[b] Symmetric.

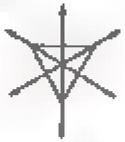
[c] Symmetric.



[d] Not symmetric. [e] Not symmetric.
[f] Symmetric.



[g] Symmetric.



[h] Symmetric.

[i] Not symmetric.

3

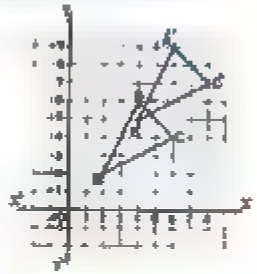
The point	The translation	The image
(2 , 3)	(x , y) ⟶ (x + 3 , y + 1)	(5 , 4)
(−5 , 4)	(x , y) ⟶ (x + 2 , y − 1)	(−3 , 3)
(0 , −3)	(x , y) ⟶ (x + 0 , y + 3)	(0 , 0)
(−4 , −1)	(x , y) ⟶ (x + 3 , y + 1)	(−1 , 0)

1 [a] (1) magnitude (2) direction
[b] (4 , 6) [c] (−1 , −1) [d] (5 , 0)
[e] (−6 , 1) [f] (−1 , 4) [g] (2 , 6)
[h] (1 , 2) [i] (1 , 5) [j] (−4 , −6)
[k] (3 , 1) [l] (0 , 1) [m] (3 , −1)
[n] (x + 3 , y − 1) [o] (0 , 6) [p] MN
[q] 4 , $\overrightarrow{MN}$

2 [a] (a) [b] (d) [c] (b) [d] (a)
[e] (a) [f] (d) [g] (a) [h] (a)
[i] (b) [j] (b) [k] (b) [l] (a)
[m] (a) [n] (a) [o] (b)

3 [a] A (6 , 2) ⟶ $\acute{A}$ (2 , −2)
B (2 , 6) ⟶ $\acute{B}$ (−1 , 1)

[b] A (4 , 6) ⟶ $\acute{A}$ (6 , 8)
B (2 , 2) ⟶ $\acute{B}$ (4 , 5)
C (6 , 4) ⟶ $\acute{C}$ (8 , 7)

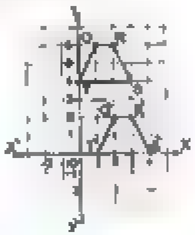


(c) A (− 1 , 1) ⟶ Á (3 , 1)
B (3 , 2) ⟶ B́ (5 , 2)
C (1 , 4) ⟶ Ć (3 , 4)

(d) A (2 , 2) ⟶ Á (5 , 0)
B (4 , 2) ⟶ B́ (7 , 0)
C (3 , 4) ⟶ Ć (6 , 2)
D (2 , 3) ⟶ D́ (6 , 1)

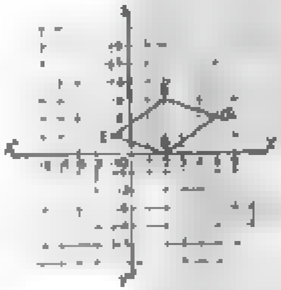
(e) A (4 , 5) ⟶ Á (7 , 1)
B (4 , 2) ⟶ B́ (7 , − 2)
C (1 , 2) ⟶ Ć (4 , − 2)
D (1 , 4) ⟶ D́ (4 , 0)

(f) Q (1 , 6) ⟶ Q́ (2 , 2)
R (2 , 6) ⟶ Ŕ (3 , 2)
S (3 , 4) ⟶ Ś (4 , 0)
T (0 , 4) ⟶ T́ (1 , 0)



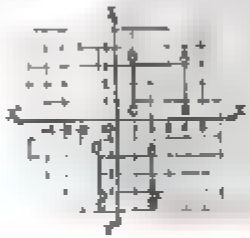
A (1 , 2) ⟶ Á (3 , 0)
B (− 2 , − 1) ⟶ B́ (0 , − 3)

(a) D (2 , 0) ⟶ D́ (5 , 2)
E (− 1 , 1) ⟶ É (2 , 3)

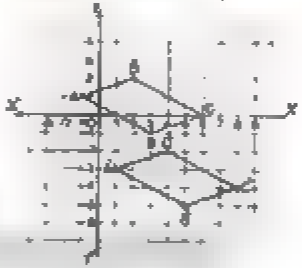


(b) Parallelogram , because $\overline{D́É}$ is the image of $\overline{DE}$ by translation (3 , 2)
i.e. $\overline{D́É} \parallel \overline{DE}$ and D́É = DE

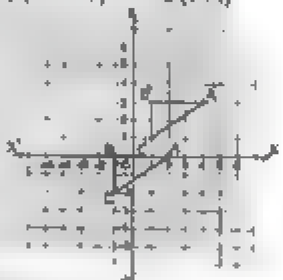
A (4 , 3) ⟶ Á (2 , − 2)
B (4 , 1) ⟶ B́ (2 , − 4)
C (1 , 1) ⟶ Ć (− 1 , − 4)
D (1 , 3) ⟶ D́ (− 1 , − 2)



(a) A (− 1 , 1) , B (3 , − 1) , C (5 , 0) , D (2 , 2)
(b) A (− 1 , 1) ⟶ Á (1 , − 3)
B (3 , − 1) ⟶ B́ (5 , − 5)
C (5 , 0) ⟶ Ć (8 , − 4)
D (2 , 2) ⟶ D́ (4 , − 2)

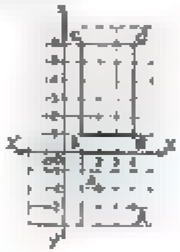


(a) A (2 , 0) , B (− 1 , 0) , C (− 1 , − 2)
(b) A (2 , 0) ⟶ Á (4 , 3)
B (− 1 , 0) ⟶ B́ (1 , 3)
C (− 1 , − 2) ⟶ Ć (1 , 1)

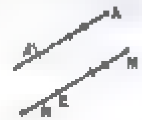


(c) 2 length units
3 length units
(d) Δ ABC is not symmetric , because its side lengths are different in length "scalene triangle"

(a) , (b) and (c) are drawn on the following coordinates plane :



(d) BC = 5 length units
BB́ = 3 length units
(e) The perimeter of the shape BB́ĆC
= (3 + 5) × 2 = 8 × 2 = 16 length units
The area of the shape BB́ĆC
= 3 × 5 = 15 square units



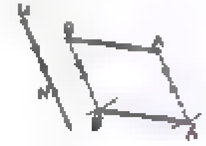


Fig. (1)

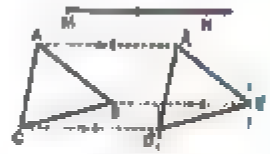
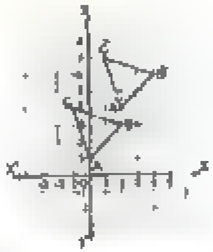


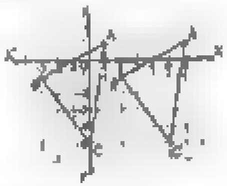
Fig. (2)

A (2 , 3) ⟶ Á (5 , 1)
B (− 2 , 0) ⟶ B́ (1 , − 2)

A (0 , 1) ⟶ Á (2 , 4)
B (2 , 3) ⟶ B́ (4 , 6)
C (−1 , 4) ⟶ Ć (1 , 7)

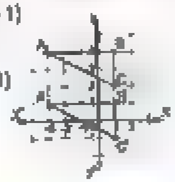


A (1 , 1) ⟶ Á (6 , 1)
B (−3 , 1) ⟶ B́ (2 , −1)
C (0 , −6) ⟶ Ć (5 , −6)

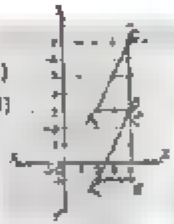


[a] OB = 2 length units
[b] O (0 , 0) ⟶ Ó (2 , 2)
B (2 , 0) ⟶ B́ (4 , 2)
C (0 , 2) ⟶ Ć (2 , 4)

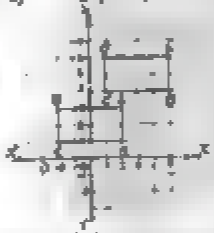
AB = 4 length units , BC = 2 length units.
A (−3 , 4) ⟶ Á (−3 , 1)
B (1 , 4) ⟶ B́ (1 , 1)
C (1 , 2) ⟶ Ć (1 , −1)



BC = 4 length units.
AB = 2 length units.
A (2 , 3) ⟶ Á (2 , −1)
B (4 , 3) ⟶ B́ (4 , −1)
C (4 , 7) ⟶ Ć (4 , 3)
The area of △ ABC
$= \frac{1}{2} \times 2 \times 4$
= 4 square units.

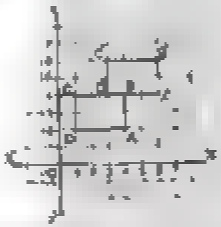


A (2 , 3) ⟶ Á (5 , 6)
B (2 , 1) ⟶ B́ (5 , 4)
C (−2 , 1) ⟶ Ć (1 , 4)
D (−2 , 3) ⟶ D́ (1 , 6)



The figure Á B́ Ć D́ is a rectangle.

[a] A (4 , 2) ⟶ Á (6 , 4)
B (4 , 4) ⟶ B́ (6 , 6)
C (1 , 4) ⟶ Ć (3 , 6)
D (1 , 2) ⟶ D́ (3 , 4)



[b] The perimeter = (3 + 2) × 2
= 10 length units.

[a] (4 , −2) $\xrightarrow[(2\ ,\ -3)]{\text{by translation}}$ (6 , −5) $\xrightarrow[(-5\ ,\ 1)]{\text{by translation}}$ (3 , −4)

[b] (−1 , 3) $\xrightarrow[(2\ ,\ -3)]{\text{by translation}}$ (1 , 0) $\xrightarrow[(-3\ ,\ 1)]{\text{by translation}}$ (−2 , 1)

[c] (0 , 2) $\xrightarrow[(2\ ,\ -3)]{\text{by translation}}$ (2 , −1) $\xrightarrow[(-3\ ,\ 1)]{\text{by translation}}$ (−1 , 0)

Let the point be A (x , y)
A (x , y) ⟶ Á (x + 2 , y + 3)
∵ Á (2 , 3) ∴ (x + 2 , y + 3) = (2 , 3)
∴ x + 2 = 2 ∴ x = 0
, y + 3 = 3 ∴ y = 0
∴ The point is (0 , 0)

(a + 2 , b − 3) = (5 , −4)
∴ a + 2 = 5 ∴ a = 3 , b − 3 = −4
∴ b = −1 (a , b) = (3 , −1)

Á (2 , 2) is the image of the point A (1 , 1)
The mapping rule of this translation
is (x , y) ⟶ (x + 1 , y + 1)
The image of O (0 , 0) is (1 , 1)
The image of B (−1 , 3) is (0 , 4)
The image of C (−3 , 5) is (−2 , 6)

(x , y) ⟶ (x + 4 , y − 3)

Let A (x , y)
∴ A (x , y) ⟶ Á (x − 1 , y − 4)
∵ Á (3 , −3) ∴ x − 1 = 3
∴ x = 4 , y − 4 = −3 ∴ y = 1
∴ A (4 , 1)

Fig. (1) Reflection and the reflection axis is y-axis
Fig. (2) Translation (x , y) ⟶ (x − 5 , y + 3)

Exercise 12

[a] 28.26 cm² [b] 78.5 m²
[c] 153.86 cm² [d] 19.625 cm²
[e] 84.985 m² [f] 50.24 cm²

[a] 200.96 cm² [b] 40.6944 m² ≈ 40.69 m²
[c] 78.5 km² [d] 6.1544 m² ≈ 6.15 m²
[e] 124.6266 mm² ≈ 124.63 mm²
[f] 19300.1984 dm² ≈ 19300.2 dm²

[a] 200.96 cm²
[b] 346.185 m² ≈ 346.19 m²
[c] 1.5386 cm² ≈ 1.54 cm²
[d] 1256 mm²
[e] 277.4504 cm² ≈ 277.45 cm²
[f] 615.44 km²

The area of the circle $= \pi r^2 = \frac{22}{7} \times (7)^2$
= 154 cm²

The area of the circle $= \pi r^2 = 3.14 \times (4)^2$
= 50.24 cm²

The radius length $= \frac{1}{2} \times$ the diameter length
$= \frac{1}{2} \times 12 = 6$ cm.
The area of the circle $= 3.14 \times (6)^2$
= 113.04 cm²

The radius length $= \frac{1}{2} \times$ the diameter length
$= \frac{1}{2} \times 20 = 10$ cm.
The area of the circle $= 3.14 \times (10)^2$
= 314 cm²

The radius length $= \frac{1}{2} \times$ the diameter length
$= \frac{1}{2} \times 17.5 = 8.75$ cm.
The area of the circle $= \frac{22}{7} \times (8.75)^2$
= 240.625 cm²

The radius length $= \frac{1}{2} \times$ the diameter length $= \frac{1}{2} \times 14 = 7$ cm.

The area of the circle $= \frac{22}{7} \times (7)^2 = 154$ cm²

The circumference $= 2 \times \frac{22}{7} \times 7 = 44$ cm

The area of the circle M $= 3.14 \times (4)^2 = 50.24$ cm²

The area of one sector $= 50.24 \div 5 = 10.048$ cm²

The area of the circle M $= \frac{22}{7} \times (7)^2 = 154$ cm²

The area of one sector $= 154 \div 8 = 19.25$ cm²

The radius length $= \frac{1}{2} \times 25 = 12.5$ cm.

The area of upper base of tent $= 3.14 \times (12.5)^2 = 490.625$ cm²

The area of one sector $= 490.625 \div 8 = 61.328125$ cm² $\simeq 61$ cm²

(a) The area $= \frac{1}{2} \pi r^2 = \frac{1}{2} \times \frac{22}{7} \times (3.5)^2 = 19.25$ cm²

(b) The area $= \frac{1}{4} \pi r^2 = \frac{1}{4} \times \frac{22}{7} \times (7)^2 = 38.5$ cm²

(c) The area $= \frac{1}{2} \times \frac{22}{7} \times (2)^2 + \frac{1}{2} \times 4 \times 5 = 6\frac{2}{7} + 10 = 16\frac{2}{7}$ cm²

(d) The area $= \frac{1}{2} \times \frac{22}{7} \times (3.5)^2 + 7 \times 9 = 19.25 + 63 = 82.25$ cm²

(e) The area $= \frac{22}{7} \times (3.5)^2 + 7 \times 10 = 38.5 + 70 = 108.5$ cm²

(f) The area $= \frac{1}{2} \times \frac{22}{7} \times (2.5)^2 + \frac{1}{2} \times 3 \times 4 = 9\frac{23}{28} + 6 = 15\frac{23}{28}$ cm²

(g) The area $= \frac{22}{7} \times (5.25)^2 + 10.5 \times 10.5 = 86.625 + 110.25 = 196.875$ cm²

(h) The area $= \frac{1}{2} \times \frac{22}{7} \times (3)^2 + \frac{1}{2} \times \frac{22}{7} \times (8)^2 + 8 \times 12 = 14\frac{1}{7} + 100\frac{4}{7} + 96 = 210\frac{5}{7}$ cm²

(i) The area $= \frac{1}{2} \times \frac{22}{7} \times (3)^2 + \frac{1}{2} \times \frac{22}{7} \times (4)^2 + \frac{1}{2} \times \frac{22}{7} \times (5)^2 + \frac{1}{2} \times 6 \times 8 = 14\frac{1}{7} + 25\frac{1}{7} + 39\frac{2}{7} + 24 = 102\frac{4}{7}$ cm²

The area of the square $= 14 \times 14 = 196$ cm²

The area of the circle $= \frac{22}{7} \times (7)^2 = 154$ cm²

The area of the coloured part $= 196 - 154 = 42$ cm²

The area of the rectangle $= 12 \times 7 = 84$ cm²

The area of the circle $= \frac{22}{7} \times (3.5)^2 = 38.5$ cm²

The area of the coloured part $= 84 - 38.5 = 45.5$ cm²

The area of the rectangle $= 34 \times 15 = 510$ cm²

The diameter length of the semicircle $= 34 - (10 + 10) = 14$ cm.

The radius length of the semicircle $= \frac{1}{2} \times 14 = 7$ cm.

The area of the semicircle $= \frac{1}{2} \times \frac{22}{7} \times (7)^2 = 77$ cm²

The area of the coloured part $= 510 - 77 = 433$ cm²

(a) The area of the large circle $= 3.14 \times (8)^2 = 200.96$ cm²

The area of the small circle $= 3.14 \times (5)^2 = 78.5$ cm²

The area of the coloured part $= 200.96 - 78.5 = 122.46$ cm²

(b) The area of the rectangle $= 18 \times 8 = 144$ cm²

The area of the circle $= 3.14 \times (3)^2 = 28.26$ cm²

The area of the coloured part $= 144 - 28.26 = 115.74$ cm²

(c) The area of the rectangle $= 16 \times 33 = 528$ cm²

The area of the circle $= 3.14 \times (8)^2 = 200.96$ cm²

The area of the coloured part $= 528 - 200.96 = 327.04$ cm²

(d) The area of the circle $= 3.14 \times (7)^2 = 153.86$ cm²

The area of the triangle $= \frac{1}{2} \times 7 \times 7 = 24.5$ cm²

The area of the coloured part $= 153.86 - 24.5 = 129.36$ cm²

(e) The area of the large semi-circle $= \frac{1}{2} \times 3.14 \times (21)^2 = 692.37$ cm²

The area of the small semi-circle $= \frac{1}{2} \times 3.14 \times (14)^2 = 307.72$ cm²

The area of the coloured part $= 692.37 - 307.72 = 384.65$ cm²

(f) The area of the square $= 60 \times 60 = 3600$ m²

The area of the circle $= 3.14 \times (21)^2 = 1384.74$ m²

The area of the coloured part $= 3600 - 1384.74 = 2215.26$ m²

(g) The area of the triangle $= \frac{1}{2} \times 10 \times 8.5 = 42.5$ cm²

The area of the circle $= 3.14 \times (2)^2 = 12.56$ cm²

The area of the coloured part $= 42.5 - (3 \times 12.56) = 4.82$ cm²

(h) The area of the circle $= 3.14 \times (10)^2 = 314$ cm²

The area of the square $= \frac{1}{2} \times (20)^2 = 200$ cm²

The area of the coloured part $= 314 - 200 = 114$ cm²

(i) The area of the square $= 14 \times 14 = 196$ m²

The area of the circle $= 3.14 \times (3.5)^2 = 38.465$ m²

The area of the coloured part $= 196 - 38.465 = 157.535$ m²

(a) πr^2 (b) 38, 616 (c) 62.8, 314 (d) 40 (e) 5 (f) 225π cm² (g) 49.625 (h) 115.5 (i) 38.5 (j) 71.75 (k) 441.5625

(a) πr^2 (b) $2\pi r$ (c) 38.5 (d) π (e) 16 (f) 9π (g) 14 (h) $\pi \div 4$ (i) 4π (j) $\pi - 2$

$\because 14\pi = 2 \times \pi \times r$ $\therefore r = \frac{14\pi}{2\pi} = 7$ m.

The area $= \pi \times (7)^2 = 49\pi$ m²

$2\pi = 2 \times \pi \times r$ $\therefore r = \frac{2\pi}{2\pi} = 1$ cm.

$\therefore$ The area $= \pi \times (1)^2 = \pi$ cm²

$88 = 2 \times \pi \times r$ $\therefore r = \frac{88}{2 \times \frac{22}{7}} = 14$ cm.

$\therefore$ The area $= \frac{22}{7} \times (14)^2 = 616$ cm²

$62.8 = 2\pi r$ $\therefore r = \frac{62.8}{2 \times 3.14} = 10$ cm

$\therefore$ The area $= 3.14 \times (10)^2 = 314$ cm²

$\because 44 = 2\pi r$ $\therefore r = \frac{44}{2 \times \frac{22}{7}} = 7$ cm.

$\therefore$ The area $= \frac{22}{7} \times (7)^2 = 154$ cm²

The area of the coloured part = the area of half the large circle $= \frac{1}{2} \times 3.14 \times (2.4)^2 = 9.0432$ cm²

The radius length $= 1.5 \div 2 = 0.75$ m

$\therefore$ The area of the circle $= 3.14 \times (0.75)^2 = 1.76625$ m²

$\therefore$ The cost price of the glass $= 1.76625 \times 60 = 105.975$ pounds.

$\because$ The length of the outer radius $= 12 \div 2 = 6$ cm.

, the length of the inner radius $= 1.5 \div 2 = 0.75$ cm.

The area of the outer circle $= 3.14 \times (6)^2 = 113.04$ cm²

, the area of the inner circle $= 3.14 \times (0.75)^2 = 1.76625$ cm²

$\therefore$ The area of the CD $= 113.04 - 1.76625 = 111.27375$ cm²

[a] $132 = d\,\pi$ $\therefore d = \frac{132}{\frac{22}{7}} = 42$ m.

[b] The radius length $= 42 \div 2 = 21$ m.

, The area of the garden $= \frac{22}{7} \times (21)^2$
$= 1386\ m^2$

The area of the circle $= 3.14 \times (5)^2 = 78.5\ cm^2$
The area of the rectangle $= 6 \times 8 = 48\ cm^2$
The area of the shaded part $= 78.5 - 48$
$= 30.5\ cm^2$

The area of the large circle $= 3.14 \times (6)^2$
$= 113.04\ cm^2$
The area of the small circle $= 3.14 \times (2)^2$
$= 12.56\ cm^2$
The coloured area $= 113.04 - (4 \times 12.56)$
$= 62.8\ cm^2$

$616 = \pi\, r^2$ $r^2 = \frac{616}{\frac{22}{7}} = 196$
$\therefore r \times r = 14 \times 14$ $r = 14$ cm.
The circumference $= 2 \times \frac{22}{7} \times 14 = 88$ cm.

Exercise 13

1 [a] The area of one face $= 15 \times 15 = 225\ cm^2$
The lateral area $= 225 \times 4 = 900\ cm^2$
The total area $= 225 \times 6 = 1350\ cm^2$
[b] The area of one face $= 8 \times 8 = 64\ dm^2$
The lateral area $= 64 \times 4 = 256\ dm^2$
The total area $= 64 \times 6 = 384\ dm^2$
[c] The area of one face $= 12 \times 12 = 144\ m^2$
The lateral area $= 144 \times 4 = 576\ m^2$
The total area $= 144 \times 6 = 864\ m^2$

2 The area of one face $= 6 \times 6 = 36\ cm^2$
The lateral area $= 36 \times 4 = 144\ cm^2$
The total area $= 36 \times 6 = 216\ cm^2$

3 The area of one face $= 8 \times 8 = 64\ cm^2$
The lateral area $= 64 \times 4 = 256\ cm^2$
The total area $= 64 \times 6 = 384\ cm^2$

4 The area of one face $= 1.5 \times 1.5 = 2.25\ cm^2$
The total area $= 2.25 \times 6 = 13.5\ cm^2$

5 The total area $= 49 \times 6 = 294\ cm^2$

6 The area of one face $= 36 \div 4 = 9\ cm^2$
The total area $= 9 \times 6 = 54\ cm^2$

7 The area of one face $= 48 \div 6 = 8\ m^2$
The lateral area $= 8 \times 4 = 32\ m^2$

8 [a] The edge length $= 48 \div 12 = 4$ cm.
[b] The area of one face $= 4 \times 4 = 16\ cm^2$
The lateral area $= 16 \times 4 = 64\ cm^2$
[c] The total area $= 16 \times 6 = 96\ cm^2$

9 The edge length $= 84 \div 12 = 7$ cm.
The area of one face $= 7 \times 7 = 49\ cm^2$
[a] The lateral area $= 49 \times 4 = 196\ cm^2$
[b] The total area $= 49 \times 6 = 294\ cm^2$

10 The edge length $= 120 \div 12 = 10$ cm.
The area of one face $= 10 \times 10 = 100\ cm^2$
[a] The lateral area $= 100 \times 4 = 400\ cm^2$
[b] The total area $= 100 \times 6 = 600\ cm^2$
[c] The volume $= 10 \times 10 \times 10 = 1000\ cm^3$

11 [a] The edge length $= 24 \div 4 = 6$ cm.
[b] The area of one face $= 6 \times 6 = 36\ cm^2$
The lateral area $= 36 \times 4 = 144\ cm^2$
[c] The total area $= 36 \times 6 = 216\ cm^2$

12 The edge length $= 28 \div 4 = 7$ cm.
The area of one face $= 7 \times 7 = 49\ cm^2$
The lateral area $= 49 \times 4 = 196\ cm^2$
The total area $= 49 \times 6 = 294\ cm^2$

13 [a] The area of one face $= 384 \div 6 = 64\ cm^2$
[b] edge $\times$ edge $= 8 \times 8$
$\therefore$ The edge length $= 8$ cm.
[c] Its lateral area $= 64 \times 4 = 256\ cm^2$

14 The area of one face $= 216 \div 6 = 36\ m^2$
The lateral area $= 36 \times 4 = 144\ m^2$
The area of one face $= 36\ m^2$
$\therefore$ edge $\times$ edge $= 6 \times 6$
$\therefore$ The edge length $= 6$ m.
$\therefore$ The volume $= 6 \times 6 \times 6 = 216\ m^3$

15 The area of one face $= 8 \times 8 = 64\ cm^2$
The lateral area $= 64 \times 4 = 256\ cm^2$
The total area $= 64 \times 6 = 384\ cm^2$
$\therefore \frac{\text{The lateral area}}{\text{The total area}} = \frac{256}{384} = \frac{2}{3} = 2 : 3$

16 [a] a cube [b] $144\ cm^2$ [c] $216\ cm^2$

17

Cube	Edge length	Lateral area	Total area
A	8	256	384
B	3	36	54
C	5	100	150

18 [a] area of one face $\times 4$
[b] area of one face $\times 6$
[c] $26\ cm^2$ [d] 144 [e] 96
[f] 150 [g] 144 [h] 30
[i] 6 [j] 4 [k] 64
[l] 8 [m] 120 [n] 600
[o] $\frac{1}{4}$ [p] $\frac{1}{6}$ [q] $\frac{2}{3}$
[r] 1 4 6

19 [a] (b) [b] (b) [c] (d)
[d] (d) [e] (c) [f] (a)
[g] (a) [h] (a) [i] (c)
[j] (b) [k] (b) [l] (c)
[m] (b)

20 The area of one face $= 1.5 \times 1.5 = 2.25\ m^2$
The total area $= 6 \times 2.25 = 13.5\ m^2$
The cost of painting $= 13.5 \times 15 =$ L.E. 202.5

21 The total area of the solid
= The total area of the greater cube + the lateral area of the smaller cube
$= 2 \times 2 \times 6 + 1 \times 1 \times 4 = 28\ cm^2$

22 The total area of the cube $= 6 \times 0.3 \times 0.3$
$= 0.54\ m^2$
The used area $= 120 \times 90\% = 108\ m^2$
The number of boxes $= 108 \div 0.54$
[redacted]

23 [a] The perimeter of the base $= 2 \times (20 + 12)$
$= 2 \times 32 = 64$ cm.
The lateral area $= 64 \times 17 = 1088\ cm^2$
The base area $= 12 \times 20 = 240\ cm^2$
The total area $= 1088 + (240 \times 2)$
$= 1088 + 480 = 1568\ cm^2$

[b] The perimeter of the base $= 8 \times 4 = 32$ cm.
The lateral area $= 32 \times 25 = 800\ cm^2$
The base area $= 8 \times 8 = 64\ cm^2$
The total area $= 800 + (64 \times 2)$
$= 800 + 128 = 928\ cm^2$

[c] The perimeter of the base $= 2 \times (7.5 + 2.5)$
$= 2 \times 10 = 20$ dm.
The lateral area $= 20 \times 2 = 40\ dm^2$
The area of the base $= 2.5 \times 7.5 = 18.75\ dm^2$
The total area $= 40 + (2 \times 18.75)$
$= 40 + 37.5 = 77.5\ dm^2$

The lateral area $= 24 \times 10 = 240\ cm^2$

The perimeter of the base $= (6 + 4) \times 2 = 20$ cm.
The lateral area $= 20 \times 8 = 160\ cm^2$
The base area $= 6 \times 4 = 24\ cm^2$
The total area $= 160 + 24 \times 2 = 208\ cm^2$

The perimeter of the base $= 6 \times 4 = 24$ cm.
The lateral area $= 24 \times 10 = 240\ cm^2$

[a] The perimeter of the base $= 8 \times 4 = 32$ cm.
The lateral area $= 32 \times 10 = 320\ cm^2$
[b] The base area $= 8 \times 8 = 64\ cm^2$
The total area $= 320 + (2 \times 64) = 448\ cm^2$

The perimeter of the base $= 9 \times 4 = 36$ cm.
The lateral area $= 36 \times 20 = 720\ cm^2$
The base area $= 9 \times 9 = 81\ cm^2$
The total area $= 720 + (81 \times 2)$
$= 720 + 162 = 882\ cm^2$

The base area $= \frac{\text{the total area} - \text{the lateral area}}{2}$
$= \frac{132 - 112}{2} = 10\ cm^2$

The perimeter of the base $= (7 + 3) \times 2 = 20$ cm.
The height $= \frac{\text{The lateral area}}{\text{The perimeter of the base}}$
$= \frac{160}{20} = 8$ cm.

[a] The lateral area $= 20 \times 8 = 160\ cm^2$
[b] The length of its base side $= 20 \div 4 = 5$ cm.
[c] The base area $= 5 \times 5 = 25\ cm^2$
The total area $= 160 + (2 \times 25) = 210\ cm^2$

The lateral area = 32 × 10 = 320 cm^2

The width = $\frac{\text{the perimeter}}{2}$ − the length

= $\frac{32}{2}$ − 9 = 7 cm.

The base area = 7 × 9 = 63 cm^2

The total area = 320 + 2 × 63 = 446 cm^2

The area of one face of the cube = 10 × 10 = 100 cm^2

The lateral area of the cube = 100 × 4 = 400 cm^2

The perimeter of the cuboid base = 2 × (8 + 5) = 26 cm.

The lateral area of the cuboid = 26 × 17 = 442 cm^2

The difference between their lateral areas = 442 − 400 = 42 cm^2

The perimeter of the base = 2 × (18 + 7) = 50 cm.

The lateral area = 50 × 12 = 600 cm^2

The base area = 18 × 7 = 126 cm^2

The total area = 600 + 126 = 726 cm^2

The perimeter of the base = 2 × (16 + 7) = 46 cm.

The lateral area = 46 × 19 = 874 cm^2

The base area = 16 × 7 = 112 cm^2

The total area = 874 + 112 = 986 cm^2

The perimeter of the base = 32 × 4 = 128 cm

The height = $\frac{3}{8}$ × 32 = 12 cm.

The lateral area = 128 × 12 = 1 536 cm^2

The base area = 32 × 32 = 1 024 cm^2

The total area = 1 536 + (2 × 1 024)

= 1 536 + 2 048 = 3584 cm^2

The height of the cuboid = $\frac{180}{5 \times 12}$ = 3 cm.

The lateral area = 2 × (5 + 12) × 3 = 102 cm^2

The base area = 5 × 12 = 60 cm^2

The total area = 102 + 2 × 60 = 222 cm^2

---

[a] a cuboid [b] 200 cm^2 [c] 350 cm^2

Cuboid	W	L	H	L.A.	T.A.
A	6	9.5	9	348	462
B	5	10	4	120	220
C	4	7	10	220	276

---

[a] perimeter of base × height

[b] lateral area + 2 × base area

[c] 160 [d] 200 [e] 2000

[f] 75 [g] 202 [h] 6

---

[a] (a) [b] (c) [c] (c)

[d] (d) [e] (b) [f] (d)

[g] (b)

---

[a] The perimeter of the base = 2 × (40 + 10) = 100 m.

The lateral area = 100 × 2.5 = 250 m^2

[b] The base area = 40 × 10 = 400 m^2

The total area = 250 + 400 = 650 m^2

The lateral area = 2 × (2.5 + 4) × 1 = 13 m^2

The base area = 2.5 × 4 = 10 m^2

The total area = 13 + 10 = 23 m^2

The cost of painting = 23 × 8 = L.E. 184

The lateral area = 2 × (5 + 2.5) × 1.6 = 24 m^2

The total area = 24 + 5 × 2.5 = 36.5 m^2

The cost price = 36.5 × 12 = L.E. 438

The lateral area = 2 × (4 + 2.5) × 1.8 = 23.4 m^2

The area of ceiling = 4 × 2.5 = 10 m^2

The total area = 23.4 + 10 = 33.4 m^2

The cost price = 33.4 × 15 = L.E. 501

The lateral area = 2 × (1.5 + 1.6) × 0.8 = 4.96 m^2

The base area = 1.5 × 1.6 = 2.4 m^2

The total area = 4.96 + 2.4 = 7.36 m^2

The cost price = 7.36 × 10 = L.E. 73.6

The perimeter of the base = 2 × (3 + 2) = 10 m.

The lateral area = 10 × 1 $\frac{1}{2}$ = 15 m^2

The base area = 3 × 2 = 6 m^2

The total area = 15 + 6 × 2 = 27 m^2

The cost of painting = 27 × 10 = L.E. 270

The lateral area = 2 × (5 + 4) × 3.2 = 57.6 m^2

Area of the ceiling = 4 × 5 = 20 m^2

The total area = 57.6 + 20 = 77.6 m^2

The total area of the painted part of the room = 77.6 − 8 = 69.6 m^2

The cost of painting = 69.6 × 8 = L.E. 556.8

The lateral area = 4 × 4 × 3 = 48 m^2

The area of the door = 0.9 × 2 = 1.8 m^2

The area of one window = 0.81 × 1 = 0.81 m^2

The total area of the painted part of the room = 48 − (1.8 + 2 × 0.81) = 44.58 m^2

The cost of painting = 44.58 × 9 = L.E. 401.22

The lateral area = 5 × 4 × 2.8 = 56 m^2

Area of the ceiling = 5 × 5 = 25 m^2

The total area = 56 + 25 = 81 m^2

The area of the door = 0.9 × 2.2 = 1.98 m^2

The area of one window = 1 × 0.6 = 0.6 m^2

The total area of the painted part of the room = 81 − (1.98 + 0.6 × 2) = 77.82 m^2

The cost of painting = 77.82 × 10 = L.E. 778.2

The lateral area = 2 × (25 + 12) × 2.25 = 166.5 m^2

Area of the floor = 25 × 12 = 300 m^2

The total area = 166.5 + 300 = 466.5 m^2

The area of one tile = 0.25 × 0.25 = 0.0625 m^2

The number of tiles = 466.5 ÷ 0.0625 = 7 464 tiles.

The lateral area = 2 × (25 + 16) × 3.5 = 287 m^2

Area of the floor = 25 × 16 = 400 m^2

The total area = 287 + 400 = 687 m^2

The area of one tile = 0.2 × 0.2 = 0.04 m^2

The number of tiles = 687 ÷ 0.04 = 17 175 tiles.

---

[a] The lateral area = 2 × (40 + 10) × 2.05 = 205 m^2

Area of the floor = 40 × 10 = 400 m^2

The total area = 205 + 400 = 605 m^2

The area of one tile = 0.25 × 0.25 = 0.0625 m^2

The number of tiles = 605 ÷ 0.0625 = 9 680 tiles.

The number of boxes = 9 680 ÷ 25 = 387.2 boxes.

[b] The price of ceramic = 45 × 605 = 27 225 pounds.

The price of covering = 5 × 605 = 3 025 pounds.

The total cost = 27 225 + 3 025 = 30 250 pounds.

The area of the cardboard = 120 × 80 = 9 600 cm^2

The total area = 6 × 30 × 30 = 5 400 cm^2

The remaind paper area = 9 600 − 5 400 = 4 200 cm^2

Assume the dimensions are 1 cm., 2 cm. and 3 cm

L.A. = [2 × (1 + 2)] × 3 = 18 cm^2

T.A. = 18 + [2 × (1 × 2)] = 22 cm^2

The new cuboid dimensions are 2 cm., 4 cm. and 6 cm.

L.A. = [2 × (2 + 4)] × 6 = 72 cm^2

T.A. = 72 + [2 × (2 × 4)] = 88 cm^2

$\frac{\text{T.A. of new cuboid}}{\text{T.A. of initial cuboid}}$ = 88 : 22 = 4 : 1

The area of one base = 10 × 10 = 100 cm^2

The lateral area = the total area − 2 × the base area = 400 − 2 × 100 = 200 cm^2

The perimeter of the base = 10 × 4 = 40 cm.

The height = $\frac{\text{The lateral area}}{\text{The perimeter of the base}}$ = $\frac{200}{40}$ = 5 cm.

---

length + width + height = $\frac{136}{4}$ = 34 cm.

∵ the height = 1 dm = 10 cm.

So, length + width = 34 − 10 = 24 cm.

So, width	length	sum
3	5	8
?	?	24

So, width = $\frac{3 \times 24}{8}$ = 9 cm.

, length = $\frac{5 \times 24}{8}$ = 15 cm

The base area = 9 × 15 = 135 cm^2

The perimeter of the base = 2 (15 + 9) = 48 cm.

The lateral area = 48 × 10 = 480 cm^2

The total area = 480 + 135 × 2 = 750 cm^2

Length + width = 18 cm.
, width + height = 14 cm.
height + length = 18 cm.
by adding 2 (length + width + height) = 18 + 14 + 18 = 48 cm.
∴ length + width + height = 24
The height = 24 − 18 = 6 cm.
, the width = 24 − 18 = 6 cm.
, the length = 24 − 14 = 10 cm.
The perimeter of the base = (10 + 6) × 2 = 32 cm.
The lateral area = 32 × 6 = 256 cm²
The base area = 10 × 6 = 60 cm²
The total area = 256 + 60 × 2 = 376 cm²
Its volume = 6 × 6 × 10 = 480 cm³

$\frac{1}{2}$ of the perimeter = $\frac{1}{2}$ × 20 = 10 m

Width	Length	Sum
3	7	10
?	?	10

The width = $\frac{3 \times 10}{10}$ = 3 m.
The length = $\frac{7 \times 10}{10}$ = 7 m.
The lateral area = 2 × (3 + 7) × 4 = 80 m²
The total area = 80 + (3 × 7) = 101 m²
The cost price of painting = 101 × 5 = L.E. 505

The area of one face of the cube = 384 ÷ 6 = 64 cm²
The edge length = 8 cm.
The volume of the cube = 8 × 8 × 8 = 512 cm³
The height of the cuboid = $\frac{512}{16 \times 2}$ = 16 cm.
The lateral area of the cuboid = 2 × (16 + 2) × 16 = 576 cm²
The total area = 576 + 2 × 16 × 2 = 640 cm²

The total area of the remained part of the cube = The total area of the original cube = 12 × 12 × 6 = 864 cm²

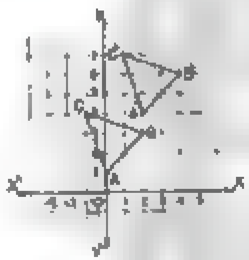
Answers of unit test

1 [a] 6 [b] (2 , 0) [c] 3
[d] 154 [e] height

2 [a] π × r² [b] 200 [c] 294
[d] (1 , 4) [e] 4 , MK

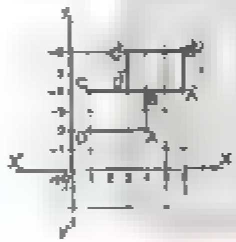
3 [a] 88 = 2 × π × r
∴ r = $\frac{88}{2 \times \frac{22}{7}}$ = 14 cm.
∴ The area = $\frac{22}{7}$ × (14)² = 616 cm²

[b] A (0 , 1) ⟶ A′ (2 , 4)
B (2 , 3) ⟶ B′ (4 , 6)
C (−1 , 4) ⟶ C′ (1 , 7)



4 [a] The lateral area = 8 × 4 × 22 = 704 cm²
The total area = 704 + 8 × 8 × 2 = 832 cm²

[b] (1) A (4 , 2) ⟶ A′ (6 , 4)
B (4 , 4) ⟶ B′ (6 , 6)
C (1 , 4) ⟶ C′ (3 , 6)
D (1 , 2) ⟶ D′ (3 , 4)



(2) AB = 2 length units
, BC = 3 length units.
The perimeter of the rectangle A′B′C′D′ = (2 + 3) × 2 = 10 length units.

5 [a] The edge length = 60 ÷ 12 = 5 cm.
(1) The lateral area = 5 × 5 × 4 = 100 cm²
(2) The total area = 5 × 5 × 6 = 150 cm²
(3) The volume = 5 × 5 × 5 = 125 cm³

[b] The area of the rectangle = 10 × 7 = 70 cm²
The area of the circle = $\frac{22}{7}$ × (3.5)² = 38.5 cm²
The area of the shaded part = 70 − 38.5 = 31.5 cm²

Unit Four

Exercise 14

1 [a] 30% [b] Social studies
[c] Social studies 10% , English 15% , French 20% , Maths 25% and Science 30%

2 [a] $\frac{18}{25}$ [b] $\frac{18}{37}$ [c] $\frac{9}{16}$
[d] The measure of the angle
$= \frac{18}{100} \times 360° = 64.8°$

3 [a] 25 %
[b] 22 %
[c] The percentage of the rangers
$= 100\% - (25\% + 22\% + 18\% + 22\%) = 13\%$
[d] The measure of the central angle
$= \frac{22}{100} \times 360° = 79.2°$
[e] Rangers. [f] Theatre.

4

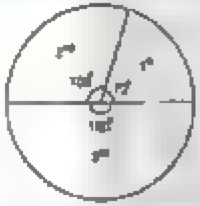
The components of the earth's surface	Water natural supplies	Valleys	Hills	Mountains
The percentage of the forming	71%	13%	4%	12%

[a] Hills.
[b] Water natural supplies
[c] The measure of the central angle
$= \frac{13}{100} \times 360° = 46.8°$

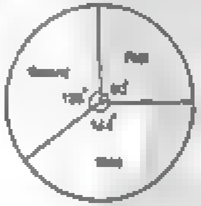
5 [a] Lion [b] Donkey and tiger
[c] 60 % [d] 30 %

6 [a] 15 [b] 15 [c] $\frac{3}{5}$
[d] 40% apples , 30% bananas , 20% oranges and 10% peaches.

7 The measure of the central angle of 1st
$= \frac{20}{100} \times 360° = 72°$
The measure of the central angle of 2nd
$= \frac{30}{100} \times 360° = 108°$
The measure of the central angle of 3rd
$= \frac{50}{100} \times 360° = 180°$

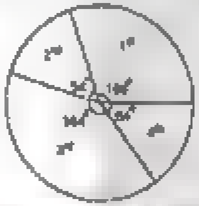


8 The measure of the central angle of first
$= \frac{25}{100} \times 360° = 90°$
The measure of the central angle of second
$= \frac{35}{100} \times 360° = 126°$
The measure of the central angle of third
$= \frac{40}{100} \times 360° = 144°$



* In the following solutions find the measures of central angles by yourself and the representations are shown as follows :

9



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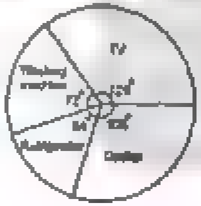

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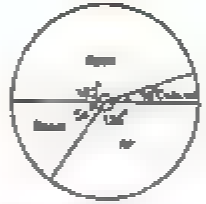

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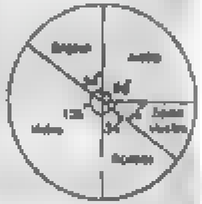
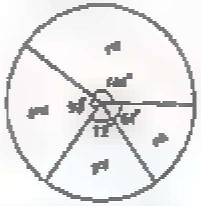


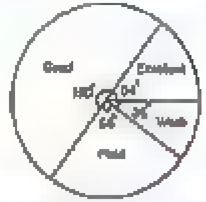
14

15 [a]



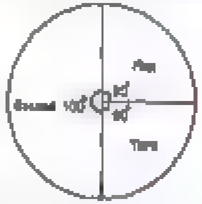
[b] The product of the first farm
$= 40\% \times 1200 = 480$ chicken.

16 [a]

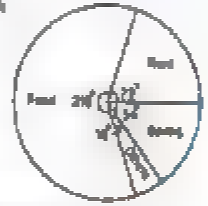


[b] The number of excellent students
$= 15\% \times 200 = 30$ students.

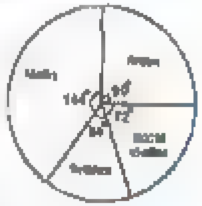
17 [a] 25%
[b]



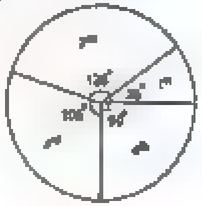
18 [a] 15%
[b]



19 [a] 20%
[b]

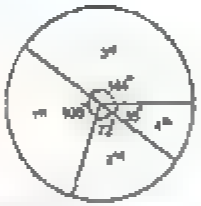


20 [a] 25%
[b]

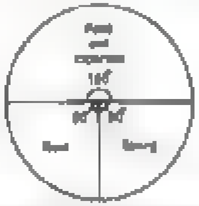


[a] 10%

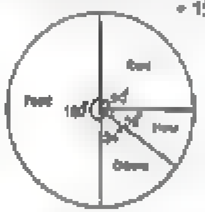
[b]



[c] the third farm

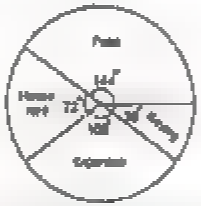


[a] The rest = 100 % − (25 % + 50 % + 15 %) = 10%



[b] The saving = 10% × 1200 = L.E. 120

The remainder = 100% − (40% + 20% + 30%) = 10%



The saved in month = 10% × 900 = L.E. 90

The saved in the year = 90 × 12 = L.E. 1080

The monthly salary of the second family = 70 ÷ 10% = L.E. 700

The measure of the central angle of Arabic = $\frac{9}{36} \times 360° = 90°$

The measure of the central angle of maths = $\frac{10}{36} \times 360° = 100°$

The measure of the central angle of science = $\frac{6}{36} \times 360° = 60°$

The measure of the central angle of English = $\frac{7}{36} \times 360° = 70°$

The measure of the central angle of social studies = $\frac{4}{36} \times 360° = 40°$

The sum of hours = 9 + 5 + 4 + 7 + 11 = 36 hours.

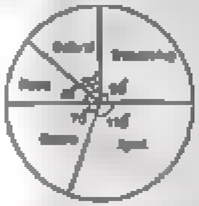
The measure of the central angle of entertaining = $\frac{9}{36} \times 360° = 90°$

The measure of the central angle of cultural = $\frac{5}{36} \times 360° = 50°$

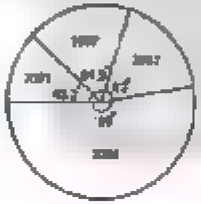
The measure of the central angle of news = $\frac{6}{36} \times 360° = 60°$

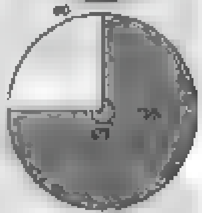
The measure of the central angle of drama = $\frac{7}{36} \times 360° = 70°$

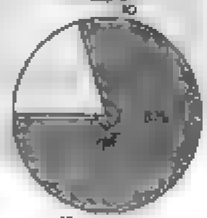
The measure of the central angle of sport = $\frac{11}{36} \times 360° = 110°$

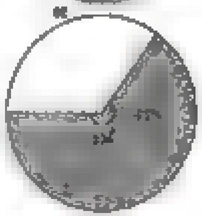


The most is sport, the least is news









[a] 6 A [b] 6 E

[c] Do it by yourself

[a] 360°	[b] 180°	[c] 90°
[d] 60°	[e] 45°	[f] 130°
[g] (1) 10 %	(2) 384	(3) 108°

Exercise 15

1 [a] S = {3 , 4 , 5 , 6 , 7} , n (S) = 5

[b] S = {1 , 2 , 3 , 7 , 8} , n (S) = 5

[c] S = {22 , 24 , 26 , 28} , n (S) = 4

[d] S = {11 , 13 , 17 , 19} , n (S) = 4

[e] S = {green , yellow , black} , n (S) = 3

2 S = {1 , 2 , 3 , 4 , 5 , 6 , 7 , 8 , 9}

3 S = {30 , 31 , 32 , 33}

4 S = {boy , girl}

5 S = {HH , HT , TH , TT}

6 S = {red , yellow}

7 S = {12 , 21 , 11 , 22}

8 S = {33 , 34 , 39 , 43 , 44 , 49 , 93 , 94 , 99} , n (S) = 9

9 S = {(girl , girl , girl) , (girl , boy , girl) , (girl , girl , boy) , (girl , boy , boy) , (boy , girl , girl) , (boy , boy , girl) , (boy , girl , boy) , (boy , boy , boy)}

10 S = {(H , H , H) , (H , T , H) , (H , H , T) , (H , T , T) , (T , H , H) , (T , T , H) , (T , H , T) , (T , T , T)}

11 S = {1 , 2 , 3} , n (S) = 3

12 S = {(H , 1) , (H , 2) , (H , 3) , (H , 4) , (H , 5) , (H , 6) , (T , 1) , (T , 2) , (T , 3) , (T , 4) , (T , 5) , (T , 6)}

13 [a] an experiment in which we can determine all its possible outcomes before carrying it out , but we can't predict in certainty which of these outcomes will occur when the experiment is carried out.

[b] sample space [c] {H , T} , 2

[d] {HH , HT , TH , TT}

[e] {1 , 2 , 3 , 4 , 5 , 6}

Exercise 16

1 [a] $\frac{2}{3}$ [b] $\frac{1}{3}$ [c] $\frac{1}{2}$ [d] $\frac{1}{6}$ [e] $\frac{1}{5}$

[f] zero [g] 1 [h] $\frac{1}{2}$ [i] $\frac{1}{6}$ [j] $\frac{1}{8}$ [k] $\frac{1}{8}$

$S = \{1, 2, 3, 4, 5, 6\}$
[a] zero [b] 1 [c] $\frac{1}{6}$

[a] $P(A) = \frac{4}{6} = \frac{2}{3}$
[b] $P(B) = \frac{4}{6} = \frac{2}{3}$

The number of all balls $= 8 + 12 = 20$ balls.
[a] $\frac{8}{20} = \frac{2}{5}$ [b] $\frac{12}{20} = \frac{3}{5}$
[c] $\frac{0}{20} = 0$

The number of all balls $= 4 + 3 + 8 = 15$ balls.
[a] $\frac{0}{15} = 0$ [b] $\frac{4+3}{15} = \frac{7}{15}$

The number of all balls $= 8 + 5 + 7 = 20$ balls.
[a] $\frac{8}{20} = \frac{2}{5}$ [b] $\frac{8+7}{20} = \frac{15}{20} = \frac{3}{4}$
[c] $\frac{8+5+7}{20} = \frac{20}{20} = 1$

[a] $\frac{14}{25}$ [b] $\frac{14+4}{25} = \frac{18}{25}$
[c] $\frac{7+14}{25} = \frac{21}{25}$ [d] $\frac{0}{25} = 0$ [e] $\frac{7}{25}$

$S = \{1, 2, 3, 4, 5, 6, 7\}$, $n(S) = 7$
[a] $P(A) = \frac{3}{7}$ [b] $P(B) = \frac{4}{7}$
[c] $P(C) = \frac{2}{7}$

$S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
[a] $P(A) = \frac{5}{10} = \frac{1}{2}$ [b] $P(B) = \frac{2}{10} = \frac{1}{5}$
[c] $P(C) = \frac{4}{10} = \frac{2}{5}$

$S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$, $n(S) = 15$
[a] $P(A) = \frac{8}{15}$ [b] $P(B) = \frac{6}{15} = \frac{2}{5}$
[c] $P(C) = \frac{5}{15} = \frac{1}{3}$

[a] $P(A) = \frac{5}{10} = \frac{1}{2}$ [b] $P(B) = \frac{10}{10} = 1$
[c] $P(C) = \frac{3}{10}$

[a] $P(A) = \frac{13}{40}$ [b] $P(B) = \frac{5}{40} = \frac{1}{8}$
[c] $P(C) = \frac{8}{40} = \frac{1}{5}$ [d] $P(D) = \frac{6}{40} = \frac{3}{20}$
[e] $P(E) = \frac{6}{40} = \frac{3}{20}$ [f] $P(F) = \frac{4}{40} = \frac{1}{10}$

[a] $P(A) = \frac{3}{10}$ [b] $P(C) = \frac{5}{10} = \frac{1}{2}$
[c] $P(B) = \frac{10}{10} = 1$
[d] $\because X - 4 = 2$ $X = 4 + 2 = 6$ $P(D) = \frac{1}{10}$

[a] $P(A) = \frac{35}{40} = \frac{7}{8}$ [b] $P(C) = \frac{8}{40} = \frac{1}{5}$

$\because \frac{\text{The number of girls}}{\text{The number of all students}} = \frac{4}{9}$
$\therefore \frac{\text{The number of girls}}{63} = \frac{4}{9}$
$\therefore$ The number of girls $= \frac{4 \times 63}{9} = 28$ girls.

$\because$ The probability of drawing a red ball $= \frac{1}{4}$
$\therefore$ The probability of drawing a blue ball $= 1 - \frac{1}{4} = \frac{3}{4}$
$\frac{\text{The number of blue balls}}{\text{The number of all balls}} = \frac{3}{4}$
$\frac{\text{The number of blue balls}}{80} = \frac{3}{4}$
The number of blue balls $= \frac{3 \times 80}{4} = 60$ balls.

$\because \frac{\text{The number of men}}{\text{The number of workers}} = \frac{3}{5}$
$\therefore \frac{\text{The number of men}}{100} = \frac{3}{5}$
$\therefore$ The number of men $= \frac{3 \times 100}{5} = 60$ men.
$\therefore$ The number of women $= 100 - 60 = 40$ women.

[a] sample space [b] 1
[c] 0, 1 [d] 0, 1 [e] 1
[f] $\frac{4}{12}$ "or $\frac{1}{3}$" [g] $\frac{1}{2}$ [h] $\frac{1}{2}$
[i] $\frac{2}{7}$ [j] 14

[a] (d) [b] (d) [c] (a) [d] (b)
[e] (a) [f] (c) [g] (b) [h] (b)
[i] (c) [j] (b) [k] (d) [l] (b)
[m] (a) [n] (c) [o] (b)

$S = \{55, 56, 65, 66\}$, $n(S) = 4$
[a] $P(A) = \frac{2}{4} = \frac{1}{2}$ [b] $P(B) = \frac{2}{4} = \frac{1}{2}$
[c] $P(C) = \frac{2}{4} = \frac{1}{2}$

[a] 30 [b] 570 [c] $\frac{19}{20}$
[d] $\frac{1}{20}$ [e] 125

$S = \{22, 32, 52, 33, 23, 53, 55, 25, 35\}$, $n(S) = 9$
[a] The probability that the tens digit is odd $= \frac{6}{9} = \frac{2}{3}$
[b] The probability that the units digit is odd $= \frac{6}{9} = \frac{2}{3}$

[c] The probability that the sum of the two digits $7 = \frac{2}{9}$
[d] The probability that the product of the two digits $15 = \frac{2}{9}$

$S = \{12, 13, 21, 23, 31, 32\}$, $n(S) = 6$
[a] The probability of getting an odd prime number $= \frac{3}{6} = \frac{1}{2}$
[b] The probability of getting an even number $= \frac{2}{6} = \frac{1}{3}$

[a] $S = \{1, 2, 3\}$, $n(S) = 3$
[b] The probability that the apparent number on the upper face is $2 = \frac{1}{3}$
[c] The probability that the apparent number on the upper face is odd $= \frac{2}{3}$

[a] $P(A) = \frac{30}{40} = \frac{3}{4}$
[b] $P(B) = \frac{10}{40} = \frac{1}{4}$

[a] The probability of entrance of a lady of weight less than 110 kg. $= \frac{4}{10} = \frac{2}{5}$
[b] The probability of entrance of a lady of weight more than 110 kg. $= \frac{6}{10} = \frac{3}{5}$
[c] The probability of entrance of a lady of weight 90 kg. $= 0$

[a] $\frac{10}{21}$ [b] 200 students

$\because$ The probability that the first player scores $= \frac{18}{21} = \frac{6}{7}$
The probability that the second player scores $= \frac{25}{32}$ $\therefore \frac{6}{7} > \frac{25}{32}$
$\therefore$ The best is choosing the first player because his probability is the greater.

[a] (1) $\frac{1}{6}$ (2) $\frac{5}{6}$ (3) $\frac{1}{2}$
[b] $\frac{7}{8}$

[a] $\frac{1}{4}$ [b] $\frac{1}{6}$ [c] $\frac{3}{8}$

[a] $\frac{20}{200} = \frac{1}{10}$ [b] $\frac{30}{200} = \frac{3}{20}$
[c] $\frac{70}{200} = \frac{7}{20}$ [d] $\frac{45}{200} = \frac{9}{40}$
[e] $\frac{35}{200} = \frac{7}{40}$

The number of all students $= 8 + 12 + 16 + 12 + 8 = 56$ students.
The probability that this student got "good" $= \frac{16}{56} = \frac{2}{7}$

[a] $P(A) = \frac{5}{40} = \frac{1}{8}$ [b] $P(B) = \frac{10}{40} = \frac{1}{4}$
[c] $P(C) = \frac{15}{40} = \frac{3}{8}$

[a] $\frac{1}{80}$ [b] $\frac{9}{80}$ [c] $\frac{33}{100}$
[d] $\frac{13}{80}$ [e] $\frac{9}{80}$

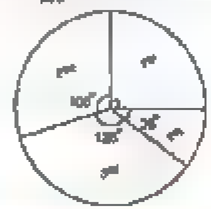
Let the number of boys $= X$
$\therefore$ The number of girls $= X - 10$
$\because X + (X - 10) = 50$ $\therefore 2X = 50 + 10$
$\therefore 2X = 60$ $\therefore X = 30$
The number of boys $= 30$
, the number of girls $= 20$
$\therefore$ The probability that the chosen student is a boy $= \frac{30}{50} = \frac{3}{5}$

Answers of unit test

[a] 360° [b] $\frac{1}{2}$ [c] 12
[d] 45 [e] 0

[a] $\{H, T\}$ [b] (1) 20 (2) 108
[c] 40 [d] 1 [e] $\frac{1}{6}$

[a] 10 %
[b] The measure of the central angle of 1st form $= \frac{25}{100} \times 360° = 90°$
The measure of the central angle of 2nd form $= \frac{30}{100} \times 360° = 108°$
The measure of the central angle of 3rd form $= \frac{35}{100} \times 360° = 126°$
The measure of the central angle of 4th form $= \frac{10}{100} \times 360° = 36°$



(1) $\frac{3}{15} = \frac{1}{5}$ (2) $\frac{7}{15}$ (3) 0 (4) $\frac{11}{15}$

The measure of the central angle of Arabic = $\frac{6}{36} \times 360^\circ = 60^\circ$

The measure of the central angle of maths = $\frac{10}{36} \times 360^\circ = 100^\circ$

The measure of the central angle of science = $\frac{7}{36} \times 360^\circ = 70^\circ$

The measure of the central angle of English = $\frac{9}{36} \times 360^\circ = 90^\circ$

The measure of the central angle of social studies = $\frac{4}{36} \times 360^\circ = 40^\circ$

Answers of TIMSS Questions

First :

(1) b (2) c (3) b (4) b (5) c
(6) c (7) d (8) c (9) a (10) d
(11) c (12) b (13) a (14) b (15) b
(16) a (17) c (18) d (19) b (20) a

Second :

(1) 1 (2) 12 (3) ten thousands
(4) 2499 (5) 6 (6) a height
(7) 6 (8) 30 (9) 16 (10) 2
(11) 135 (12) 2 or 3 or 5 or 7 (13) 26
(14) 3200 (15) $\frac{5}{7}$ (16) 4 (17) 6.25, 6.3
(18) December 23 (19) 89.3 (20) 5

Third

(1) 24 (2) 30 (3) $\frac{3}{5}$ (4) 6
(5) 3 hours, 15 minutes

Guide Answers of Worksheets

First — Worksheets on unit 1 and unit 2

Sheet 1

1. (a) ∅ (b) 13 (c) −7 (d) ⊇ (e) ℕ
2. (a) ∉ (b) ∈ (c) ⊂ (d) ∈ (e) ⊄
3. (a) −2 (b) 7 (c) 9 (d) 80 (e) 95
4. (a) 6 (b) 5 (c) 33 (d) 16 (e) 0
5. (a) 
(b) |−6| = 6 
(c) −2, 2 
(d) 
(e) 

Sheet 2

1. (a) < (b) > (c) = (d) > (e) >
2. (a) −18, −6, 0, |−9|, 15
(b) 17, 18, |−9|, −9, −15
3. (a) zero (b) −1, 0, 1 (c) ℕ (d) 1 (e) 33
4. (a) −26, 26
(b) −4, −3, −2, −1, 0, 1, 2
5. (a) {−3, −2, −1, 0, 1, 2, ...}
(b) {−2, −3, −4, ...}
(c) {0, 1, 2, 3, 4, ...}
(d) {−5, −4, −3, −2, −1, 0, 1, 2, 3, 4}

Sheet 3

1. (a) −5 (b) −9 (c) −12 (d) 19 (e) 0
2. (a) associative property
(b) commutative property
(c) additive identity
(d) additive inverse
3. (a) −5 (b) 5 (c) −7 (d) −5 (e) 5
4. (a) −15 + 29 + 15
$= 15 + 15 + 29$
$= 0 + 29 = 29$
(b) $55 + (-255) + 45 + 255$
$= 55 + 45 + (-255) + 255$
$= (55 + 45) + ((-255) + 255)$
$= 100 + 0 = 100$
5. (a) The order is −3, 0, −31, 4 and 5
(b) The order is −11, −1, 1 and 11

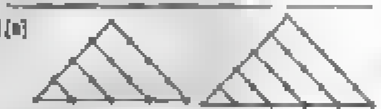
Sheet 4

1. (a) a positive integer (b) −5
(c) −2 (d) ℕ (e) −10
2. (a) −27 (b) 0 (c) 18 (d) −54
3. (a) (1) $(50 \times 2) \times 14 = 100 \times 14 = 1400$
(2) $(8 \times 125) \times ((-9) \times 3) = 1000 \times (-27)$
$= -27000$
(b) (1) $3(-2 + 5) = 3 \times 3 = 9$
(2) $112(98 + (-97)) = 112 \times 1 = 112$
4. (a) $54 \times (100 + 1) = 54 \times 100 + 54 \times 1$
$= 5400 + 54 = 5454$
(b) $73 \times (100 - 1) = 73 \times 100 - 73 \times 1$
$= 7300 - 73 = 7227$
5. (a) (1) {−2, −1, 0, 1, 2, ...}
(2) {−3, −2, −1, 0, 1}
(b) (1) $5 + (-5) + 4$
$= [5 + (-5)] + 4$
$= 0 + 4$
$= 4$
(2) $45 + 55 + 36 + 64$
$= (45 + 55) + (36 + 64)$
$= 100 + 100 = 200$

Sheet 5

1. (a) ∈ (b) −9 (c) 81 (d) 4 (e) 0
2. (a) 5^3 (b) $9^2 = 81$ (c) 6^6 (d) $(-7)^2 = 49$
3. (a) $\frac{3^5}{3^3} = 3^2 = 9$ (b) $\frac{6^4}{6^4} = 6^0 = 1$
4. (a) < (b) > (c) = (d) <
5. $(-2)^5 = -32$, $(-4)^0 = 1$, $(-3)^4 = 81$, $(-1)^{15} = -1$, $3^2 = 9$
The order is $(-3)^4$, 3^2, $(-4)^0$, $(-1)^{15}$ and $(-2)^5$

Sheet 6

1. (a) 18, 22 (b) 18, 26 (c) $\frac{1}{32}$, $\frac{1}{64}$
(d) 10, 1 (e) 8, 8
2. (a) 37, 45, 63 (b) 9, 5, 1
(c) 37, 64, 128
3. (a) 
(b) 15, 21
4. (a) −14, −5, 0, |−5|, 8, $(-4)^2$
(b) −11, −|7|, 0, 5^0, 11, 3^3
5. (a) ⊄ (b) 0 (c) −10 (d) ℤ (e) 19

Sheet 7

1. (a) {7} (b) ∅ (c) {−4} (d) {0}
2. (a) {0, 1} (b) {0, 1, 2}
(c) {2, 3, 4, 5, 6} (d) {1, 0}
3. (a) {3} (b) {2, 3}
4. (a) 4 (b) 12 (c) 32, 64, 128
(d) ∅ (e) 1
5. (a) $5^{4-2} \times 3^{6-4} = 5^2 \times 3^2 = 25 \times 9 = 225$
(b) (1) 1st degree (2) 3rd degree
(3) 1st degree (4) 6th degree

Sheet 8

1. (a) ∴ 2x = 9 + 1 ∴ 2x = 10
x = 5 The S.S. = {5}
(b) 3x = 17 − 2 , 3x = 15
, x = 5 The S.S. = {5}
(c) 3x = 11 + 4 ∴ 3x = 15
∴ x = 5 The S.S. = {5}
(d) 4x = −7 + 3 ∴ 4x = −4
x = −1 The S.S. = ∅
2. (a) ∴ x = −3 − 8 ∴ x = −11
∴ The S.S. = {−11}
(b) 3x = −19 − 2 ∴ 3x = −21
x = −7 ∴ The S.S. = {−7}
(c) 2x = −4 − 4 ∴ 2x = −8
∴ x = −4 ∴ The S.S. = {−4}
(d) ∴ 2x = 13 − 1 ∴ 2x = 12
∴ x = 6 The S.S. = {6}
3. (a) second (b) −7, 7 (c) ∅
(d) 0 (e) 243, 729
4. (a) (1) $13 + 26 + (-26) = 13 + 0 = 13$
(2) $5 + 7 + (-3) + (-9) = 12 + (-12) = 0$
(b) $|-6| = 6$, $(-2)^2 = 4$, $-(3)^2 = -9$
The order is
$-(3)^2$, -6, 0, $(-2)^2$ and $|-6|$
5. (a) (1) $(50 \times 2) \times (-31) = 100 \times (-31)$
$= -3100$
(2) $(-25) \times (-4) \times 9 = 100 \times 9 = 900$
(b) $\frac{2^{11}}{2^9} = 2^2 = 4$

Sheet 9

1. (a) ∵ 2x < 7 − 1 ∴ 2x < 6 ∴ x < 3
∴ The S.S = {2, 1, 0}
(b) ∴ 2x ≥ 5 + 3 2x ≥ 8 ∴ x ≥ 4
∴ The S.S. = {4, 5, 6, 7, ...}

[c] ∵ $3x + 1 \le 13$ ∴ $3x \le 13 - 1$
∴ $3x \le 12$ ∴ $x \le 4$
∴ The S.S. = {0 , 1 , 2 , 3 , 4}

[d] ∵ $1 - 2x > 5$ ∴ $-2x > 5 - 1$
∴ $-2x > 4$ ∴ $x < -2$
The S.S = {−3 , −4 , −5 , …}

∵ $x + 2 \le 5$ ∴ $x \le 5 - 2$ ∴ $x \le 3$
∴ The S.S. in N = {3 , 2 , 1 , 0}

0 1 2 3

, the S.S. in Z = {3 , 2 , 1 , 0 , −1 , −2 , …}

−3 −2 −1 0 1 2 3 4

[a] 23 [(−121) + 21] = 23 × −100 = −2300
[b] (−38) [(−72) + 62] = (−35) × 10 = −350

[a] 1^{st} [b] −1 [c] 1 [d] −1 [e] $\frac{1}{3}$, $\frac{1}{3}$

[a] (4 × 25) × (−16) = 100 × (−16) = −1600
[b] (−16 + 16) + 29 = 0 + 29 = 29

Second Worksheets on unit 3 and unit 4

Sheet 1

1 [a] AB = |4 − 1|
= |3| = 3 units.
, BC = |5 − (−1)|
= |5 + 1| = |6|
= 6 units.

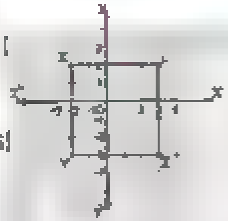
[b] Δ ABC is scalene and right-angled at B.

[c] The area of Δ ABC = $\frac{1}{2}$ × 3 × 6
= 9 square units.

[a] A(−1 , 1) , B(1 , −2) , C(3 , 1) , D(1 , 4)
[b] AC = |3 − (−1)| = |3 + 1| = |4| = 4 units.
[c] BD = |4 − (−2)| = |4 + 2| = |6| = 6 units.
[d] The area = $\frac{1}{2}$ × 4 × 6 = 12 square units.

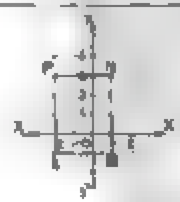
5 XY = |2 − (−3)|
= |2 + 3| = |5|
= 5 units

, YZ = |3 − (−2)|
= |3 + 2| = |5|
= 5 units

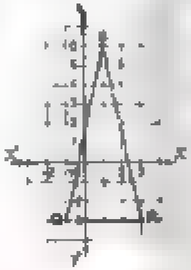


So ,
[a] The shape is a square
[b] The perimeter = 5 × 4 = 20 units.
The area = 5 × 5 = 25 square units.
[c] The number of axes of symmetry = 4

4

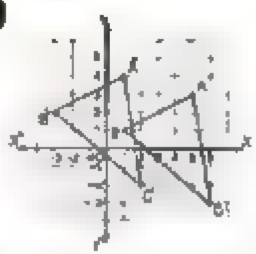


[a] LP = |3 − (−1)| = |3 + 1| = |4| = 4 units.
PN = |1 − (−2)| = |1 + 2| = |3| = 3 units.
[b] The perimeter = (4 + 3) × 2 = 7 × 2
= 14 units.
The area = 4 × 3 = 12 square units.

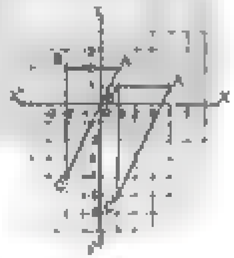


[a] QR = |3 − (−1)| = |3 + 1| = |4| = 4 units
[b] Δ QRS is isosceles.
[c] The number of axes of symmetry = 1

Sheet 2

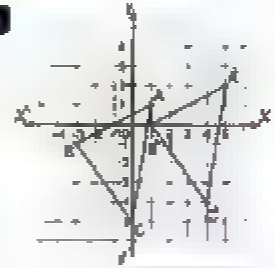


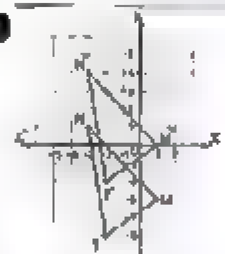
The image of A (5 , 3) is Á (5 − 4 , 3 + 1)
= (1 , 4)
The image of B (1 , 1) is B́ (1 − 4 , 1 + 1)
= (−3 , 2)
The image of C (6 , −3) is Ć (6 − 4 , −3 + 1)
= (2 , −2)
So , Δ ÁB́Ć is the image of Δ ABC by translation (−4 , 1)



[a] AB = |−2 − 1| = |−3| = 3 units.
[b] BC = |−4 − 2| = |−6| = 6 units.
[c] The image of A (1 , 2) is Á (1 + 3 , 2 − 1)
= (4 , 1)
The image of B (−2 , 2) is B́ (−2 + 3 , 2 − 1)
= (1 , 1)
The image of C (−2 , −4) is
Ć (−2 + 3 , −4 − 1) = (1 , −5)
So , Δ ÁB́Ć is the image of Δ ABC by translation (3 , −1)

The mapping rule is (−2 , 4)





The image of M (1 , −3) is Ḿ (1 , −3 + 3)
= (1 , 0)
The image of N (−3 , 1) is Ń (−3 , 1 + 3)
= (−3 , 4)
The image of T (−2 , −5) is T́ (−2 , −5 + 3)
= (−2 , −2)
So , Δ ḾŃT́ is the image of Δ MNT by translation of magnitude 3 units in the positive direction of y-axis.

Sheet 3

[a] The area = 3.14 × $(5)^2$ = 78.5 m^2
[b] The area = 3.14 × $(4)^2$ = 50.24 cm^2
[c] The area = 3.14 × $(6)^2$ = 113.04 m^2
[d] The area = 3.14 × $(10)^2$ = 314 dm^2

The area = 3.14 × $(10)^2$ = 314 cm^2
The area of one sector = 314 ÷ 8 = 39.25 cm^2

[a] (1) The circumference of the circle
= 14 × $\frac{22}{7}$ = 44 cm.
(2) The radius length = 14 ÷ 2 = 7 cm
The area = $(7)^2$ × $\frac{22}{7}$ = 154 cm^2

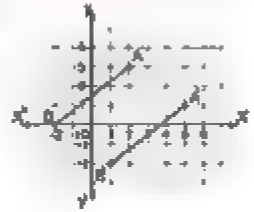
[b] The radius length $= 7 \div 2 = 3.5$ cm.

The area $= \frac{1}{2} \times (3.5)^2 \times \frac{22}{7} = 19.25$ cm^2

4 The area of the shaded part = the area of square − the area of circle

$= 12 \times 12 - 3.14 \times (6)^2 = 30.96$ cm^2

5 [a]



[b] $\because 88 = 2 \pi r$ $\therefore r = \frac{88}{2 \times \frac{22}{7}} = 14$ cm.

$\therefore$ The area $= \frac{22}{7} \times (14)^2 = 616$ cm^2

Sheet 4

1 [a] The area of one face $= 5 \times 5 = 25$ cm^2

The lateral area $= 25 \times 4 = 100$ cm^2

The total area $= 25 \times 6 = 150$ cm^2

[b] The perimeter of the base $= (7 + 5) \times 2 = 24$ cm.

The area of the base $= 7 \times 5 = 35$ cm^2

The lateral area = the perimeter of the base × height $= 24 \times 8 = 192$ cm^2

The total area = lateral area + area of the two bases $= 192 + 35 \times 2 = 262$ cm^2

2 [a] The edge length $= \frac{108}{12} = 9$ cm.

The area of one face $= 9 \times 9 = 81$ cm^2

The lateral area $= 81 \times 4 = 324$ cm^2

The total area $= 81 \times 6 = 486$ cm^2

[b] The perimeter of the base $= 3 \times 4 = 12$ cm.

The area of the base $= 3 \times 3 = 9$ cm^2

The lateral area = the perimeter of the base × height $= 12 \times 6 = 72$ cm^2

The total area = lateral area + area of the two bases $= 72 + 2 \times 9 = 90$ cm^2

[a] The lateral area $= 20 \times 6 = 120$ cm^2

[b] The area of one face $= 100 \div 4 = 25$ cm^2

The total area $= 25 \times 6 = 150$ cm^2

[a] (1) The lateral area $= 20 \times 8 = 160$ cm^2

(2) The length of base side $= 20 \div 4 = 5$ cm.

(3) The area of the base $= 5 \times 5 = 25$ cm^2

The total area $= 160 + 2 \times 25$

$= 210$ cm^2

[b] The area $= \frac{22}{7} \times (7)^2 = 154$ cm^2

[a] The area of the two bases $= 132 - 112$

$= 20$ cm^2

The area of the base $= 20 \div 2 = 10$ cm^2

[b] The perimeter of the base $= (6 + 4) \times 2$

$= 20$ cm.

The height $= 140 \div 20 = 7$ cm.

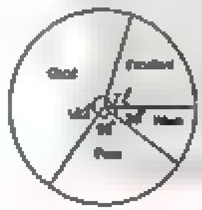
Sheet 5

1 [a] • The measure of the central angle of excellent sector $= \frac{20}{100} \times 360° = 72°$

• The measure of the central angle of good sector $= \frac{45}{100} \times 360° = 162°$

• The measure of the central angle of pass sector $= \frac{25}{100} \times 360° = 90°$

• The measure of the central angle of weak sector $= \frac{10}{100} \times 360° = 36°$



[b] The number of excellent pupils $= \frac{20}{100} \times 300$

$= 60$ pupils.

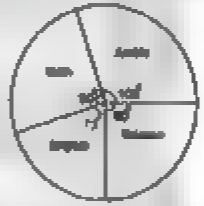
[a] 25%

[b] • The measure of the central angle of Arabic sector $= \frac{30}{100} \times 360° = 108°$

• The measure of the central angle of Math sector $= \frac{25}{100} \times 360° = 90°$

• The measure of the central angle of English sector $= \frac{20}{100} \times 360° = 72°$

• The measure of the central angle of Science sector $= \frac{25}{100} \times 360° = 90°$

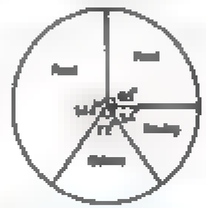


[a] • The measure of the central angle of rent sector $= \frac{25}{100} \times 360° = 90°$

• The measure of the central angle of food sector $= \frac{40}{100} \times 360° = 144°$

• The measure of the central angle of others sector $= \frac{20}{100} \times 360° = 72°$

• The measure of the central angle of saving sector $= \frac{15}{100} \times 360° = 54°$



[b] The family saves monthly $= \frac{15}{100} \times 1800$

$=$ L.E. 270

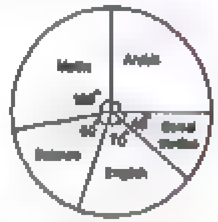
4 The measure of the central angle of Arabic $= \frac{9}{36} \times 360° = 90°$

The measure of the central angle of maths $= \frac{10}{36} \times 360° = 100°$

The measure of the central angle of science $= \frac{6}{36} \times 360° = 60°$

The measure of the central angle of English $= \frac{7}{36} \times 360° = 70°$

The measure of the central angle of social studies $= \frac{4}{36} \times 360° = 40°$



[a] The lateral area $= (3 + 2) \times 2 \times 4 = 40$ cm^2

The total area $= 40 + 2 \times 3 \times 2 = 52$ cm^2

[b] The area $= 3.14 \times (10)^2 = 314$ cm^2

Sheet 6

1 S = {1 , 2 , 3 , 4 , 5 , 6 , 7 , 8}

2 S = {11 , 15 , 17 , 51 , 55 , 57 , 71 , 75 , 77}

, n (S) = 9

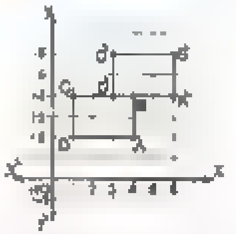
3 S = {HHH , HHT , HTH , HTT , THH , THT , TTH , TTT}

4 [a] (1) A (4 , 2) ⟶ Á (6 , 4)

B (4 , 4) ⟶ B́ (6 , 6)

C (1 , 4) ⟶ Ć (3 , 6)

D (1 , 2) ⟶ D́ (3 , 4)



(2) The perimeter of rectangle $\acute{A}\acute{B}\acute{C}\acute{D}$

$= (3 + 2) \times 2 = 10$ units.

[b] The radius length $= 28 \div 2 = 14$ cm.

The area $= (14)^2 \times \frac{22}{7} = 616$ cm^2

4 [a] The perimeter of the base $= 6 \times 4 = 24$ cm.

The area of the base $= 6 \times 6 = 36$ cm^2

The lateral area = the perimeter of the base × height $= 24 \times 8 = 192$ cm^2

The total area of = the lateral area + area of the two bases $= 192 + 2 \times 36$

$= 264$ cm^2

[b] • The measure of the central angle of 1st sector $= \frac{40}{100} \times 360° = 144°$

• The measure of the central angle of 2nd sector $= \frac{15}{100} \times 360° = 54°$

• The measure of the central angle of 3rd sector $= \frac{30}{100} \times 360° = 108°$

• The measure of the central angle of 4th sector $= \frac{15}{100} \times 360° = 54°$

Sheet 7

1 $S = \{1, 2, 3, ..., 14, 15\}$, $n(S) = 15$

[a] $\frac{7}{15}$ [b] $\frac{1}{5}$ [c] $\frac{2}{5}$

2 [a] $\frac{12}{25}$ [b] $\frac{18}{25}$ [c] $\frac{13}{25}$ [d] zero

3 The probability $= \frac{\text{The number of red marbles}}{\text{The number of all marbles}}$

$\therefore \frac{3}{5} = \frac{\text{The number of red marbles}}{20}$

$\therefore$ The number of red marbles $= 12$

4 [a] The lateral area $= (3 + 2) \times 4 = 40$ cm^2

The total area $= 40 + 2 \times 3 \times 2 = 52$ cm^2

[b] (1) The area of the circle

$= \frac{22}{7} \times 7^2 = 154$ cm^2

The area of one sector $= 154 \div 8$

$= 19.25$ cm^2.

(2) The measure of central angle of the sector $= \frac{1}{8} \times 360° = 45°$

5 [a] The edge length $= \frac{20}{4} = 5$ cm.

The area of one face $= 5 \times 5 = 25$ cm^2

The lateral area $= 25 \times 4 = 100$ cm^2

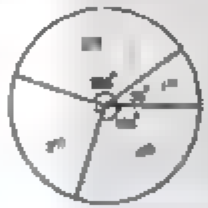
The total area $= 25 \times 6 = 150$ cm^2

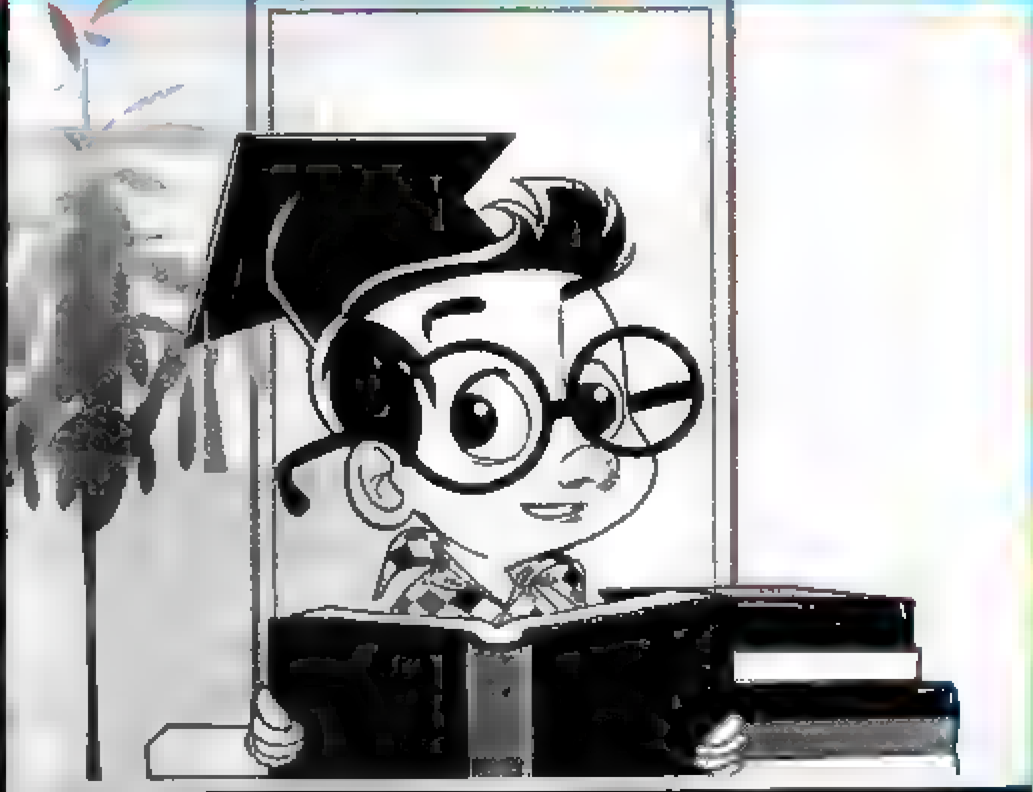
[b] The measure of the central angle of 1st term $= \frac{10}{100} \times 360° = 36°$

The measure of the central angle of 2nd term $= \frac{33}{100} \times 360° = 120°$

The measure of the central angle of 3rd term $= \frac{25}{100} \times 360° = 90°$

The measure of the central angle of 4th term $= \frac{30}{100} \times 360° = 108°$





Guide Answers of Final Examinations

Answers of model examinations of the school book

Model 1

1 (1) zero (2) (−3 , 0) (3) ⊂ (4) zero

2 (1) ∈ (2) −4 (3) 6 (4) $\frac{1}{2}$

3 (a) $4 \times 9 \div 6 - 21 = 36 \div 6 - 21 = 4 - 21 = -17$

(b) ∴ $x - 2 \geq 3$ ∴ $x \geq 3 + 2$ ∴ $x \geq 5$

∴ The S.S. = {5 , 6 , 7 , ...}

4 (a) The lateral area = $10 \times 4 \times 7 = 280$ cm²

(b) ∴ $88 = 2\pi r$ ∴ $r = \frac{88}{2 \times \frac{22}{7}} = 14$ cm.

∴ The area = $\frac{22}{7} \times (14)^2 = 616$ cm²

5 (a) ∵ $3x + 9 = 3$ ∴ $3x = 3 - 9$

∴ $3x = -6$ ∴ $x = \frac{-6}{3}$ ∴ $x = -2$

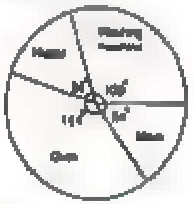
∴ The S.S. = {−2}

(b) The measure of the central angle of washing machine = $\frac{30}{100} \times 360° = 108°$

The measure of the central angle of heater = $\frac{15}{100} \times 360° = 54°$

The measure of the central angle of oven = $\frac{40}{100} \times 360° = 144°$

The measure of the central angle of mixer = $\frac{15}{100} \times 360° = 54°$



Model 2

1 (1) 2 (2) 2r (3) $\frac{1}{2}$ (4) −1

2 (1) 84 (2) ⊂ (3) 6 (4) $\frac{1}{2}$

3 (a) $30 - 6 \div 3 = 30 - 2 = -32$

(b) $x - 2 \geq 3$ ∴ $x \geq 3 + 2$ ∴ $x \geq 5$

The S.S = {5 , 6 , 7 , ...}

$$-1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7$$

4 (a) ∴ $2x + 9 = 5$ $2x = 5 - 9$

∴ $2x = -4$ $x = \frac{-4}{2}$ $x = -2$

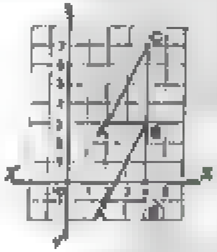
∴ The S.S. = {−2}

(b) The area of rectangle = $8 \times 7 = 56$ cm²

The area of circle = $\frac{22}{7} \times (3.5)^2 = 38.5$ cm²

The area of the shaded part = $56 - 38.5 = 17.5$ cm²

5 (a) (1) BC = 4 length units



(2) A (2 , 3) ⟶ A′ (2 , −1)

B (4 , 3) ⟶ B′ (4 , −1)

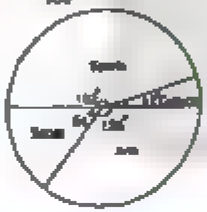
C (4 , 7) ⟶ B (4 , 3)

(b) The measure of the central angle of cultural = $\frac{5}{100} \times 360° = 18°$

The measure of the central angle of sports = $\frac{45}{100} \times 360° = 162°$

The measure of the central angle of social = $\frac{15}{100} \times 360° = 54°$

The measure of the central angle of arts = $\frac{35}{100} \times 360° = 126°$



Model examination for the special needs students

1 (1) 3 (2) 0 (3) 1 (4) 40

2 (1) 2^4 (2) r^2 (3) π (4) $\frac{1}{2}$

3 (1) (✓) (2) (✗) (3) (✗) (4) (✓)

4 (1) 360° (2) ∈ (3) (0 , 1 , 2) (4) (4 , 4)

5 (a) The total area = $6 \times 4^2 = 96$ cm²

The lateral area = $4 \times 4^2 = 64$ cm²

(b) $\frac{2^{4+1}}{2^3} = 2^2 = 4$

Answers of model examinations

Model 1

1 (1) $\frac{1}{6}$ (2) $\varnothing$ (3) $\subset$ (4) -3
(5) -2 (6) $(10)^7$ (7) sum

2 (1) $\{0\}$ (2) 213 (3) 108
(4) $(2, 2)$ (5) -4 (6) -17
(7) -12 (8) $\frac{5}{3}$, 2

3 (1) 1 (2) $\varnothing$ (3) πr^2 (4) 0
(5) -25 (6) 3 (7) $120°$

4 (1) $\because 2\pi r = 88$ $\therefore 2 \times \frac{22}{7} \times r = 88$

$\therefore r = \frac{88}{\frac{22}{7} \times 2} = 14$ cm.

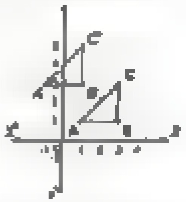
$\therefore$ The area $= \frac{22}{7} \times (14)^2 = 616$ cm²

(2) $2x + 1 \leq 7$ $2x \leq 7 - 1$

$\therefore 2x \leq 6$ $\therefore x \leq \frac{6}{2}$ $\therefore x \leq 3$

$\therefore$ The S.S. $= \{3, 2, 1, 0, -1, ...\}$

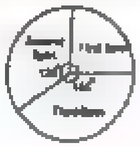
(3) $A(1, 1) \longrightarrow \acute{A}(-1, 3)$
$B(3, 1) \longrightarrow \acute{B}(1, 3)$
$C(3, 3) \longrightarrow \acute{C}(1, 5)$



(4) The measure of the central angle of first term $= \frac{25}{100} \times 360° = 90°$

The measure of the central angle of second term $= \frac{35}{100} \times 360° = 126°$

The measure of the central angle of third term $= \frac{40}{100} \times 360° = 144°$



Model 2

1 (1) -5 (2) 3^{-2} (3) 54 (4) $\in$
(5) 12 (6) 1 (7) 1

2 (1) $\frac{1}{5}$ (2) 13, 16 (3) 6
(4) 4 (5) $45°$ (6) 1
(7) $(1, 5)$ (8) first

3 (1) zero (2) $\subset$ (3) 1 (4) $\in$
(5) -3 (6) 10^2 (7) $\subset$

4 (1) $2x - 3 = -9$ $2x = -9 + 3$
$2x = -6$ $\therefore x = \frac{-6}{2}$ $x = -3$
The S.S. $= \{-3\}$

(2) The perimeter of the base $= 6 \times 4$
$= 24$ cm.
The lateral area $= 24 \times 10 = 240$ cm²
The total area $= 240 + 2 \times 6 \times 6$
$= 312$ cm².

(3) $32 \times (117 - 17) = 32 \times 100 = 3200$

(4) The sum of pupils $= 9 + 14 + 10 + 7$
$= 40$ pupils

The measure of the central angle of excellent $= \frac{9}{40} \times 360° = 81°$

The measure of the central angle of very good $= \frac{14}{40} \times 360° = 126°$

The measure of the central angle of good $= \frac{10}{40} \times 360° = 90°$

The measure of the central angle of weak $= \frac{7}{40} \times 360° = 63°$

Model 3

1 (1) $(7, 0)$ (2) $90°$ (3) 5^6 (4) 6
(5) 45 (6) 1 (7) $\notin$

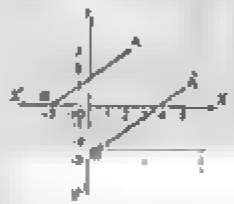
2 (1) 6 (2) 256 (3) -17 (4) 100
(5) 35 (6) 0 (7) 9, 5 (8) 7

3 (1) 6 (2) 1 (3) 3^7 (4) $\varnothing$
(5) 1 (6) $\subset$ (7) first

4 (1) The perimeter of the base
$= (16 + 7) \times 2 = 46$ cm
The lateral area $= 46 \times 19 = 874$ cm²
The total area $= 874 + 16 \times 7$
$= 986$ cm²

(2) $S = \{33, 35, 53, 55\}$
(a) $P(A) = \frac{2}{4} = \frac{1}{2}$
(b) $P(B) = \frac{4}{4} = 1$
(c) $P(C) = \frac{0}{4} = 0$

(3) $A(2, 3) \longrightarrow \acute{A}(5, 1)$
$B(-2, 0) \longrightarrow \acute{B}(1, -2)$

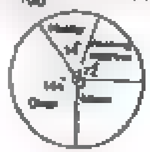


(4) The measure of the central angle of washing machine $= \frac{20}{100} \times 360° = 72°$

The measure of the central angle of heater $= \frac{15}{100} \times 360° = 54°$

The measure of the central angle of oven $= \frac{40}{100} \times 360° = 144°$

The measure of the central angle of mixer $= \frac{25}{100} \times 360° = 90°$



Model 4

1 (1) $>$ (2) 2 (3) -2 (4) $2\pi r$
(5) 1 (6) $\frac{1}{2}$ (7) 78.5

2 (1) Sample space (2) 1
(3) $\frac{1}{36}$, $\frac{1}{36}$ (4) 216 (5) $\{2\}$
(6) -2 (7) -6 (8) $\{0, 1, 2, 3, 4\}$

3 (1) -3 (2) $(6, -3)$ (3) 1
(4) zero (5) 1 (6) 4 (7) $\subset$

4 (1) The area $= \frac{1}{2} \times \frac{22}{7} \times 7^2 = 77$ cm²

(2) $\because 2 - x > 3$ $\therefore -x > 3 - 2$
$\therefore -x > 1$ $\therefore x < -1$
$\therefore$ The S.S. $= \{-2, -3, -4, ...\}$

(3) $43 \times (44 + 56) = 43 \times 100 = 4300$

(4) $\therefore 25$

- The measure of the central angle of football $= \frac{40}{100} \times 360° = 144$

The measure of the central angle of volleyball $= \frac{20}{100} \times 360° = 72°$

The measure of the central angle of basketball $= \frac{15}{100} \times 360° = 54°$

The measure of the central angle of swimming $= \frac{25}{100} \times 360° = 90°$

Model 5

1 (1) -2 (2) 4 (3) 5 (4) zero
(5) -16 (6) 9^2 (7) 21

2 (1) 270 (2) $\frac{1}{3}$ (3) $\{0, 1, 2, 3\}$
(4) zero (5) 5 (6) 9
(7) 4 (8) -1

1 (1) (−3 , 0) (2) 16 (3) 5
(4) > (5) π (6) ∈ (7) third

2 (1) $2x - 8 = -26$ ∴ $2x = -18$ ∴ $x = -9 \notin \mathbb{N}$

$2x = -26 + 8$, $x = \frac{-18}{2}$ ∴ The S.S = ∅

(2) $\frac{(2)^4 \times 2^6}{(2)^3 \times -(2)} = \frac{2^{10}}{-(2^4)} = (2^6) = -16$

(3) The sum of all balls = 4 + 6 + 5 = 15 balls.

(a) The probability that the ball is white = $\frac{4}{15}$

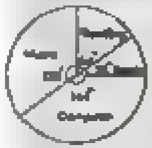
(b) The probability that the ball is not red = $\frac{4+5}{15} = \frac{9}{15} = \frac{3}{5}$

(4) The measure of the central angle of sports = $\frac{10}{100} \times 360° = 36°$

The measure of the central angle of reading = $\frac{15}{100} \times 360° = 54°$

The measure of the central angle of music = $\frac{35}{100} \times 360° = 126°$

The measure of the central angle of computer = $\frac{40}{100} \times 360° = 144°$



Answers of Schools' Examinations

1 Cairo

1 (1) 2 (2) ⊆ (3) 8
(4) (5 , −1) (5) −3 (6) 0

2 (1) ∈ (2) 324 (3) 1
(4) 90° (5) 0 (6) 3^3

3 (1) $\mathbb{Z}^-$ (2) 4 (3) 10 cm.
(4) (−1 , 0) (5) 100 cm² (6) $\frac{1}{6}$
(7) 134 (8) 18

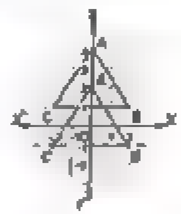
4 (a) $\frac{5^{10}}{5^{8}} = 5^2 = 25$

(b) ∵ $3x - 2 < 7$ ∴ $3x < 7 + 2$
∴ $3x < 9$ ∴ $x < \frac{9}{3}$
∴ $x < 3$
∴ The S.S. = {0 , 1 , 2}

(c) The area of the circle = 3.14 × (10)² = 314 cm²

The area of one sector = 314 ÷ 8 = 39.25 cm²

5 (a) A (0 , 4) ⟶ Á (0 , 2)
B (2 , 1) ⟶ B́ (2 , −1)
C (−2 , 1) ⟶ Ć (−2 , −1)



(b) The measure of the central angle of washing machine = $\frac{30}{100} \times 360° = 108°$

The measure of the central angle of heater = $\frac{15}{100} \times 360° = 54°$

The measure of the central angle of oven = $\frac{40}{100} \times 360° = 144°$

The measure of the central angle of mixture = $\frac{15}{100} \times 360° = 54°$

2 Cairo

1 (1) π (2) 5^{th} (3) 0
(4) $4\pi^3$ (5) −8 (6) 15

2 (7) (4 , 0) (8) 84 cm² (9) <
(10) { } (11) 27 (12) $\frac{9}{14}$

3 (1) 3 : 2 (2) 6 (3) $\frac{1}{3}$
(4) 75.36 cm. (5) 49
(6) {−2 , −1 , 0 , 1 , ...} (7) 216
(8) 40°

4 (1) ∵ $2x - 3 = -9$ ∴ $2x = -9 + 3$
∴ $2x = -6$ ∴ $x = \frac{-6}{2}$
$x = -3$
When $x \in \mathbb{Z}$, The S.S. = {−3}
When $x \in \mathbb{N}$ The S.S. = ∅

(2) 25 (9 + 1 − 9) = 25 × 1 = 25

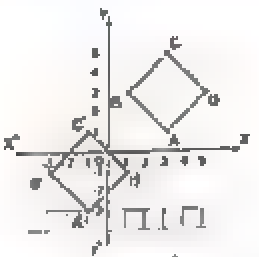
(3) The area of the circle = $\frac{22}{7} \times (7)^2$ = 154 cm²

The area of the triangle = $\frac{1}{2} \times 7 \times 14$ = 49 cm²

The area of the shaded part = 154 − 49 = 105 cm²

(4) (a) 20 % (b) 108°

(5) A (3 , 1) ⟶ Á (−1 , −3)
B (1 , 3) ⟶ B́ (−3 , −1)
C (3 , 5) ⟶ Ć (−1 , 1)
D (5 , 3) ⟶ D́ (1 , −1)



The area of the image $= \frac{1}{2} \times 4 \times 4$
$= 8$ square units.

3 Giza

1
(1) 0 (2) 5^2 (3) 12
(4) (4 , 1) (5) 0 (6) $\emptyset$
(7) 7 (8) $\in$ (9) 12
(10) $\mathbb{Z}^-$ (11) $\frac{1}{3}$ (12) 2

2
(1) 17 (2) 3 (3) 360°
(4) 2^{24} (5) $\frac{2}{5}$ (6) (2 , 2)
(7) 300 cm² (8) 45.5 cm³

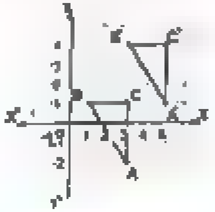
3
(1) $\frac{-(4)^{11} \times 4^2}{4^{10}} = \frac{-(4)^{13}}{4^{11}} = -(4^2) = -16$

(2) $\because 2x + 9 < 1$ $\therefore 2x < 1 - 9$
$\therefore 2x < -8$ $\therefore x < \frac{-8}{2}$
$\therefore x < -4$
The S.S. $= \{-5 , -6 , -7 , ...\}$

−7 −6 −5 −4 −3 −2 −1 0 1

(3) The total area $= 1.5 \times 1.5 \times 6 = 13.5$ m²
The cost $= 13.5 \times 15 =$ L.E. 202.5

(4) (a) BC = 2 units.
(b) A (3 , −2) ⟶ Á (5 , 1)
B (1 , 1) ⟶ B́ (3 , 4)
C (3 , 1) ⟶ Ć (5 , 4)



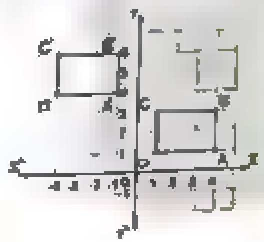
(5) The measure of the central angle of football $= \frac{45}{100} \times 360° = 162°$
The measure of the central angle of basketball $= \frac{10}{100} \times 360° = 36°$
The measure of the central angle of volleyball $= \frac{25}{100} \times 360° = 90°$
The measure of the central angle of swimming $= \frac{20}{100} \times 360° = 72°$

4 Alexandria

1
(1) $\mathbb{Z}^-$ (2) $\subset$ (3) −3
(4) = (5) < (6) {6}
(7) −1 (8) 144 (9) (−2 , −7)
(10) 4 (11) 360° (12) 0

2
(1) 12 (2) 7 (3) {0 , 1 , 2}
(4) (3 , 5) (5) 8 (6) 180
(7) 40 (8) $\frac{1}{6}$

3
(1) The order is: −15 , −9 , |−9| , 15 and 17
(2) $\frac{(-5)^6}{(-5)^5} = -5$
(3) The area $= \frac{22}{7} \times (3.5)^2 = 38.5$ cm²
(4) A (4 , 1) ⟶ Á (−1 , 4)
B (4 , 3) ⟶ B́ (−1 , 6)
C (1 , 3) ⟶ Ć (−4 , 6)
D (1 , 1) ⟶ D́ (−4 , 4)



(5) The measure of the central angle of cultural $= \frac{5}{100} \times 360° = 18°$
The measure of the central angle of sports $= \frac{45}{100} \times 360° = 162°$
The measure of the central angle of social $= \frac{15}{100} \times 360° = 54°$
The measure of the central angle of arts $= \frac{35}{100} \times 360° = 126°$

5 El-Kalyoubia

1
(1) $\not\subset$ (2) 8 (3) 7
(4) 2^{10} (5) (0 , 0) (6) 360°
(7) $\frac{1}{2}$ (8) 36 (9) 6
(10) 2^{10}

2
(1) −6 (2) 1 (3) 154
(4) $\mathbb{Z}$ (5) {2}
(6) {2 , 1 , 0 , ...}
(7) 320 cm² , 446 cm² (8) −1

3
(1) (a) The probability that the ball is white $= \frac{5}{18}$
(b) The probability that the ball is black or red $= \frac{9 + 4}{18} = \frac{13}{18}$
(c) The probability that the is yellow = 0
(d) The probability that the ball is not black $= \frac{5 + 9}{18} = \frac{14}{18} = \frac{7}{9}$
(2) The area of the square $= 14 \times 14$
$= 196$ cm²
The area of the circle $= \frac{22}{7} \times 7^2 = 154$ cm²
The area of the shaded part $= 196 - 154$
$= 42$ cm²
(3) The order is:
$(-2)^3 , (-1)^{15} , (-3)^2$ and $(-5)^2$

(4) (a) A (4 , 3) ⟶ Á (2 , 0)
B (4 , 1) ⟶ B́ (2 , −2)
C (1 , 1) ⟶ Ć (−1 , −2)
D (1 , 3) ⟶ D́ (−1 , 0)

(b) The area $= 3 \times 2 = 6$ square units
, the perimeter $= (3 + 2) \times 2$
$= 10$ length units.
(c) rectangle.

6 El-Sharkia

1
(1) zero (2) 314 (3) $\subset$
(4) 3 (5) (−3 , 0) (6) N
(7) 180° (8) final (9) $\frac{1}{6}$
(10) 216 (11) −20 (12) $(3)^2$

2
(13) {0} , $\mathbb{Z}^+$ (14) perimeter of the base
(15) 40 (16) 0 (17) −4
(18) 360° (19) 10
(20) diameter length , π

3
(21) $\because 2x + 9 = 5$ $\therefore 2x = 5 - 9$
$\therefore 2x = -4$ $\therefore x = \frac{-4}{2}$
$x = -2$
$\therefore$ The S.S. $= \{-2\}$

(22) $-17 + 19 + 17 = -17 + 17 + 19$
(Commutative property)
$= (-17 + 17) + 19$ (Associative property)
$= 0 + 19$ (Additive Inverse)
$= 19$ (Additive identity)

(23) The lateral area $= 7 \times 4 \times 10 = 280$ cm²

(24) $\because x + 4 < 7$ $\therefore x < 7 - 4$
$\therefore x < 3$
$\therefore$ The S.S. $= \{0 , 1 , 2\}$

(25) The measure of central angle of football $= \frac{40}{100} \times 360° = 144°$

The measure of central angle of basketball = $\frac{20}{100} \times 360° = 72°$

The measure of central angle of handball = $\frac{30}{100} \times 360° = 108°$

The measure of central angle of volleyball = $\frac{10}{100} \times 360° = 36°$

7 El-Monofia

1

(1) π	(2) − 1	(3) r^2
(4) $\frac{1}{6}$	(5) zero	(6) $\mathbb{Z}^-$
(7) 5	(8) zero	(9) − 20
(10) >	(11) (− 3 , 0)	(12) 4

2

(1) 0.5	(2) 16	(3) height
(4) 3^{10}	(5) 120	(6) 64
(7) {− 1 , 0}	(8) 64	

3 (1) L.A. = 10 × 4 × 7 = 280 cm²

(2) ∵ $2X + 9 = 3$ ∴ $2X = 3 - 9$

∴ $2X = -6$ ∴ $X = \frac{-6}{2}$

∴ $X = -3$

∴ The S.S. = {− 3}

(3) The area of the rectangle = 8 × 7 = 56 cm²

The area of the circle = $\frac{22}{7} \times (3.5)^2$ = 38.5 cm²

The area of the shaded part = 56 − 38.5 = 17.5 cm²

(4) 116 + 190 + (− 116) = 116 + (− 116) + 190 = [116 + (− 116)] + 190 = 0 + 190 = 190

(5) The measure of central angle of cultural = $\frac{5}{100} \times 360° = 18°$

The measure of central angle of sports = $\frac{45}{100} \times 360° = 162°$

The measure of central angle of social = $\frac{15}{100} \times 360° = 54°$

The measure of central angle of arts = $\frac{35}{100} \times 360° = 126°$

8 El-Gharbia

1

(1) $\frac{1}{6}$	(2) ∅	(3) − 3
(4) − 2	(5) 10^7	(6) sure
(7) 1	(8) ∅	(9) πr^2
(10) − 25	(11) 3	(12) 120°

2

(1) {0}	(2) 213	(3) 100
(4) 1	(5) − 4	(6) − 17
(7) − 12	(8) − 7	

3 (1) ∵ $2\pi r = 88$ ∴ $2 \times \frac{22}{7} \times r = 88$

∴ $r = \frac{88}{2 \times \frac{22}{7}} = 14$ cm.

∴ The area = $\frac{22}{7} \times (14)^2 = 616$ cm²

(2) ∵ $2X + 1 \leq 7$ ∴ $2X \leq 7 - 1$

∴ $2X \leq 6$ ∴ $X \leq \frac{6}{2}$

∴ $X \leq 3$

∴ The S.S. = {1 , 2 , 3}

(3) A (1 , 1) ⟶ A′ (− 1 , 3)

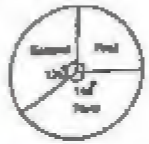
B (3 , 1) ⟶ B′ (1 , 3)

C (3 , 3) ⟶ C′ (1 , 5)

(4) The measure of central angle of first farm = $\frac{25}{100} \times 360° = 90°$

The measure of central angle of second farm = $\frac{35}{100} \times 360° = 126°$

The measure of central angle of third farm = $\frac{40}{100} \times 360° = 144°$



9 El-Dakahlia

1

(1) ∉	(2) (− 2 , − 7)	(3) second
(4) 0	(5) >	(6) 314
(7) 1	(8) 24	(9) 4
(10) 1	(11) 8	(12) − 9

2

(1) magnitude , direction		(2) 1
(3) 0	(4) 360	(5) 12
(6) 120	(7) 400	(8) − 10

3 (1) ∵ $3X - 7 \leq 5$ ∴ $3X \leq 5 + 7$

∴ $3X \leq 12$ ∴ $X \leq \frac{12}{3}$

∴ $X \leq 4$

∴ The S.S. = {4 , 3 , 2 , ...}

(2) $\frac{(-3)^{11}}{(-3)^5} = 3^6 = 729$

(3) (a) 2

(b) A (2 , 3) ⟶ A′ (2 , 1)

B (4 , 3) ⟶ B′ (4 , 1)

C (4 , 5) ⟶ C′ (4 , 3)

(4) The perimeter of the base = (18 + 8) × 2 = 50 cm.

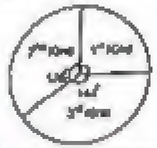
The lateral area = 50 × 5 = 250 cm²

The total area = 250 + 18 × 8 = 394 cm²

(5) The measure of central angle of 1st kind = $\frac{25}{100} \times 360° = 90°$

The measure of central angle of 2nd kind = $\frac{35}{100} \times 360° = 126°$

The measure of central angle of 3rd kind = $\frac{40}{100} \times 360° = 144°$



10 Ismailia

1

(1) ∅	(2) 0	(3) − 1
(4) ∅	(5) 3	(6) $\mathbb{Z}^+$
(7) (0 , 0)	(8) >	(9) 0
(10) r^2	(11) $\frac{1}{2}$	(12) 9

2

(1) {0}	(2) 400	(3) first
(4) 154	(5) 7	(6) 3 or − 3
(7) 10	(8) 100	

3 (1) $(-2)^2 \times 3^2 = 4 \times 9 = 36$

(2) The area of the semicircle = $\frac{1}{2} \times \frac{22}{7} \times 7^2 = 77$ cm²

The area of the rectangle = 20 × 14 = 280 cm²

The area of the figure = 77 + 280 = 357 cm²

(3) The edge length = 28 ÷ 4 = 7 cm.

The lateral area = 7 × 7 × 4 = 196 cm²

The total area = 7 × 7 × 6 = 294 cm²

(4) ∵ $X + 5 = 4$ ∴ $X = 4 - 5$

∴ $X = -1$ ∴ The S.S. = {− 1}

(5) The number of black balls = 25 − (6 + 7) = 12 balls.

(a) The probability that the ball is black = $\frac{12}{25}$

(b) The probability that the ball is not red = $\frac{12 + 6}{25} = \frac{18}{25}$

(11) Suez

1
(1) zero (2) ⊂ (3) second
(4) = (5) zero (6) 360
(7) $\mathbb{Z}^-$ (8) = (9) 8
(10) 7 (11) − 20 (12) (− 3 , 6)

2
(1) $\mathbb{Z}^-$ (2) diameter length
(3) 32 (4) − 4 (5) height
(6) 400 cm²
(7) Perimeter of the rectangle (8) $\frac{1}{2}$

3
(1) (− 7) + 19 + 17 = (− 7) + 17 + 19 (Commutative property)
= (− 7 + 17) + 19 (Associative property)
= 10 + 19 = 29

(2) ∵ $x - 2 \leq 3$ ∴ $x \leq 3 + 2$ ∴ $x \leq 5$
∴ The S.S. = {5 , 4 , 3 , ...}

(3) The area = $\frac{22}{7} \times 7^2 = 154$ cm²

(4) The perimeter of the base = 10 × 4 = 40 cm.
The lateral area = 40 × 7 = 280 cm²

(5) The measure of central angle of washing machine = $\frac{25}{100} \times 360° = 90°$
The measure of central angle of heater = $\frac{15}{100} \times 360° = 54°$
The measure of central angle of oven = $\frac{40}{100} \times 360° = 144°$
The measure of central angle of mixer = $\frac{20}{100} \times 360° = 72°$

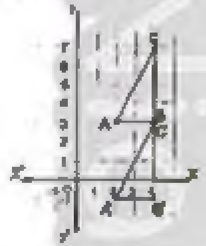
(12) Port Said

1
(1) r^2 (2) $\mathbb{Z}^-$ (3) 6
(4) zero (5) ∈ (6) 2^7
(7) zero (8) 3 (9) 6
(10) zero (11) (3 , 6) (12) − 4

2
(1) ∈ (2) 40 (3) 1
(4) 360° (5) diameter length
(6) 5 (7) ℕ (8) 3

3
(1) 4 × 9 + 9 − 21 = 36 + 9 − 21 = 4 − 21 = − 17

(2) (a) BC = 4 length units.
(b) A (2 , 3) ⟶ $\acute{A}$ (2 , − 1)
B (4 , 3) ⟶ $\acute{B}$ (4 , − 1)
C (4 , 7) ⟶ $\acute{C}$ (4 , 3)



(3) ∵ $x - 2 \geq 3$ ∴ $x \geq 3 + 2$ ∴ $x \geq 5$
∴ The S.S. = {5 , 6 , 7 , ...}

(4) The perimeter of the base = 10 × 4 = 40 cm.
The lateral area = 40 × 4 = 160 cm²

(5) The measure of central angle of washing machine = $\frac{30}{100} \times 360° = 108°$
The measure of central angle of heater = $\frac{15}{100} \times 360° = 54°$
The measure of central angle of oven = $\frac{40}{100} \times 360° = 144°$
The measure of central angle of mixer = $\frac{15}{100} \times 360° = 54°$

(13) Damietta

1
(1) ℕ (2) third (3) 16
(4) (− 2 , 3) (5) $\frac{1}{2}$ (6) zero
(7) − 4 (8) 256 (9) 120
(10) $\mathbb{Z}^-$ (11) 1 (12) {2}

2
(13) 4 (14) 10 (15) 8
(16) 8 (17) $\frac{1}{2}$ (18) 1
(19) (− 1 , 5) (20) πr^2

3
(21) ∵ $3x - 2 \geq 4$ ∴ $3x \geq 4 + 2$
∴ $3x \geq 6$ ∴ $x \geq \frac{6}{3}$ ∴ $x \geq 2$
∴ The S.S. = {2 , 3 , 4 , ...}

(22) 115 + 390 + (− 115) = 115 + (− 115) + 390 (Commutative property)
= [115 + (− 115)] + 390 (Associative property)
= 0 + 390 (Additive inverse)
= 390 (Additive identity)

(23) The total area = 12 × 12 × 6 = 864 cm²

(24) The area = $\frac{22}{7} \times 7^2 = 154$ cm²

(25) The measure of central angle of excellent = $\frac{15}{100} \times 360° = 54°$
The measure of central angle of good = $\frac{50}{100} \times 360° = 180°$
The measure of central angle of pass = $\frac{25}{100} \times 360° = 90°$
The measure of central angle of weak = $\frac{10}{100} \times 360° = 36°$

(14) Kafr El-Sheikh

1
(1) 5 (2) 40 (3) >
(4) 0 (5) (4 , 3) (6) ∅
(7) zero (8) 216 (9) $\frac{1}{6}$
(10) r^2 (11) 45° (12) 1

2
(13) − 2 (14) 40 (15) second
(16) 17 (17) 150 (18) 154
(19) 2 (20) 0

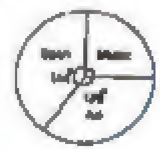
3
(21) ∵ $2x + 1 < 5$ ∴ $2x < 5 - 1$
∴ $2x < 4$ ∴ $x < \frac{4}{2}$ ∴ $x < 2$
∴ The S.S. = {0 , 1}

(22) $\frac{2^3 \times 2^4}{2^5} = \frac{2^7}{2^5} = 2^2 = 4$

(23) The side length = 36 ÷ 12 = 3 cm.
The lateral area = 3 × 3 × 4 = 36 cm²
The total area = 3 × 3 × 6 = 54 cm²

(24) The area of the circle = $\frac{22}{7} \times 7^2$ = 154 cm²
The area of one sector = 154 ÷ 8 = 19.25 cm²

(25) The measure of central angle of music = $\frac{25}{100} \times 360° = 90°$
The measure of central angle of sport = $\frac{40}{100} \times 360° = 144°$
The measure of central angle of art = $\frac{35}{100} \times 360° = 126°$



(15) El-Fayoum

1
(1) ℤ (2) − 4 (3) zero
(4) 7 (5) > (6) − 1
(7) xy (8) (1 , 0) (9) $\frac{1}{2}$
(10) $\frac{1}{4}$ (11) 2 : 3 (12) 60

2
(13) third (14) {0} (15) ∅
(16) 150 (17) 7 (18) 5
(19) radius length (20) 0 , 1

4 (21) $\frac{(-5)^6}{(-5)^4} = (-5)^2 = 25$

(22) $\because 3(x+2) = 3$ $\quad \therefore x + 2 = \frac{3}{3}$

$\therefore x + 2 = 1$ $\quad \therefore x = 1 - 2$

$\therefore x = -1$ $\quad \therefore$ The S.S. $= \{-1\}$

(23) The area $= 3.14 \times 10^2 = 314 \text{ cm}^2$

(24) The perimeter of the base

$= (10 + 5) \times 2 = 30$ cm.

The lateral area $= 30 \times 8 = 240 \text{ cm}^2$

The total area $= 240 + 2 \times 10 \times 5$

$= 340 \text{ cm}^2$

(25) The measure of central angle of

football $= \frac{40}{100} \times 360° = 144°$

The measure of central angle of

basketball $= \frac{35}{100} \times 360° = 126°$

The measure of central angle of

handball $= \frac{25}{100} \times 360° = 90°$

16 El-Menia

1 (1) 5 (2) 216 (3) $\frac{1}{6}$
(4) second (5) 0 (6) -1
(7) 16 (8) zero (9) 54 cm^2
(10) zero (11) zero (12) >

2 (1) 45° (2) 2 (3) $\mathbb{Z}^+$
(4) 40 (5) 360° (6) (1 , 2)
(7) 4 (8) -4

3 (1) $\because 3x - 5 \le 7$ $\quad \therefore 3x \le 7 + 5$

$\therefore 3x \le 12$ $\quad \therefore x \le \frac{12}{3}$ $\quad \therefore x \le 4$

$\therefore$ The S.S. $= \{1 , 2 , 3 , 4\}$

$-1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5$

(2) The perimeter of the base

$= (6 + 4) \times 2 = 20$ cm.

The lateral area $= 20 \times 8 = 160 \text{ cm}^2$

The total area $= 160 + 2 \times 6 \times 4$

$= 208 \text{ cm}^2$

(3) $\frac{2^8}{2^4} = 2^4 = 16$

(4) The probability that the ball is red $= \frac{3}{15}$

The probability that the ball is white $= \frac{8}{15}$

The probability that the ball is blue

$= \frac{0}{15} = 0$

The probability that the ball is red or

white $= \frac{7 + 8}{15} = 1$

(5) The measure of central angle of

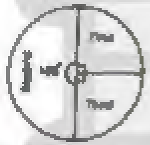
first farm $= \frac{25}{100} \times 360° = 90°$

The measure of central angle of

second farm $= \frac{50}{100} \times 360° = 180°$

The measure of central angle of

third farm $= \frac{25}{100} \times 360° = 90°$



17 Souhag

1 (1) 1 (2) Ø (3) zero
(4) 314 (5) zero (6) 0
(7) 14 (8) third (9) 0
(10) 104 (11) (−3 , −1) (12) ∈

2 (1) 40 (2) 4
(3) $\mathbb{Z}^+ \cup \{0\} \cup \mathbb{Z}^-$ (4) 100
(5) 5 (6) 35 (7) 360°
(8) (1 , 6)

3 (1) $\because 2x - 3 = -9$ $\quad \therefore 2x = -9 + 3$

$\therefore 2x = -6$ $\quad \therefore x = \frac{-6}{2}$

$\therefore x = -3$ $\quad \therefore$ The S.S. $= \{-3\}$

(2) The perimeter of the base $= 6 \times 4$

$= 24$ cm.

The lateral area $= 24 \times 10 = 240 \text{ cm}^2$

The total area $= 240 + 2 \times 6 \times 6$

$= 312 \text{ cm}^2$

(3) $\because 3x - 2 \ge 4$ $\quad \therefore 3x \ge 4 + 2$

$\therefore 3x \ge 6$ $\quad \therefore x \ge \frac{6}{3}$

$\therefore x \ge 2$

$\therefore$ The S.S. $= \{2 , 3 , 4 , ...\}$

(4) The area of the rectangle $= 10 \times 7$

$= 70 \text{ cm}^2$

The area of the circle $= \frac{22}{7} \times 3.5^2$

$= 38.5 \text{ cm}^2$

The area of the shaded part

$= 70 - 38.5 = 31.5 \text{ cm}^2$

(5) The measure of central angle of

excellent $= \frac{15}{100} \times 360° = 54°$

The measure of central angle of

good $= \frac{50}{100} \times 360° = 180°$

The measure of central angle of

pass $= \frac{25}{100} \times 360° = 90°$

The measure of central angle of

weak $= \frac{10}{100} \times 360° = 36°$

18 Qena

1 (1) 54 (2) zero
(3) absolute value (4) zero
(5) (5 , 4) (6) zero
(7) 5 (8) 144

2 (1) 1 (2) -54 (3) -1
(4) -5 (5) half (6) >
(7) $\frac{1}{6}$ (8) 216 (9) third
(10) $\mathbb{Z}^-$ (11) -22 (12) ⊂

4 (1) $\because 2\pi r = 44$ $\quad \therefore r = \frac{44}{2 \times \frac{22}{7}} = 7$ cm.

$\therefore$ The area $= \frac{22}{7} \times 7^2 = 154 \text{ cm}^2$

(2) The perimeter of the base $= (6 + 4) \times 2$

$= 20$ cm.

The lateral area $= 20 \times 8 = 160 \text{ cm}^2$

The total area $= 160 + 2 \times 6 \times 4$

$= 208 \text{ cm}^2$

(3) $\frac{(-3)^7}{(-3)^5} = (-3)^2 = 9$

(4) $\because 3x - 2 \ge 4$ $\quad \therefore 3x \ge 4 + 2$

$\therefore 3x \ge 6$ $\quad \therefore x \ge \frac{6}{3}$

$\therefore x \ge 2$

$\therefore$ The S.S. $= \{2 , 3 , 4 , ...\}$

$-1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4$

(5) The measure of central angle of

washing machine $= \frac{30}{100} \times 360° = 108°$

The measure of central angle of

heater $= \frac{15}{100} \times 360° = 54°$

The measure of central angle of

oven $= \frac{40}{100} \times 360° = 144°$

The measure of central angle of

mixer $= \frac{15}{100} \times 360° = 54°$

19 Aswan

1 (1) -1 (2) 6 (3) 12
(4) (−2 , −7) (5) -6 (6) 0
(7) 5 (8) 9 (9) 4
(10) -10 (11) zero (12) (0 , 3)

2 (1) -4 (2) 7
(3) 100 cm^2 (4) (5 , 4)
(5) Lateral area (6) $\frac{1}{3}$
(7) $\mathbb{Z}^-$ (8) 6

1 (1) $37 + 25 + 63 + 75$
$= 37 + 63 + 25 + 75$
$= (37 + 63) + (25 + 75)$
$= 100 + 100 = 200$

(2) $\because 2\pi r = 88$ $\therefore r = \frac{88}{2 \times \frac{22}{7}} = 14$ cm.

$\therefore$ The area $= \frac{22}{7} \times (14)^2 = 616$ cm^2.

(3) $\because X - 2 \geq 3$ $\therefore X \geq 3 + 2$ $\therefore X \geq 5$

$\therefore$ The S.S. $= \{5, 6, 7, ...\}$

(4) The perimeter of the base $= 9 \times 4$
$= 36$ cm.

The lateral area $= 36 \times 20 = 720$ cm^2.

The total area $= 720 + 2 \times 9 \times 9$
$= 882$ cm^2

(5) The measure of central angle of washing machine $= \frac{30}{100} \times 360° = 108°$

The measure of central angle of heater $= \frac{15}{100} \times 360° = 54°$

The measure of central angle of oven $= \frac{40}{100} \times 360° = 144°$

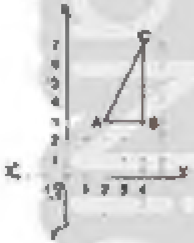
The measure of central angle of mixer $= \frac{15}{100} \times 360° = 54°$

2 (1) $\frac{2^{11}}{2^4} = 2^7 = 128$

(2) $\because 2X + 9 = 3$ $\therefore 2X = 3 - 9$
$\therefore 2X = -6$ $\therefore X = \frac{-6}{2}$
$\therefore X = -3$ $\therefore$ The S.S. $= \{-3\}$

(3) The area $= \frac{22}{7} \times 7^2 = 154$ cm^2

(4) BC = 4 length units.



(5) The measure of central angle of first farm $= \frac{25}{100} \times 360° = 90°$

The measure of central angle of second farm $= \frac{35}{100} \times 360° = 126°$

The measure of central angle of third farm $= \frac{40}{100} \times 360° = 144°$

20 South Sinai

1 (1) >   (2) [illegible]   (3) (6 , 4)
(4) $\frac{1}{6}$ (5) $\notin$ (6) -1
(7) $2r$ (8) $\varnothing$ (9) $X < -5$
(10) 8   (11) 360°   (12) 3

2 (1) 216   (2) 196   (3) -3
(4) 0   (5) $(-1, 6)$ (6) second
(7) 40   (8) 6

Notes